

Neural Network Introduction

LING 575K Deep Learning for NLP

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April 7 2021

Announcements

- HW1 due tomorrow night, upload readme and hw1.tar.gz to Canvas
 - NB: separate files!
 - Do not put readme inside of tar.gz
- indices_to_tokens: no error handling
- You can/should use `Vocabulary.from_text_files` to build your vocab object
 - Factory design pattern allows for different initialization signatures in Python
 - E.g. from_csv in pandas, from_pretrained in huggingface (later this course)
- Note on *args and **kwargs
 - https://book.pythontips.com/en/latest/args_and_kwargs.html

*args and **kwargs

```
def add(a, b):  
    return a + b  
  
print(add(1, 2))    # 3  
print(add(*(1, 2))) # 3  
  
def add_any(*args):  
    return sum(args)  
  
print(add_any(1, 2, 3))    # 6  
print(add_any(1, 2, 3, 4)) # 10
```

*args and **kwargs

```
def keywords(name="Shane", course="575k"):
    return f"{name} is teaching {course}"

print(keywords(name="Agatha"))
print(keywords(**{"name": "Agatha"}))

def keywords_any(**kwargs):
    for key, value in kwargs.items():
        print(f"{key}: {value}")

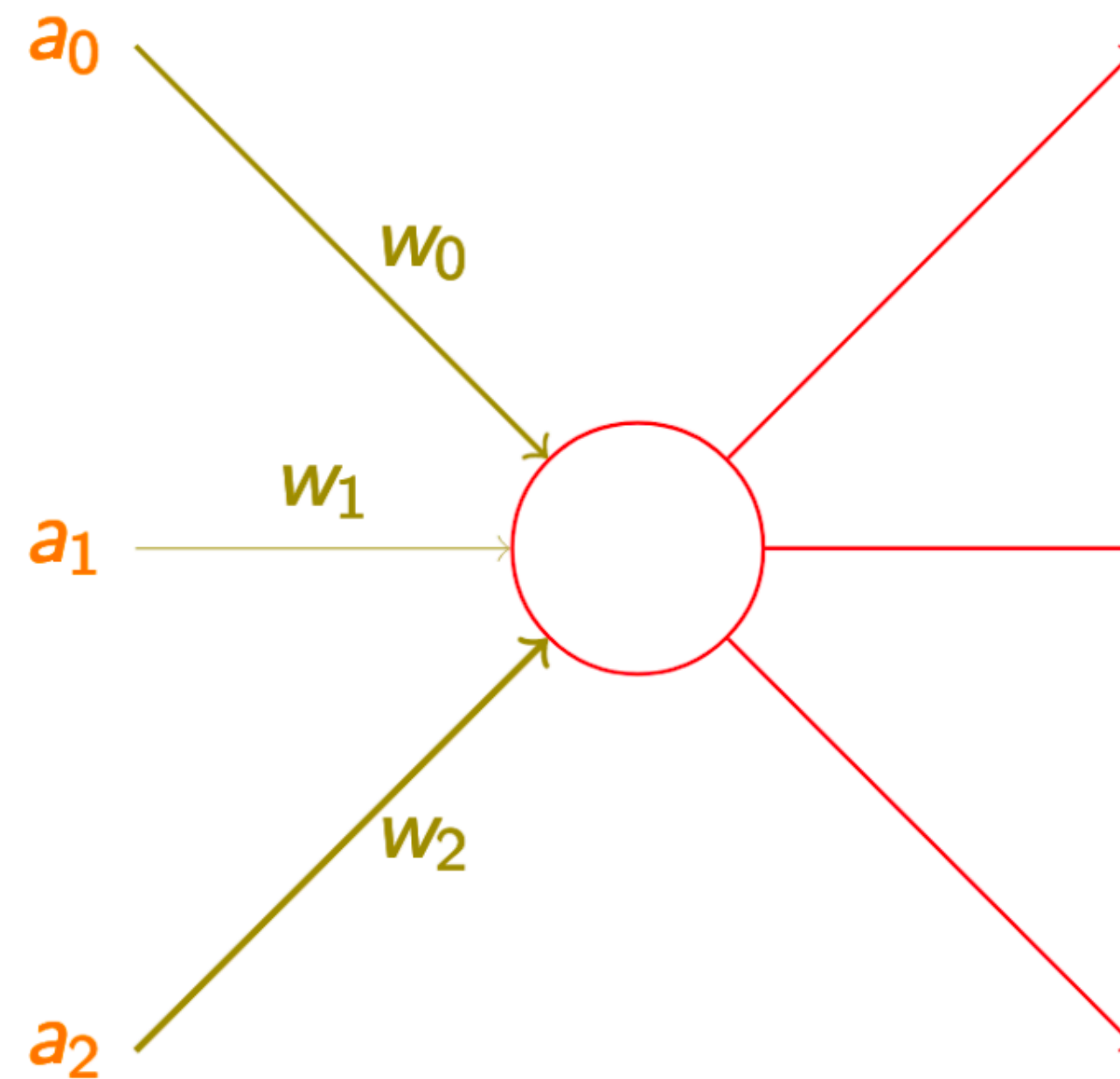
keywords_any(name="Shane", course="575k")
keywords_any(name="Shane", course="575k", foo="bar")
keywords_any(**{"name": "Shane", "course": "575k"})
```

Plan for Today

- Last time:
 - Prediction-based word vectors
 - Skip-gram with negative sampling [model + loss]
- Today: intro to feed-forward neural networks
 - Basic computation + expressive power
 - Multilayer perceptrons
 - Mini-batches
 - Hyper-parameters and regularization

Computation: Basic Example

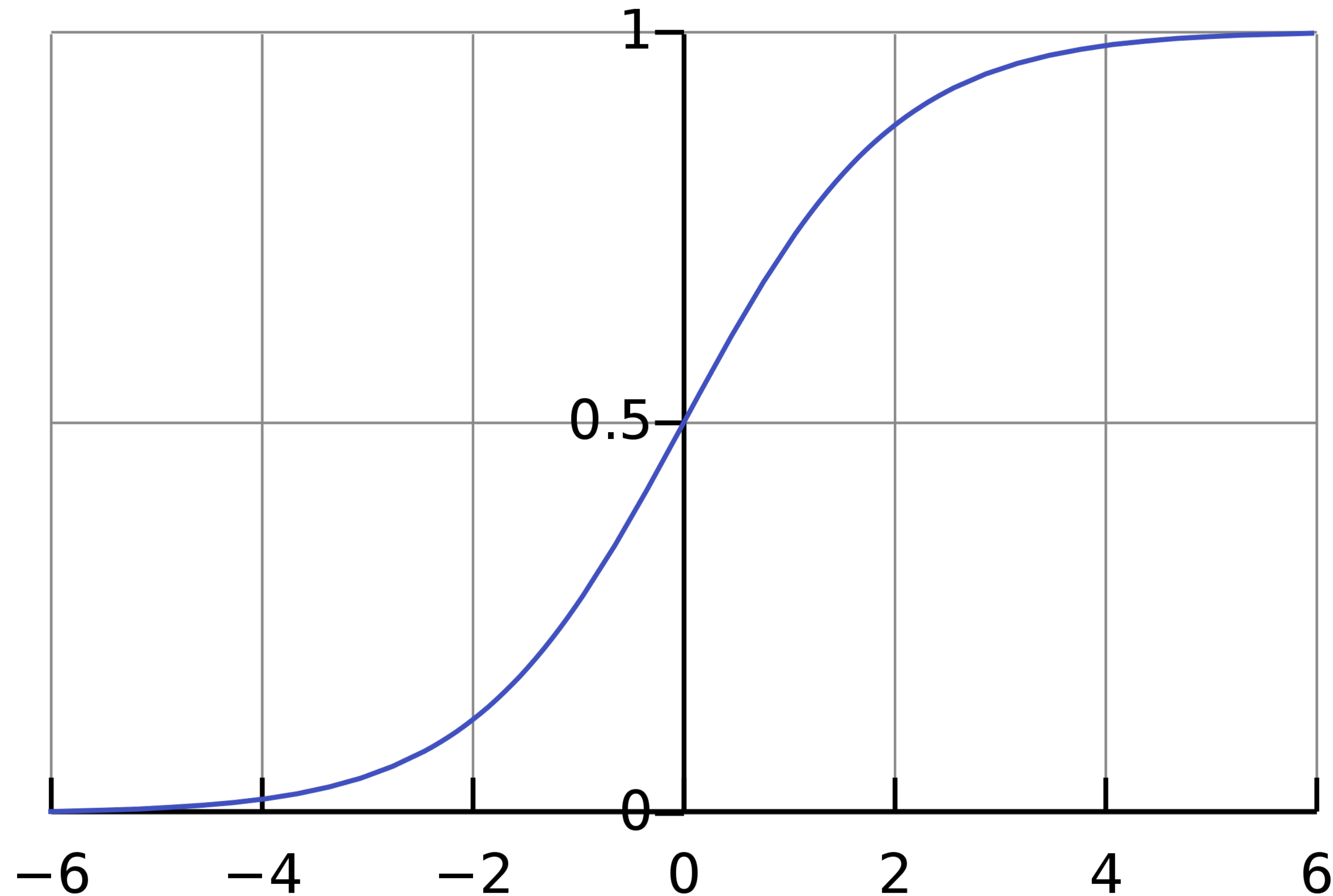
Artificial Neuron



$$a = f(a_0 \cdot w_0 + a_1 \cdot w_1 + a_2 \cdot w_2)$$

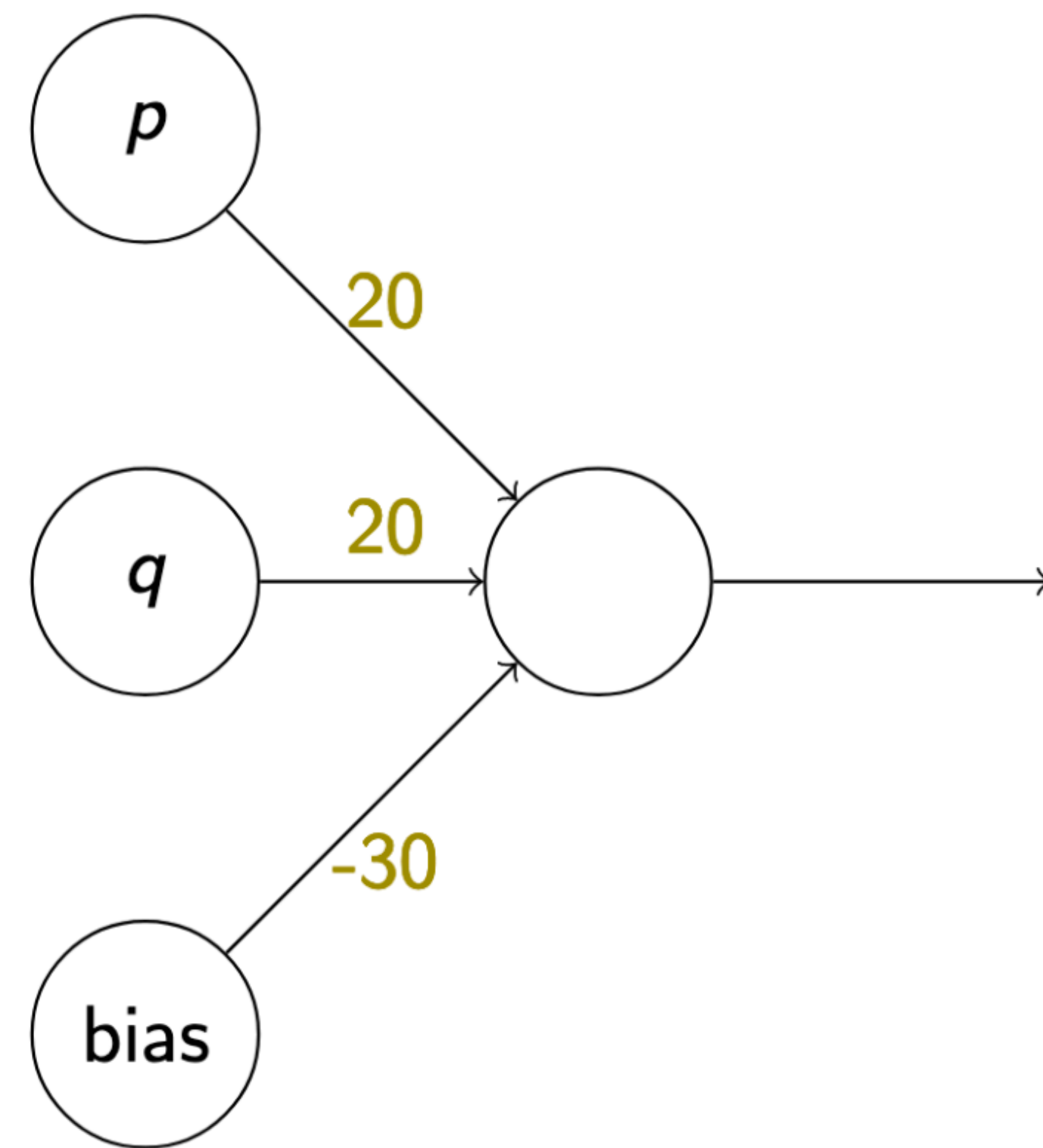
<https://github.com/shanest/nn-tutorial>

Activation Function: Sigmoid



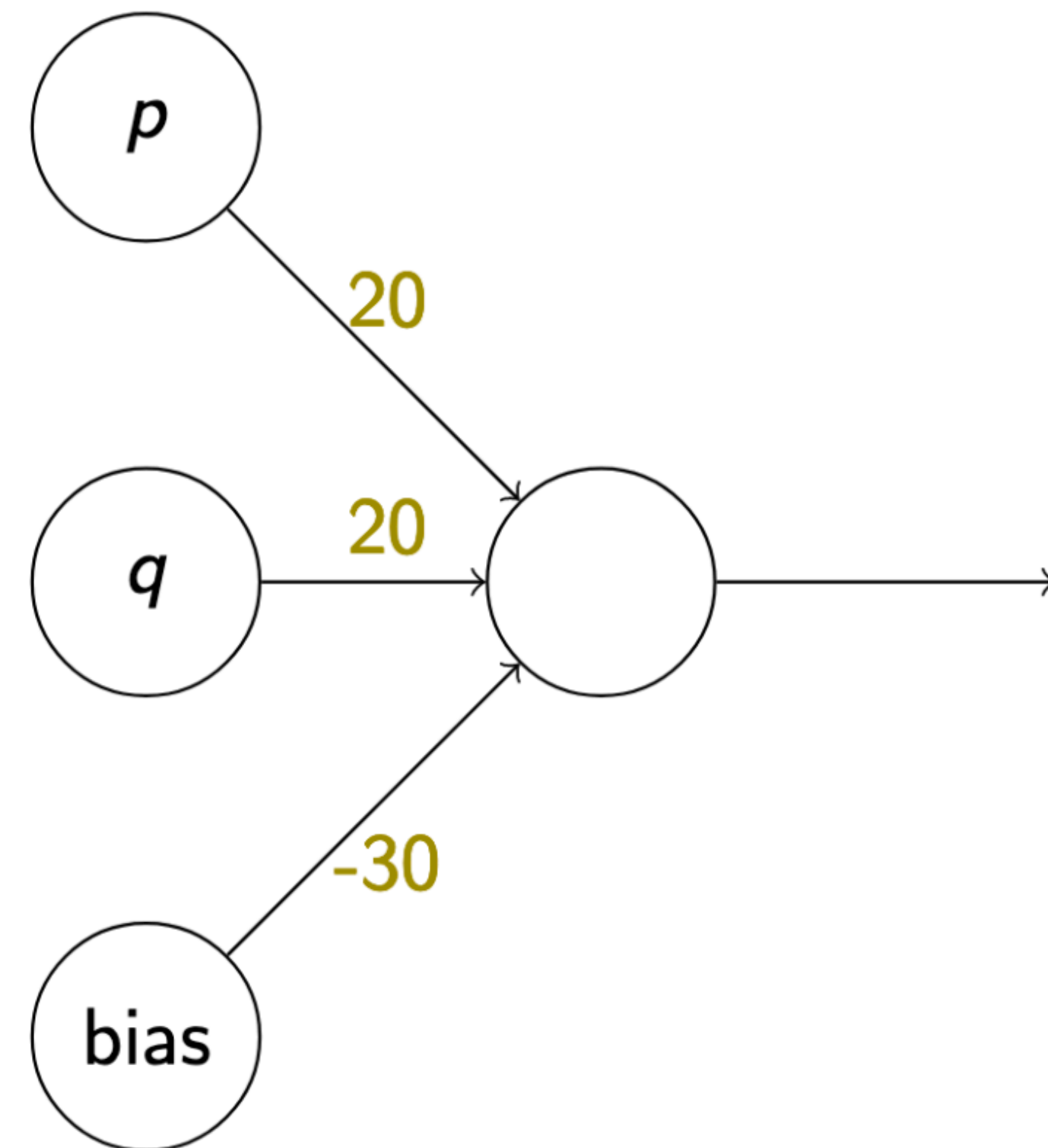
$$\sigma(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{e^x + 1}$$

Computing a Boolean function



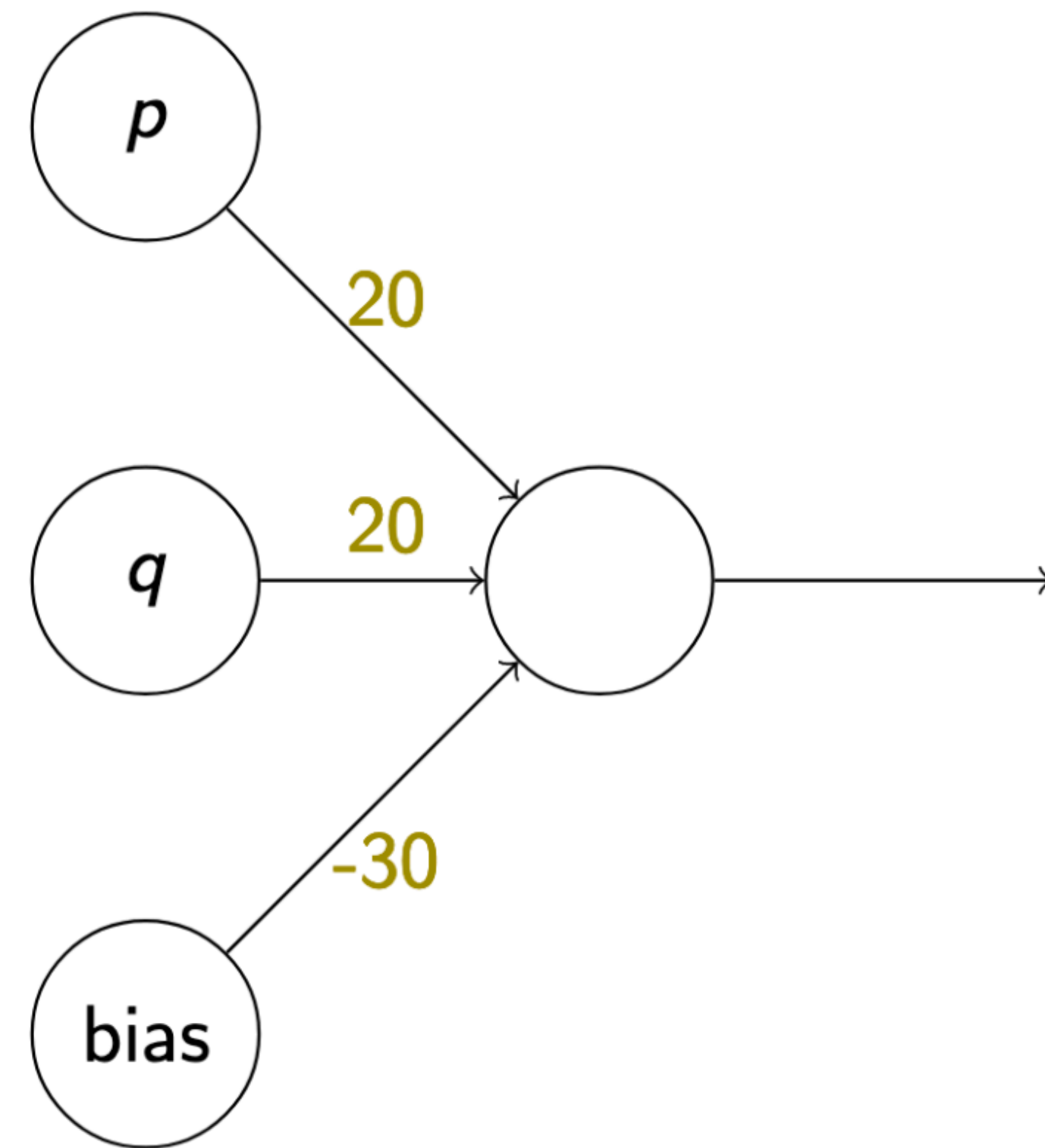
Computing a Boolean function

p	q	a
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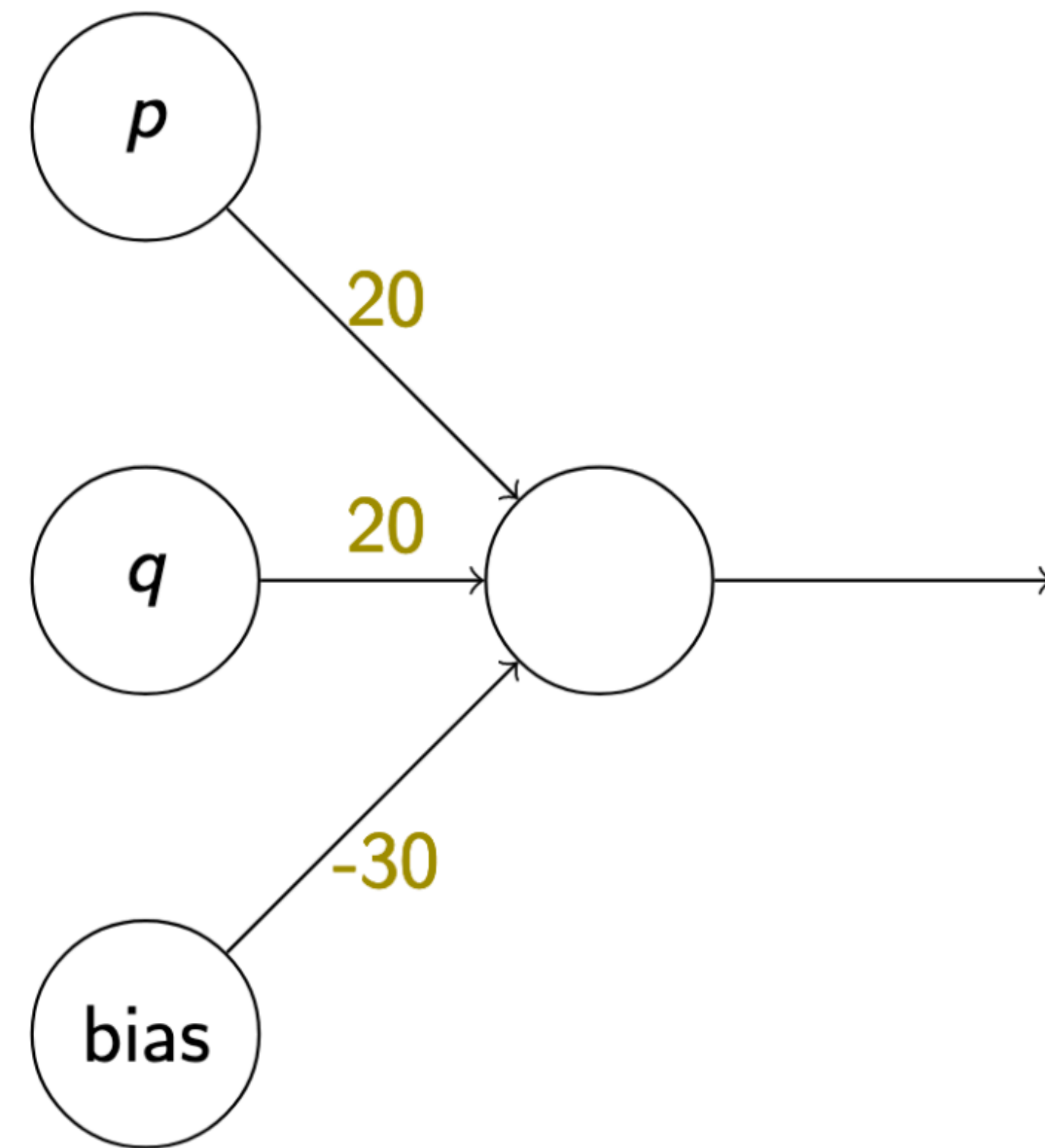
Computing a Boolean function

p	q	a
1	1	1



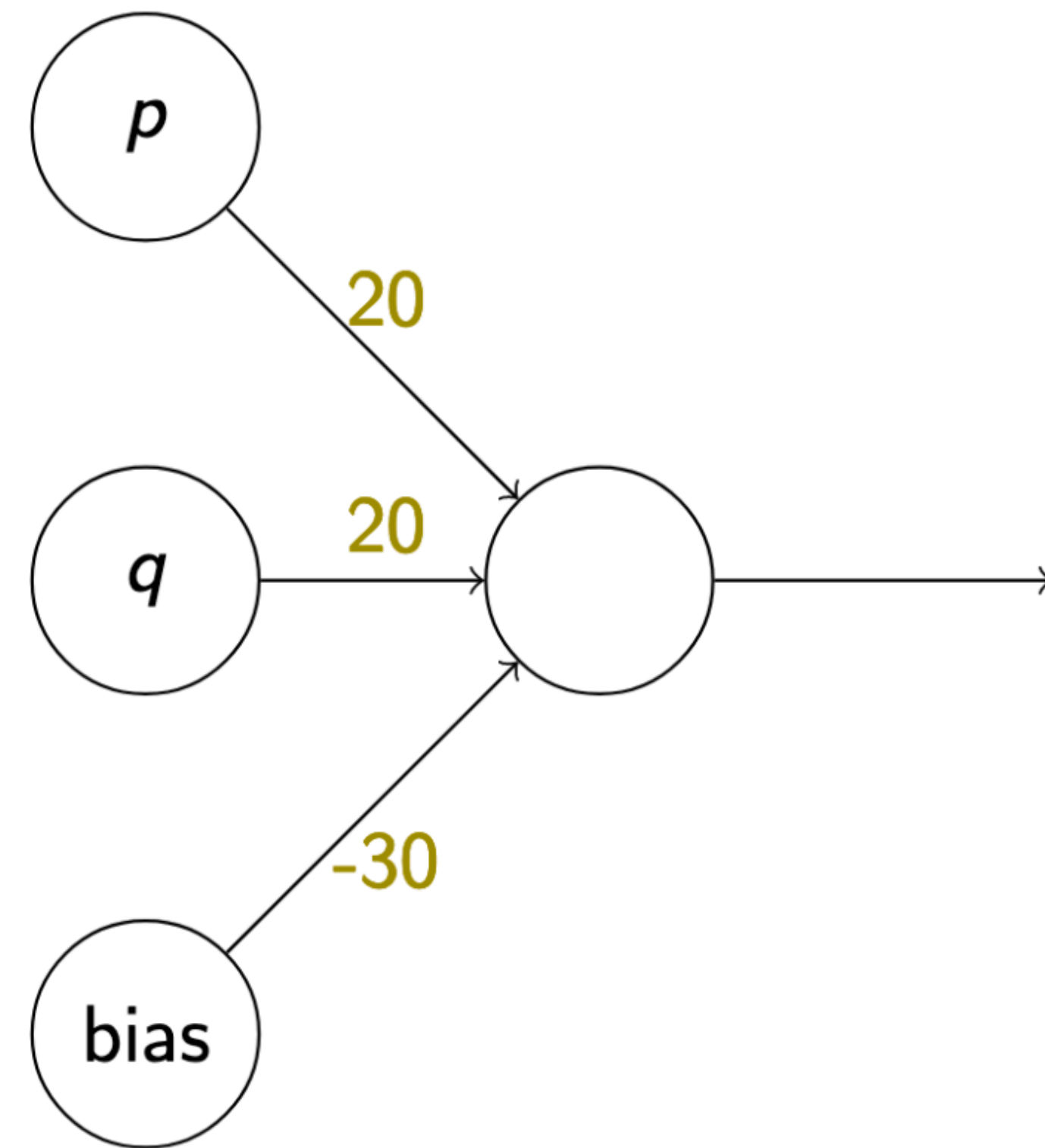
Computing a Boolean function

p	q	a
1	1	1
1	0	0



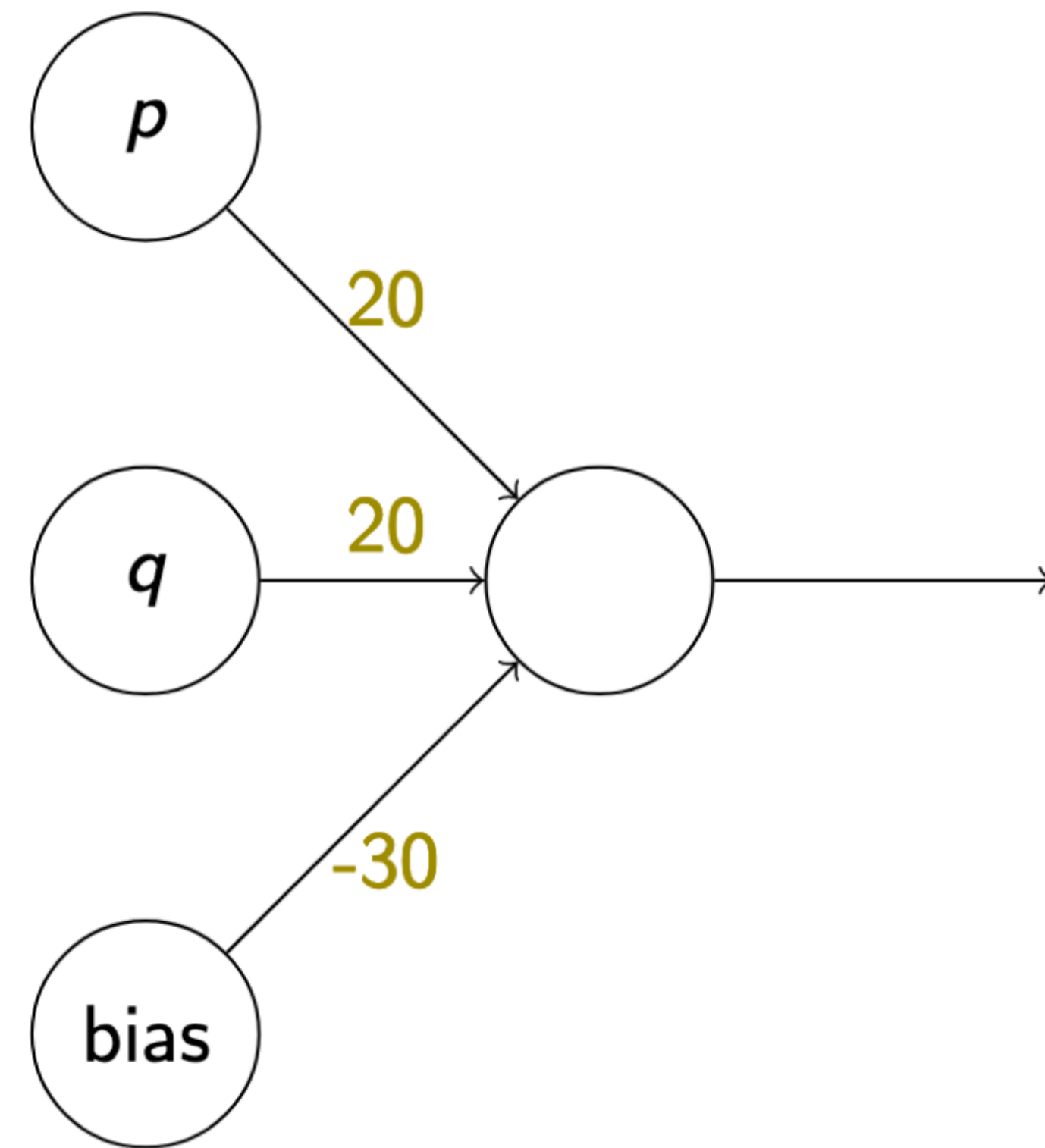
Computing a Boolean function

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1	0	0
0	1	0

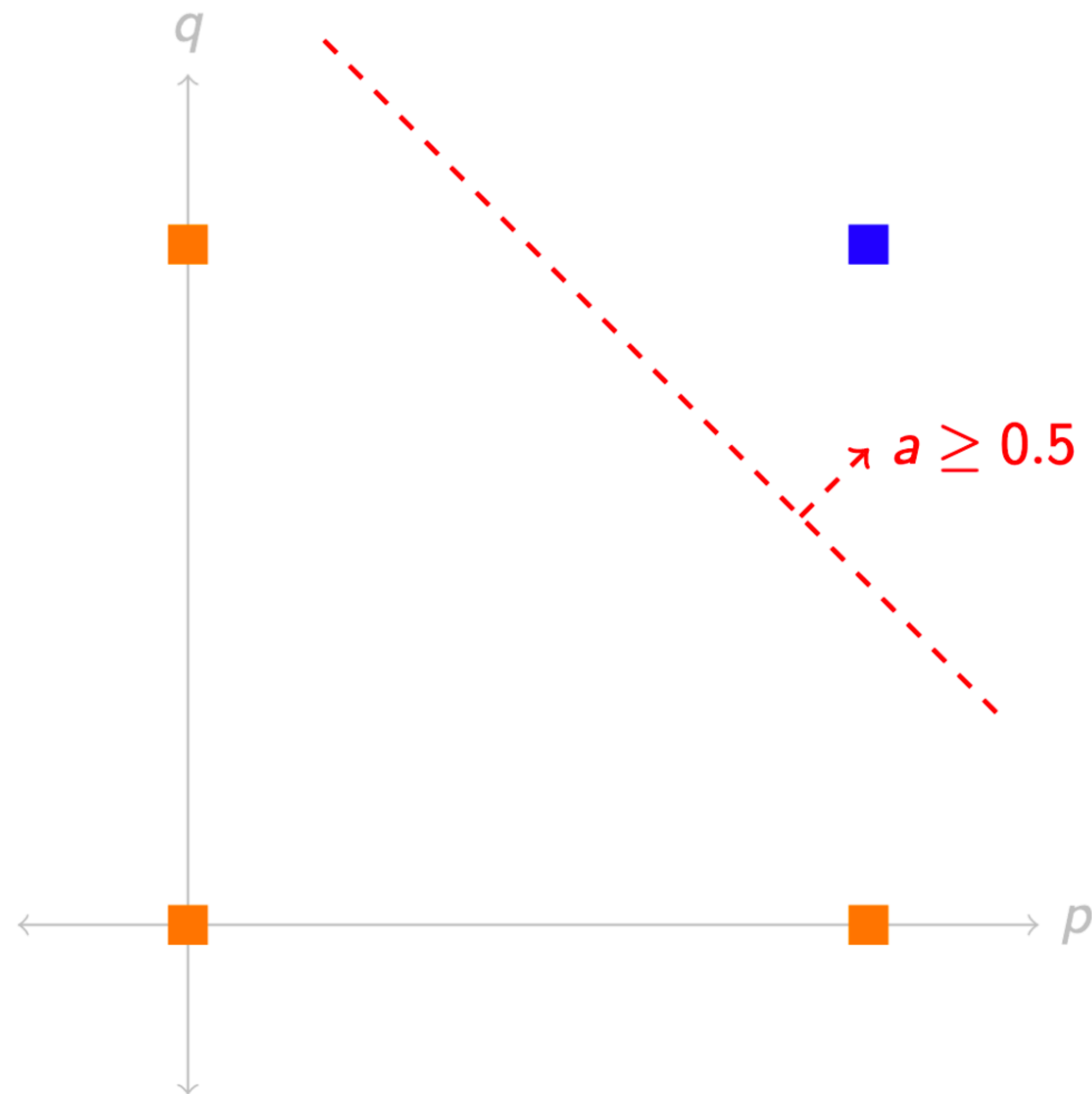


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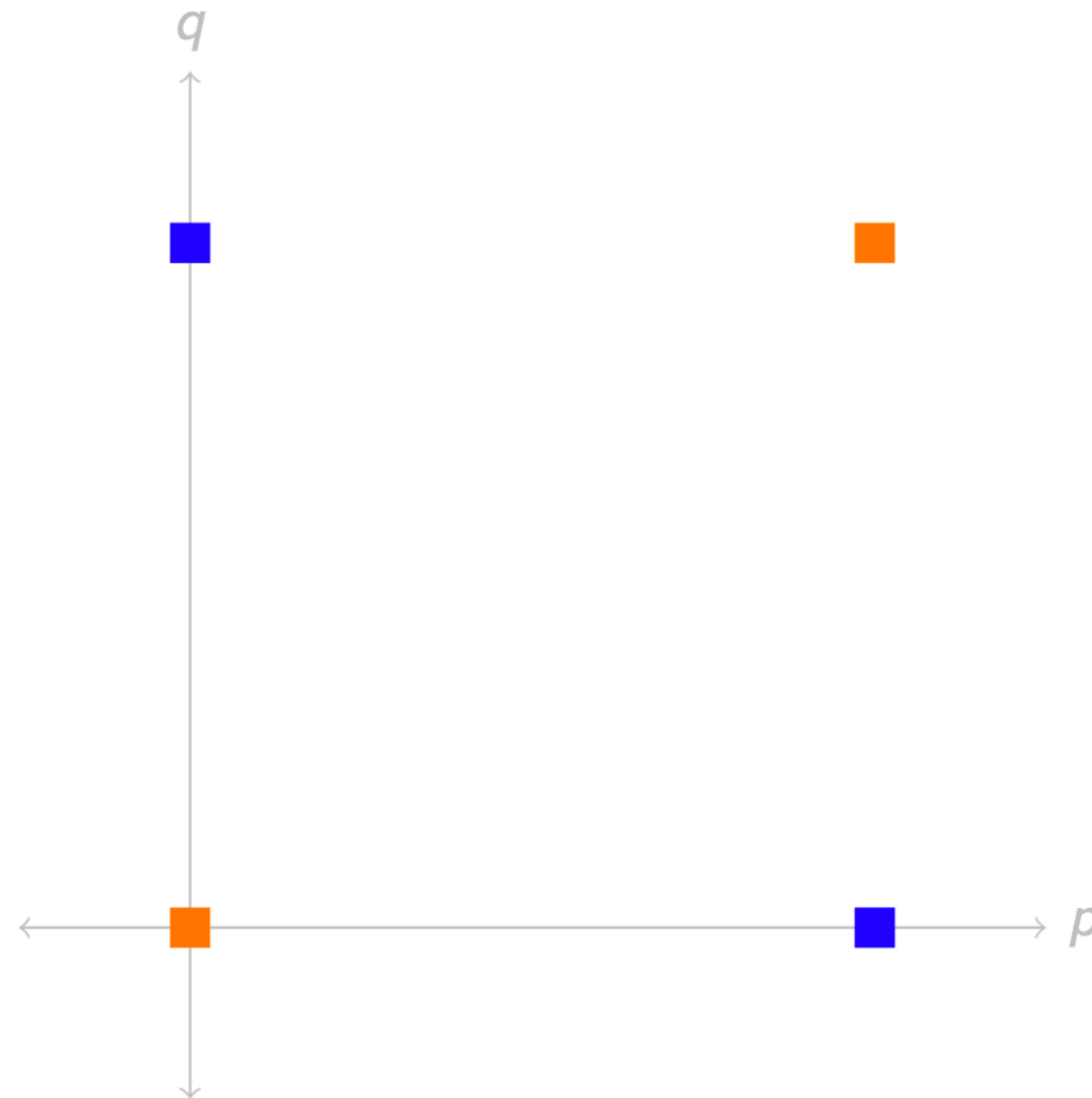
p	q	a
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1	0	0
0	1	0
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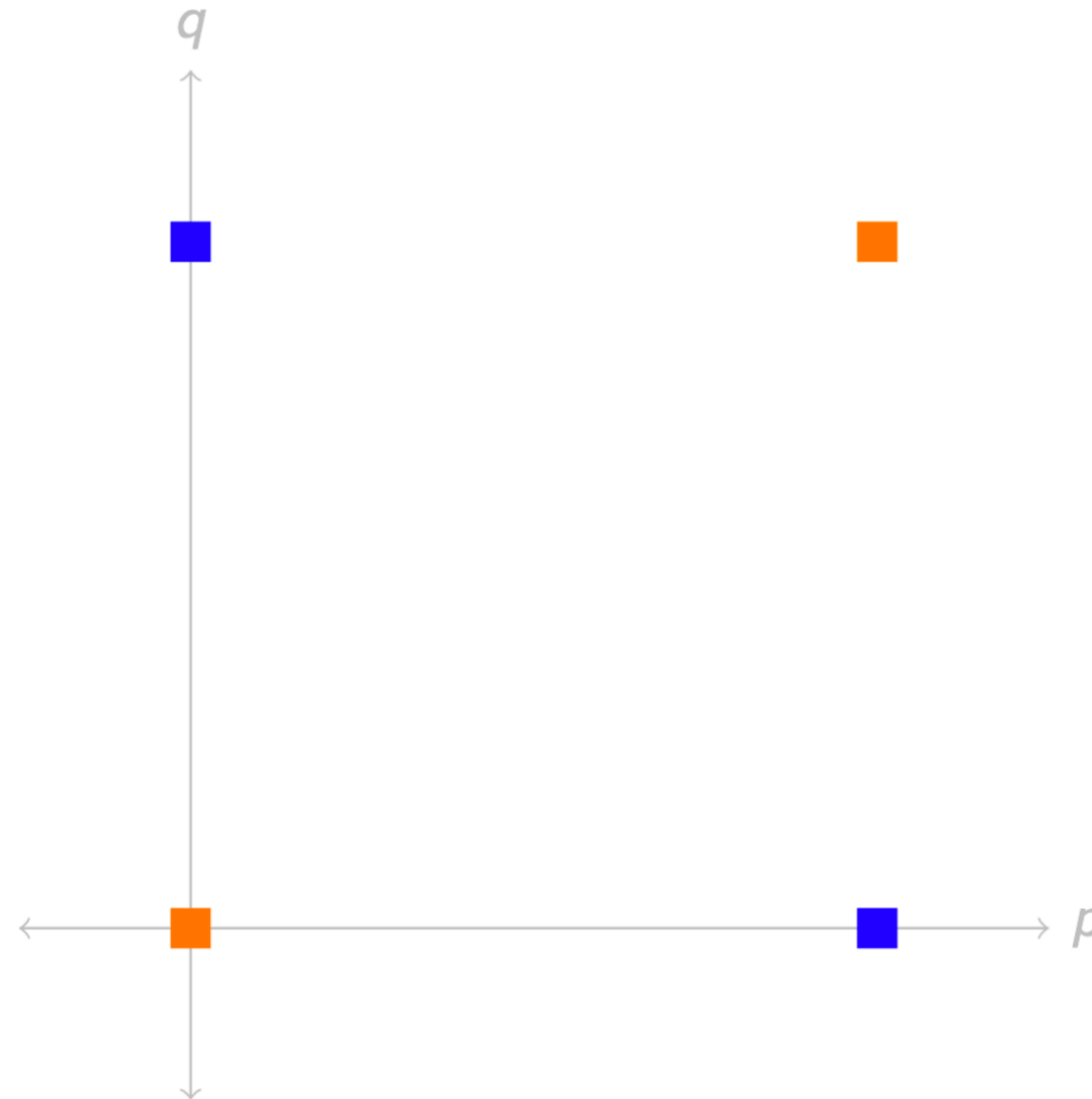
Computing 'and'



The XOR problem

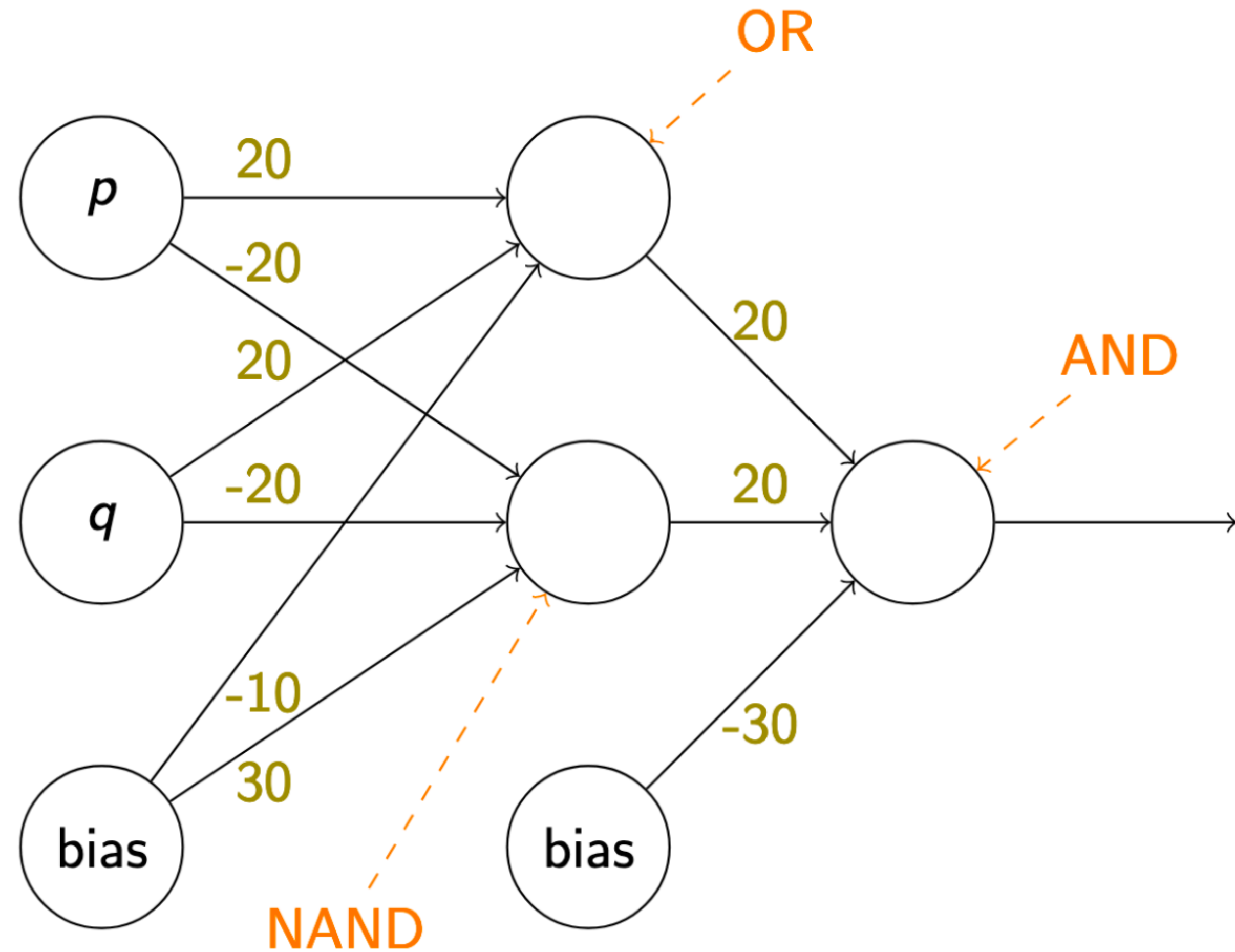


The XOR problem

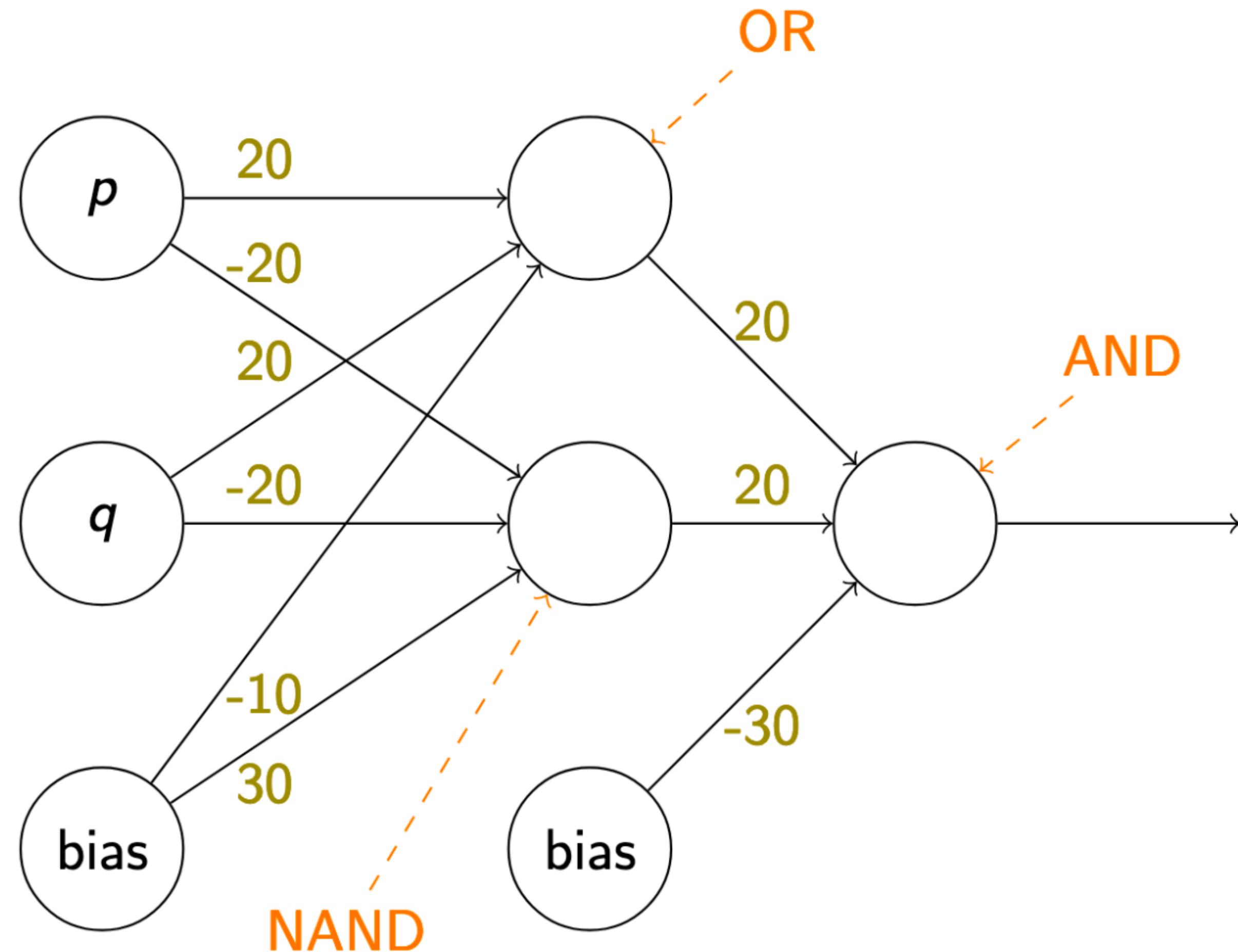


XOR is not linearly separable

Computing XOR

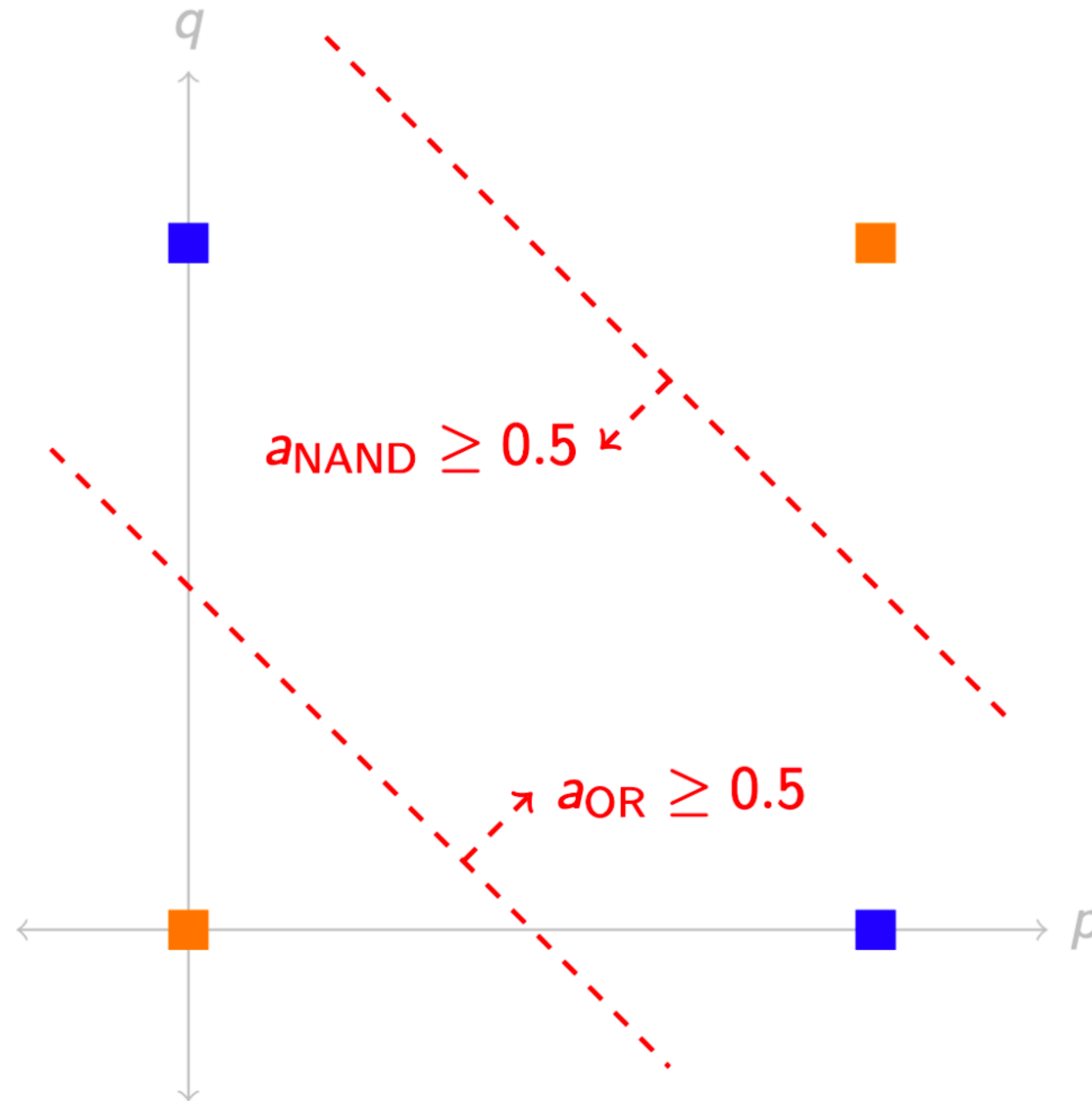


Computing XOR



Exercise: show that
NAND behaves as described.

Computing XOR



Key Ideas

- Hidden layers compute high-level / abstract features of the input
 - Via training, will *learn which features* are helpful for a given task
 - Caveat: doesn't always learn much more than shallow features
- Doing so *increases the expressive power* of a neural network
 - Strictly more functions can be computed with hidden layers than without

Expressive Power

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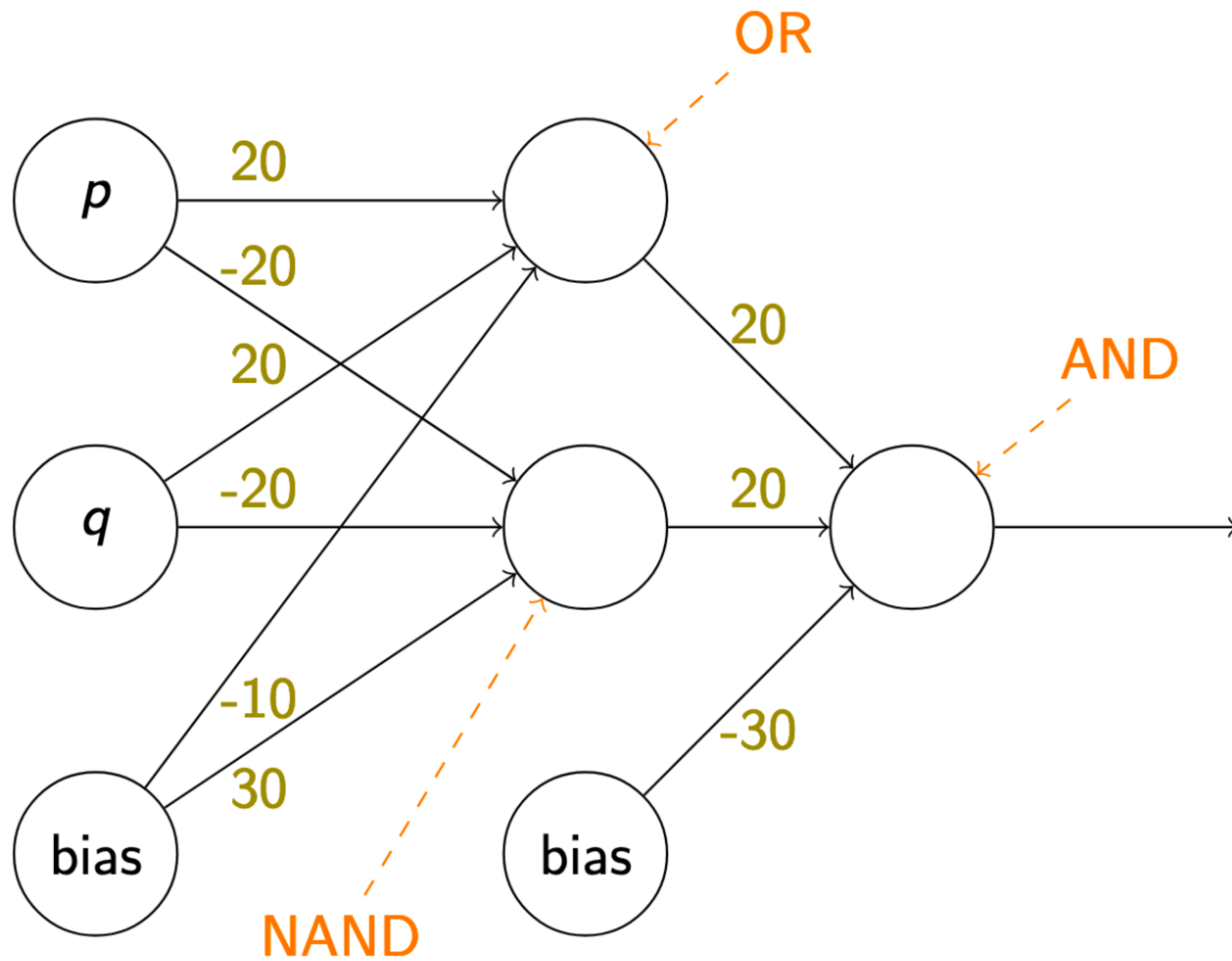
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 - Size of the hidden layer is *exponential* in m
 - How does one *find/learn* such a good approximation?

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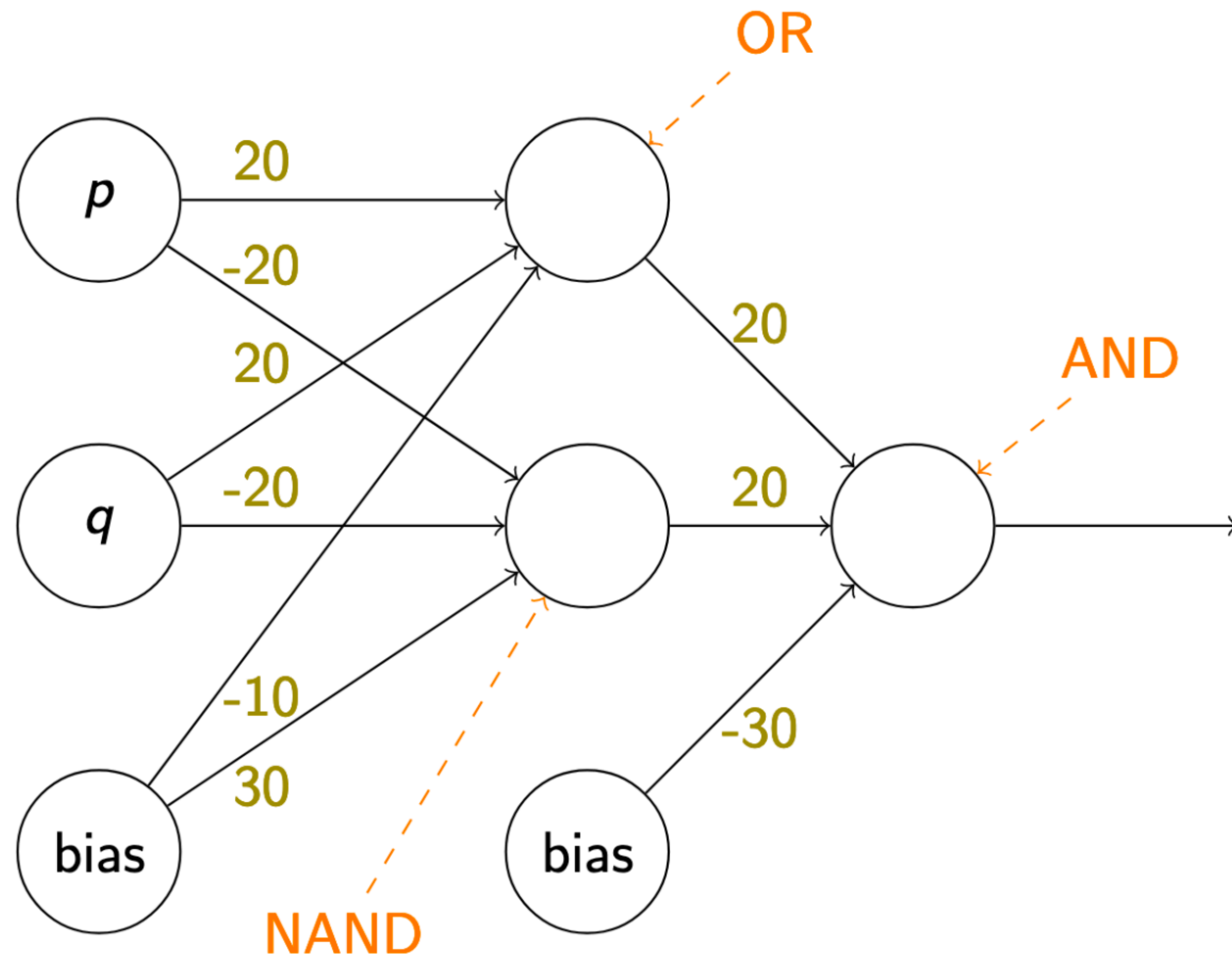
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 - Size of the hidden layer is *exponential* in m
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- Nice walkthrough: <http://neuralnetworksanddeeplearning.com/chap4.html>

Feed-forward networks aka Multi-layer perceptrons (MLP)

XOR Network

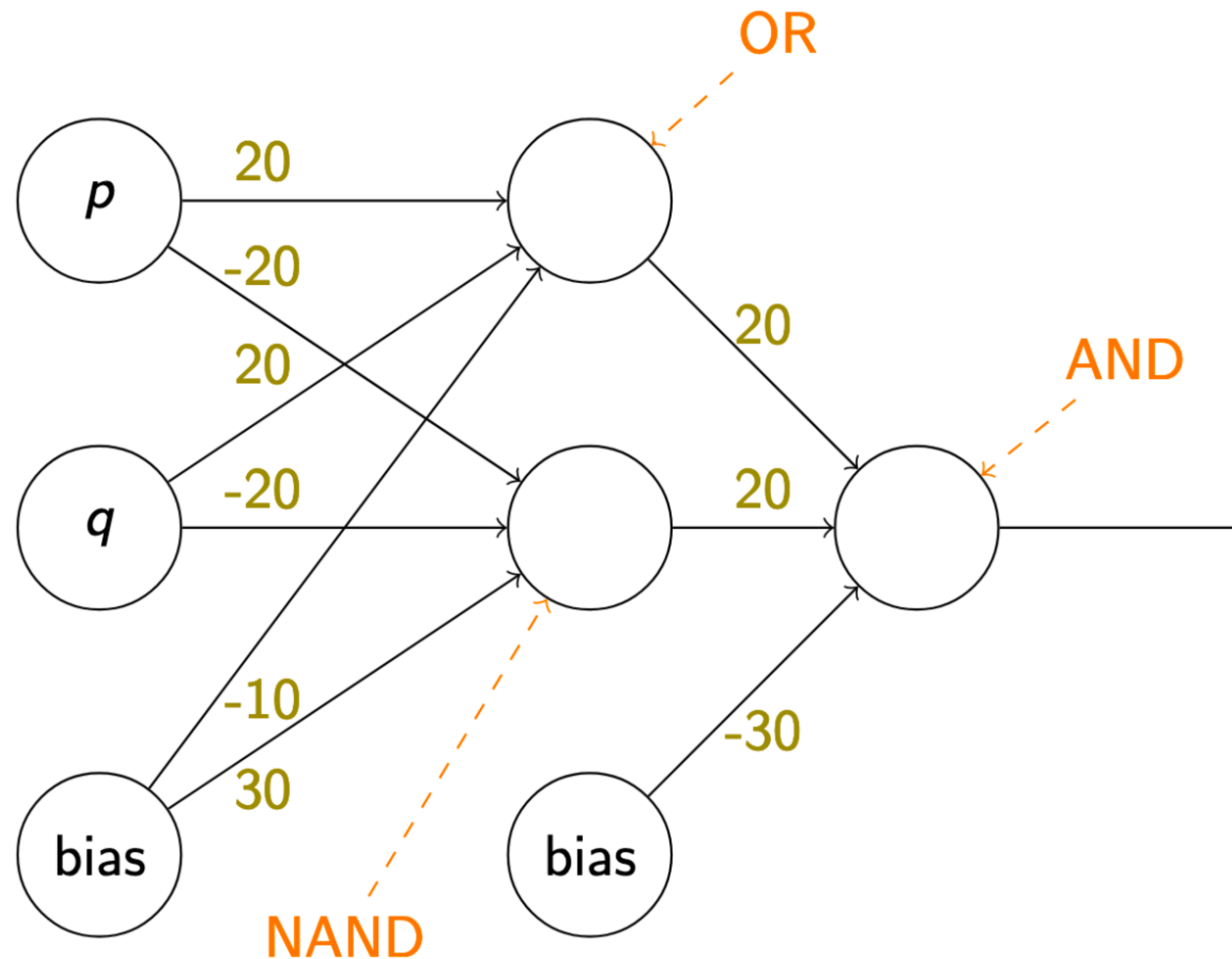


XOR Network



$$a_{\text{and}} = \sigma \left(w_{\text{or}}^{\text{and}} \cdot a_{\text{or}} + w_{\text{nand}}^{\text{and}} \cdot a_{\text{nand}} + b^{\text{and}} \right)$$

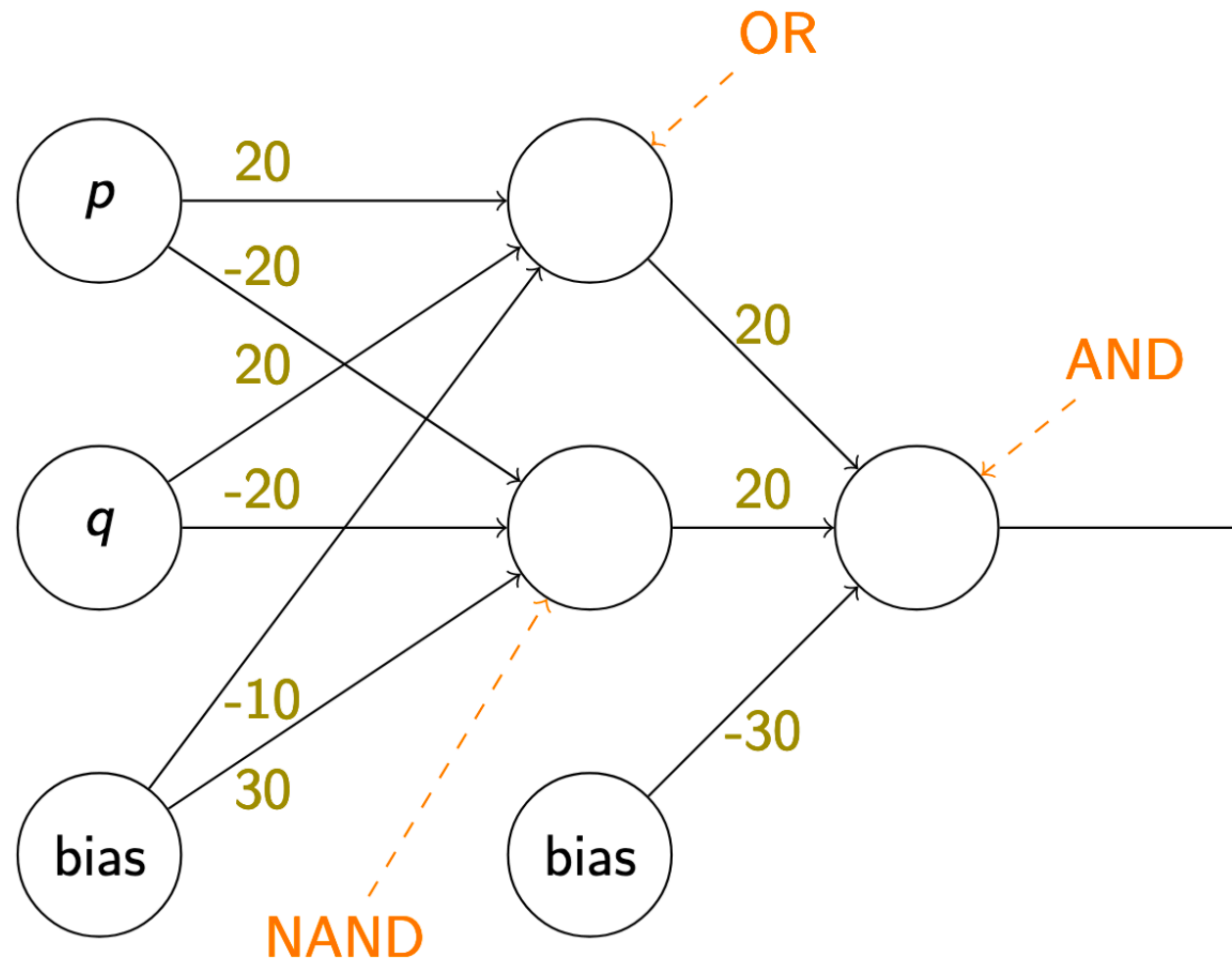
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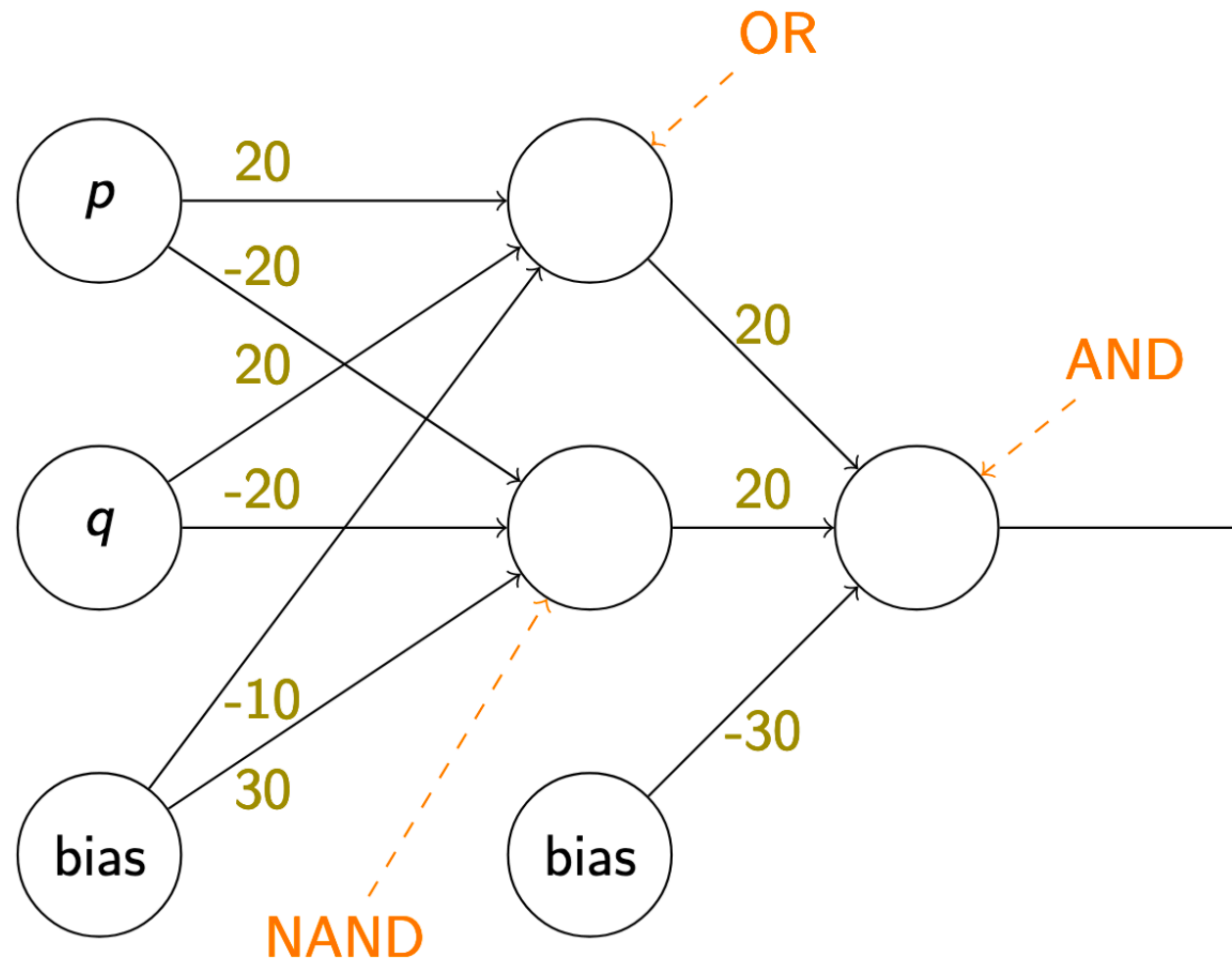
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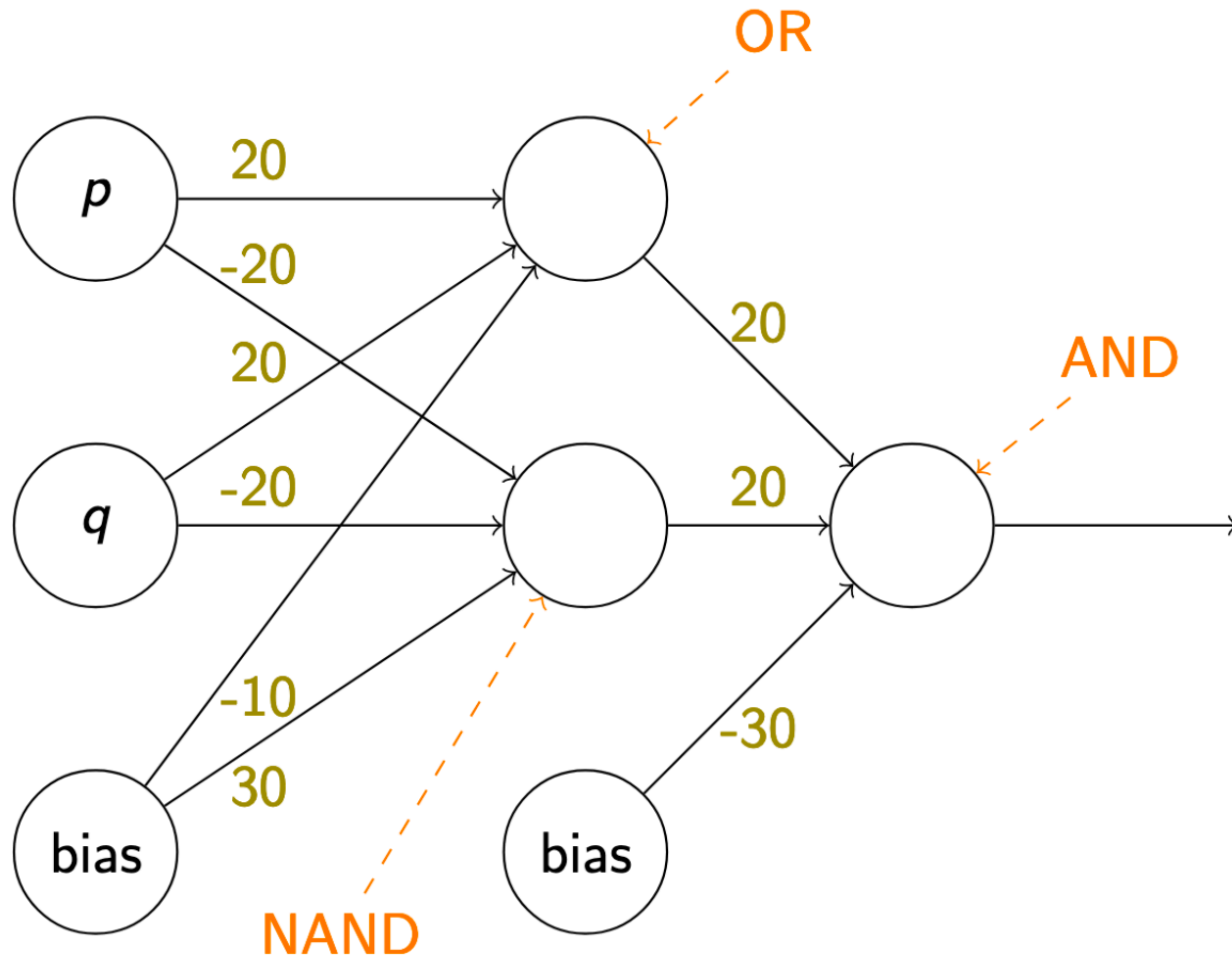


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$$a_{\text{or}} = \sigma \left(w_p^{\text{or}} \cdot a_p + w_q^{\text{or}} \cdot a_q + b^{\text{or}} \right)$$

XOR Network



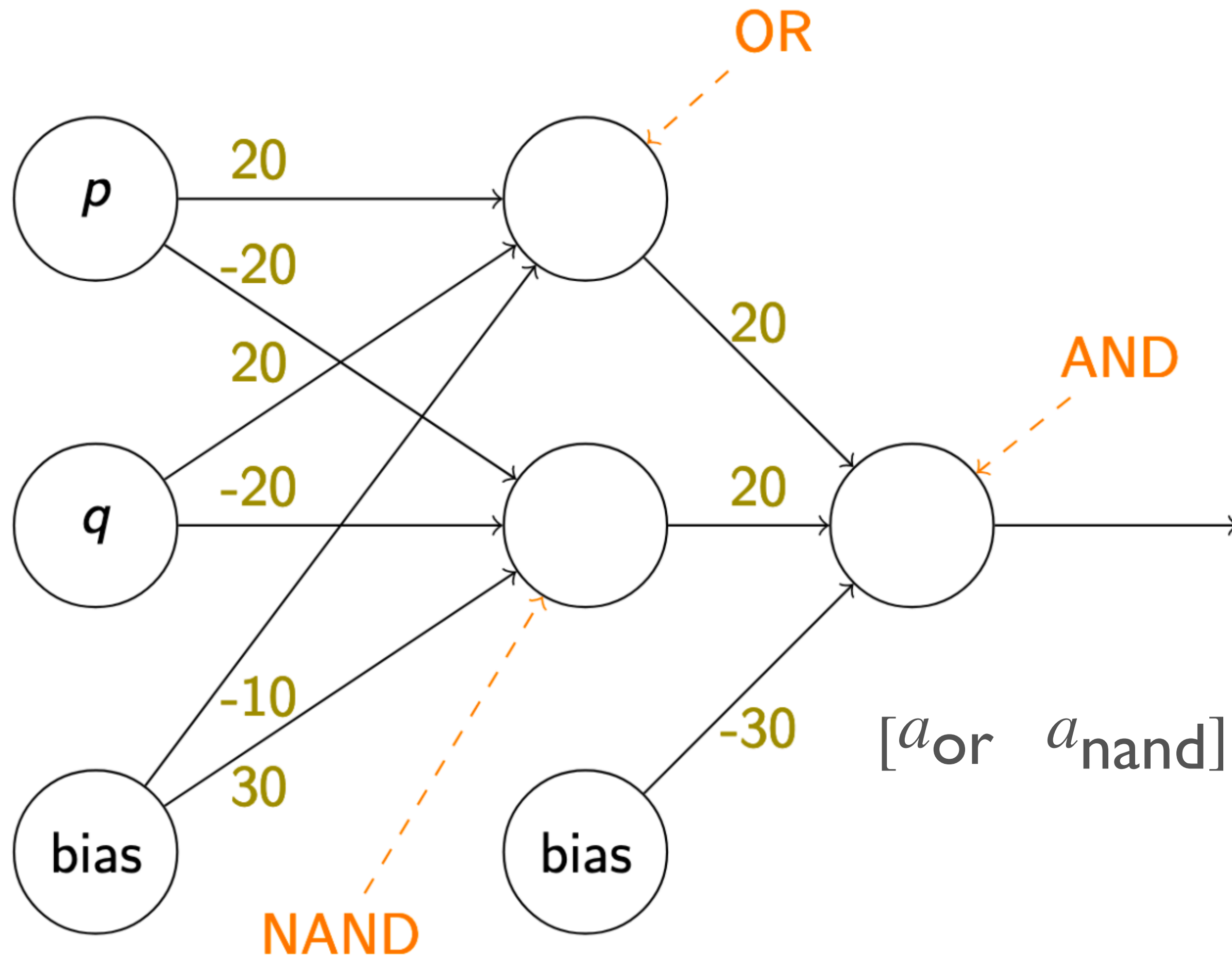
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XOR Network

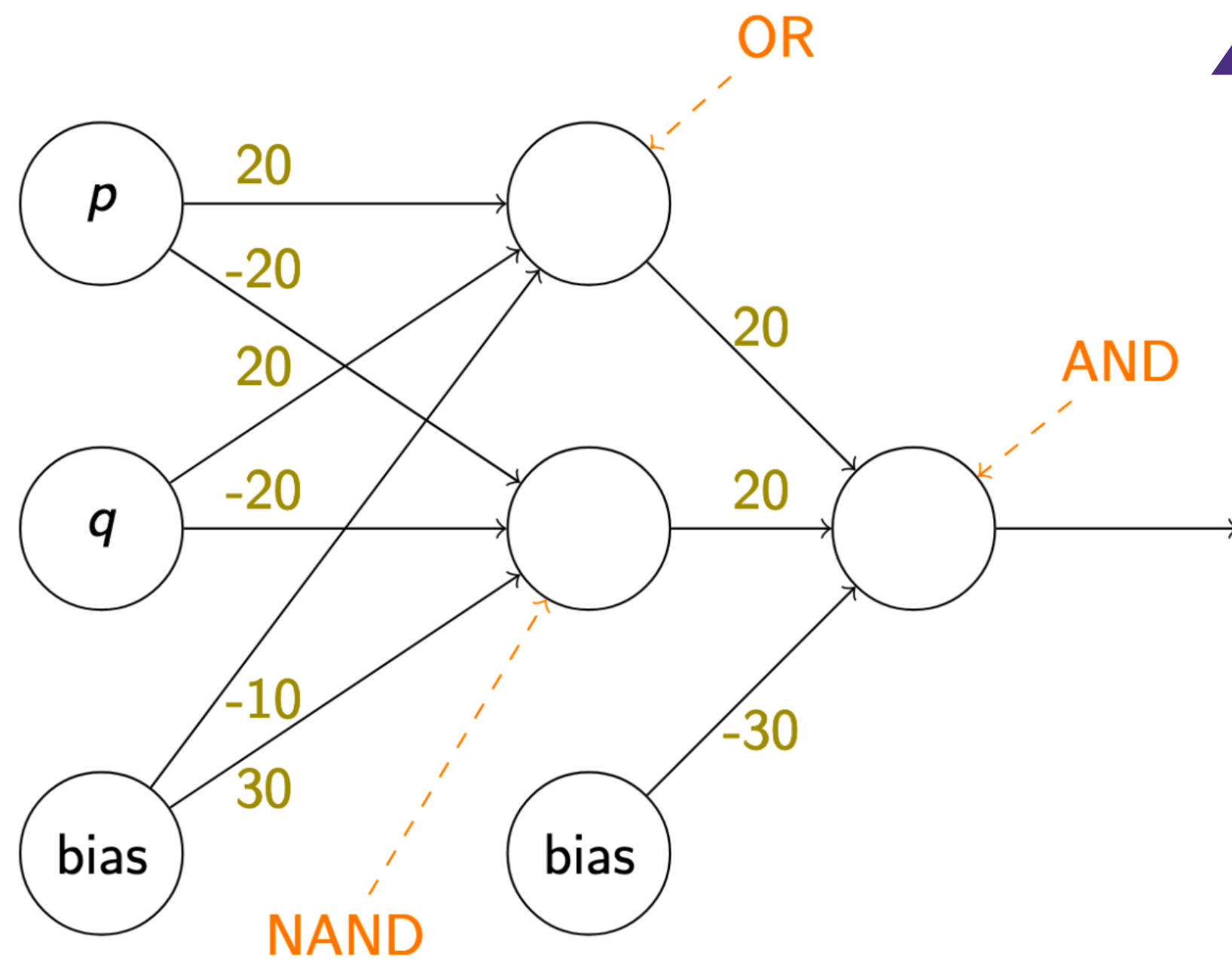


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XOR Network



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Generalizing

$$a_{\text{and}} = \sigma \left(\sigma \left(\begin{bmatrix} a_p & a_q \end{bmatrix} \begin{bmatrix} w_p^{\text{or}} & w_p^{\text{nand}} \\ w_q^{\text{or}} & w_q^{\text{nand}} \end{bmatrix} + \begin{bmatrix} b^{\text{or}} & b^{\text{nand}} \end{bmatrix} \right) \begin{bmatrix} w^{\text{and}}_{\text{or}} \\ w^{\text{and}}_{\text{nand}} \end{bmatrix} + b^{\text{and}} \right)$$

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$$\hat{y} = f_2 \left(f_1 \left(xW^1 + b^1 \right) W^2 + b^2 \right)$$

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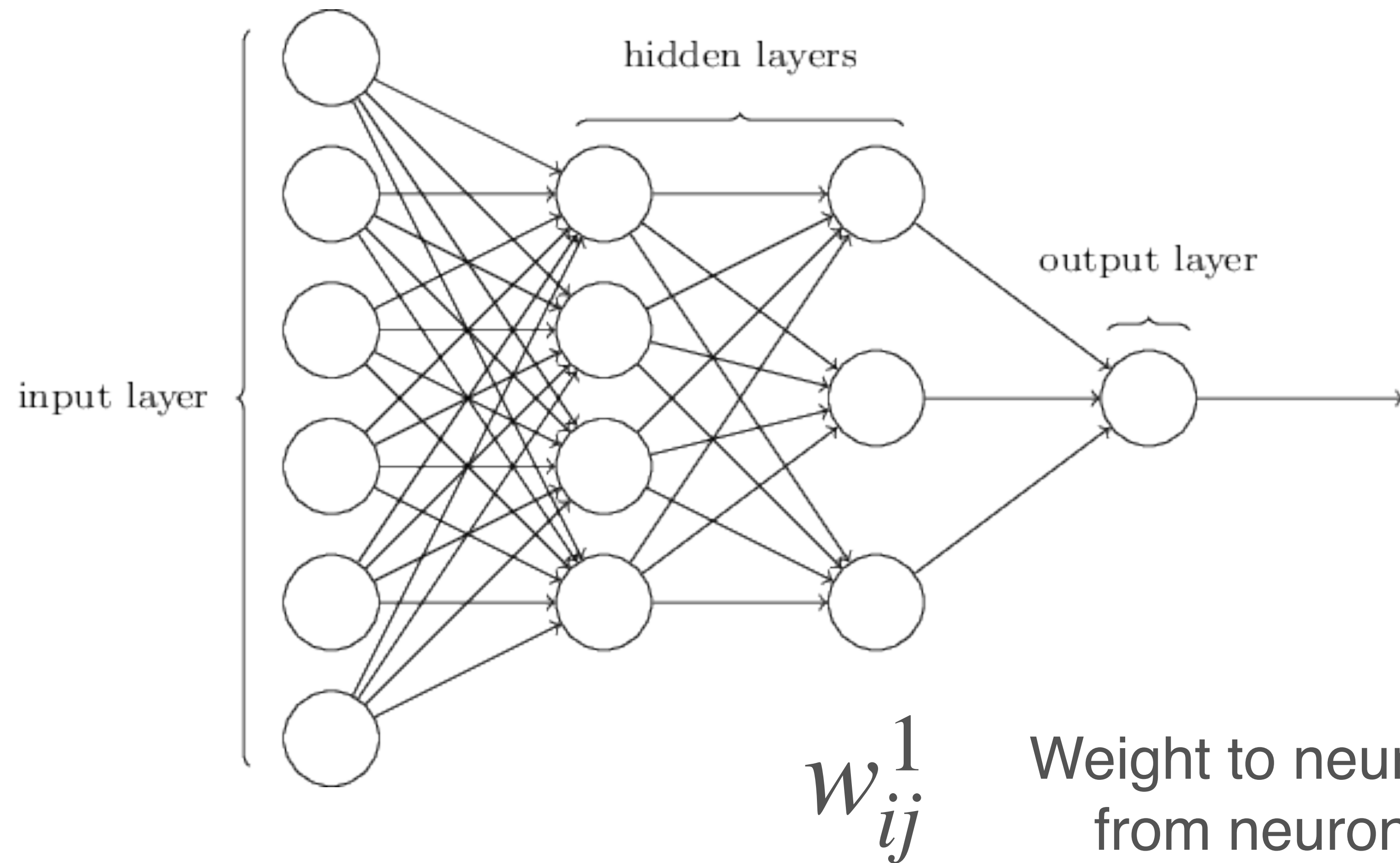
$$\hat{y} = f_n \left(f_{n-1} \left(\cdots f_2 \left(f_1 (xW^1 + b^1) W^2 + b^2 \right) \cdots \right) W^n + b^n \right)$$

Some terminology

- Our XOR network is a *feed-forward neural network* with *one hidden layer*
 - Aka a multi-layer perceptron (MLP)
- Input nodes: 2; output nodes: 1
- Activation function: sigmoid

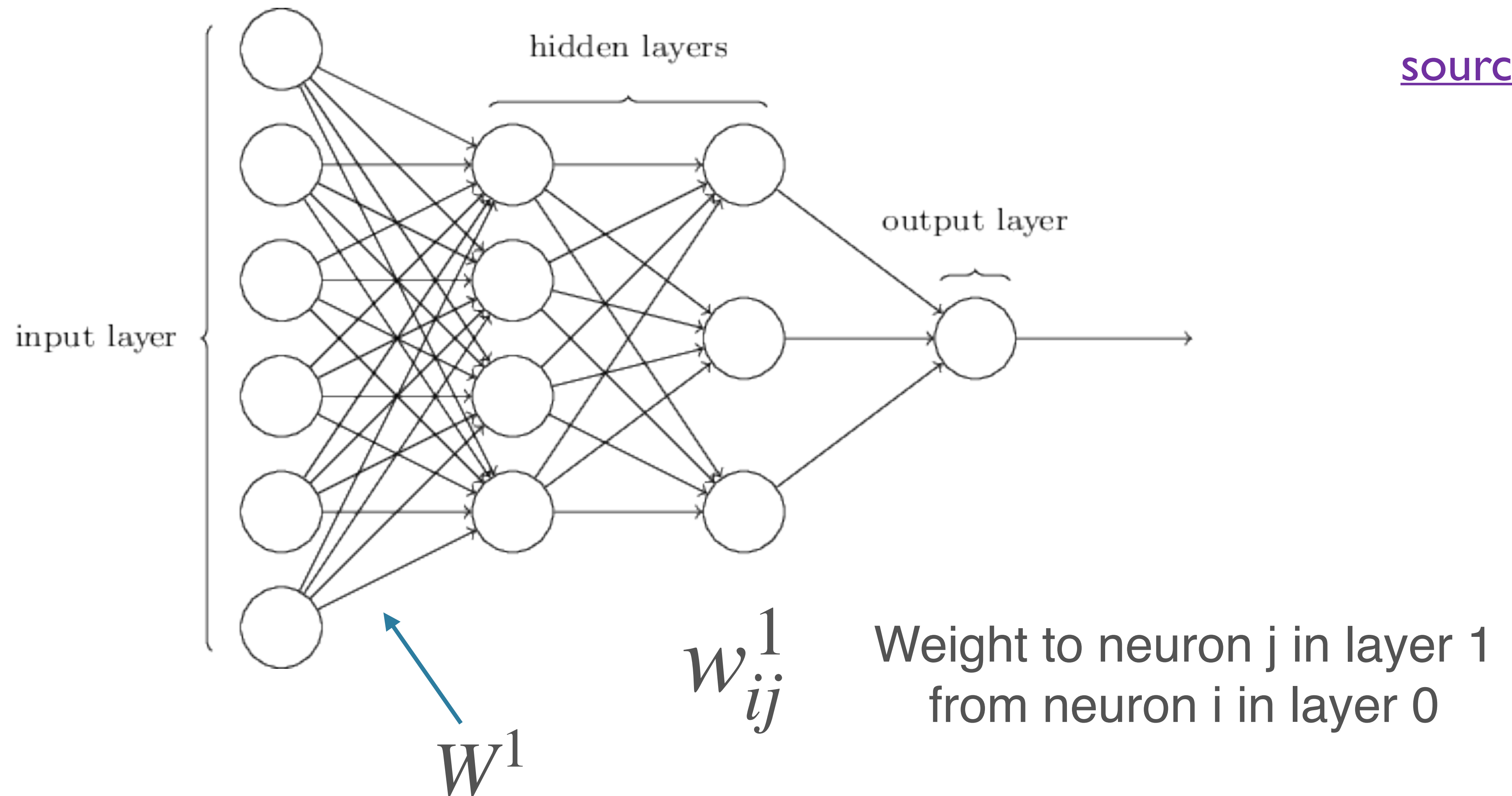
General MLP

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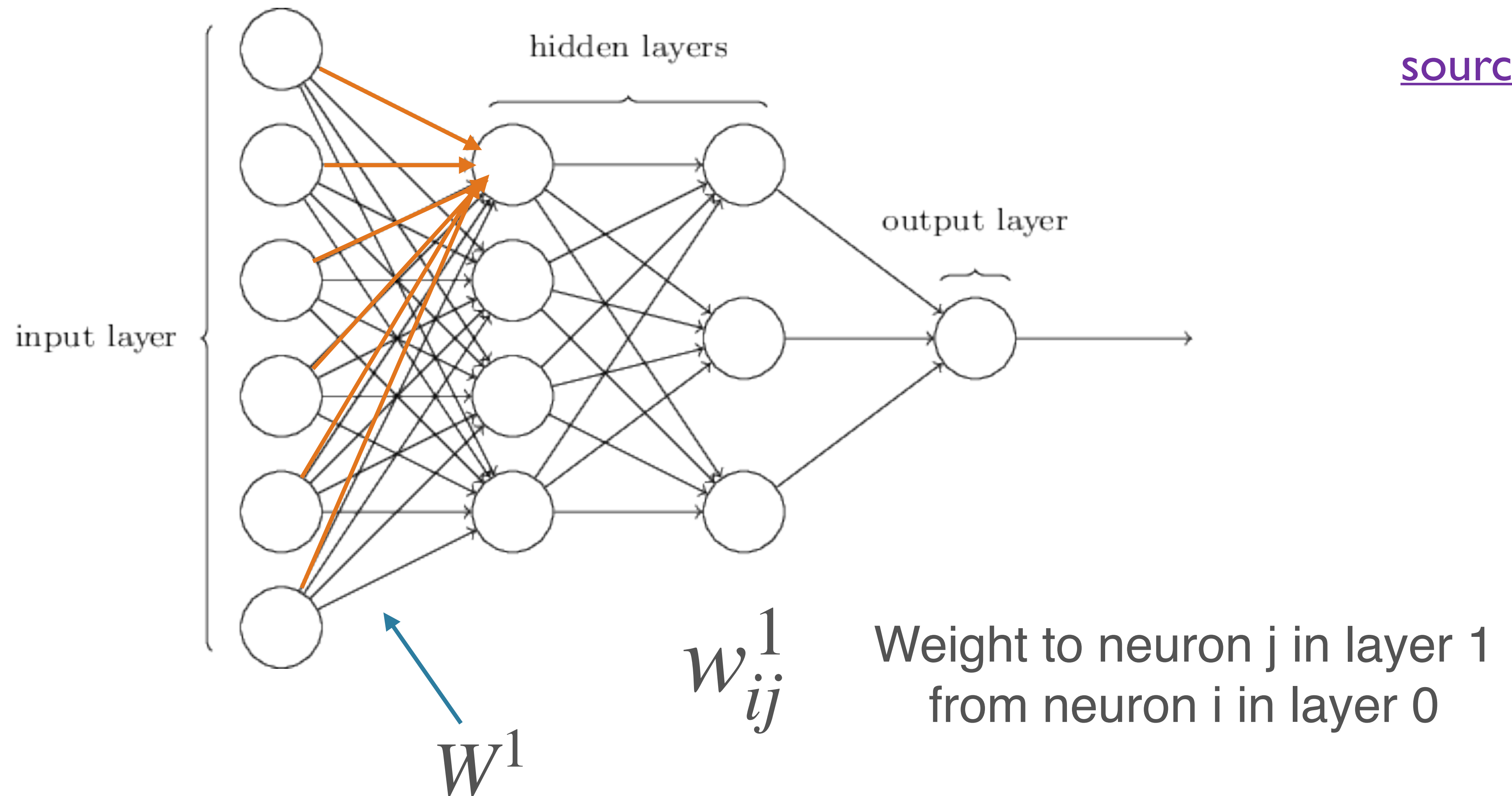
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$$x = [x_0 \quad x_1 \quad \cdots \quad x_{n_0}]$$

Shape: $(1, n_0)$

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$$W^1 = \begin{bmatrix} w_{00}^1 & w_{10}^1 & \cdots & w_{0n_1}^1 \\ w_{10}^1 & w_{11}^1 & \cdots & w_{1n_1}^1 \\ \vdots & \vdots & \ddots & \vdots \\ w_{n_00}^1 & w_{n_01}^1 & \cdots & w_{n_0n_1}^1 \end{bmatrix}$$

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Shape: (n_0, n_1)

n_0 : number of neurons in layer 0 (input)

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Parameters of an MLP

- Weights and biases
 - For each layer l : $n_l(n_{l-1} + 1)$
 - $n_l n_{l-1}$ weights; n_l biases
- With n hidden layers (considering the output as a hidden layer):

$$\sum_{i=1}^n n_i(n_{i-1} + 1)$$

Hyper-parameters of an MLP

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- Input size, output size
 - Usually fixed by your problem / dataset
 - Input: image size, vocab size; number of “raw” features in general
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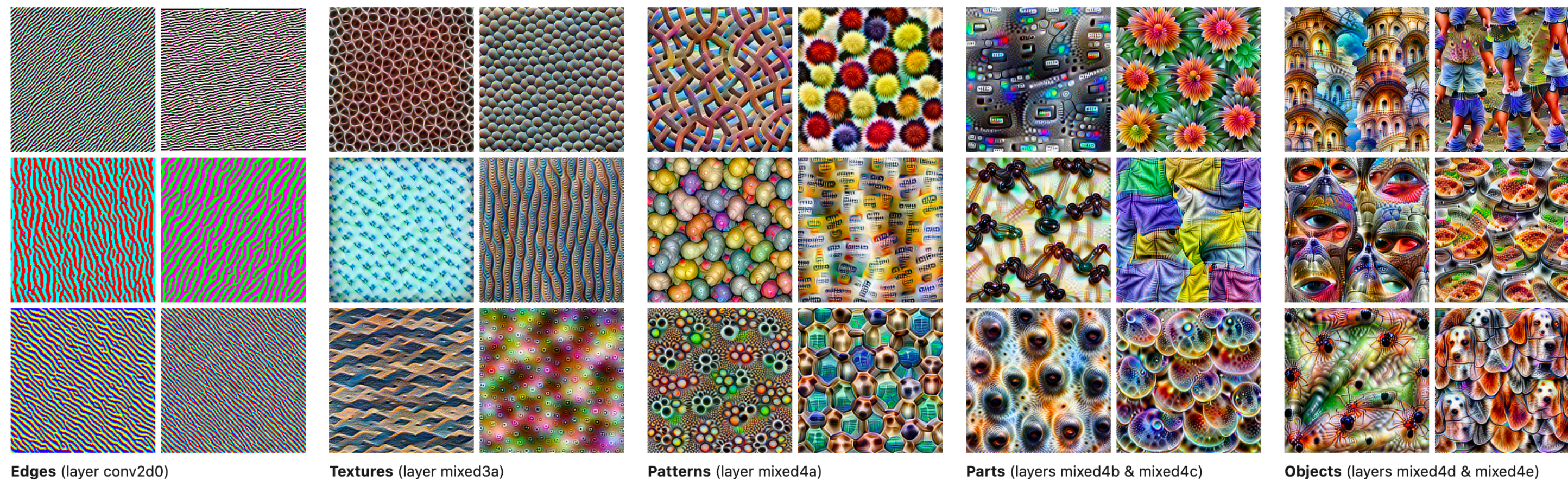
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- Others: initialization, regularization (and associated values), learning rate / training, ...

The Deep in Deep Learning

- The Universal Approximation Theorem says that one hidden layer suffices for arbitrarily-closely approximating a given function
- Empirical drawbacks: Super-exponentially many neurons; hard to discover
- “Deep and narrow” >> “Shallow and wide”
 - In principle allows hierarchical features to be learned
 - More well-behaved w/r/t optimization



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Activation Functions

- Note: *non-linear* activation functions are essential
- MLP: linear transformation, followed by a point-wise non-linearity, repeated several times over
- Without the non-linearity, would just have several linear transformations
 - Composition of linear transformations is *also* linear!

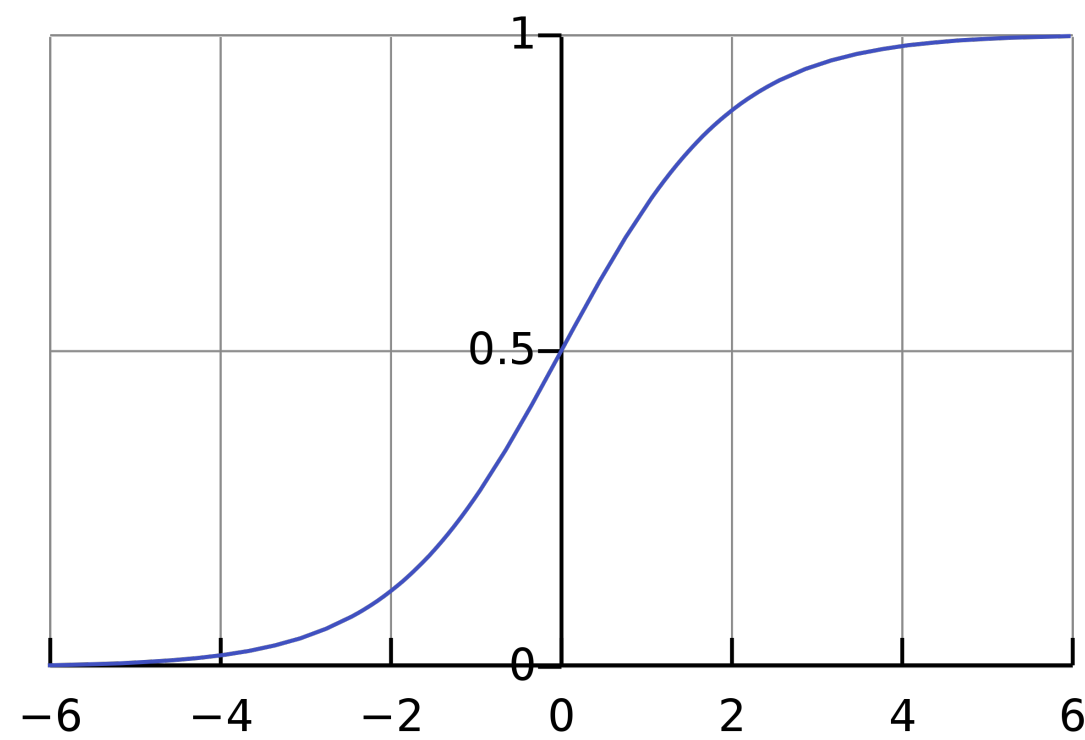
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Activation Functions: Hidden Layer

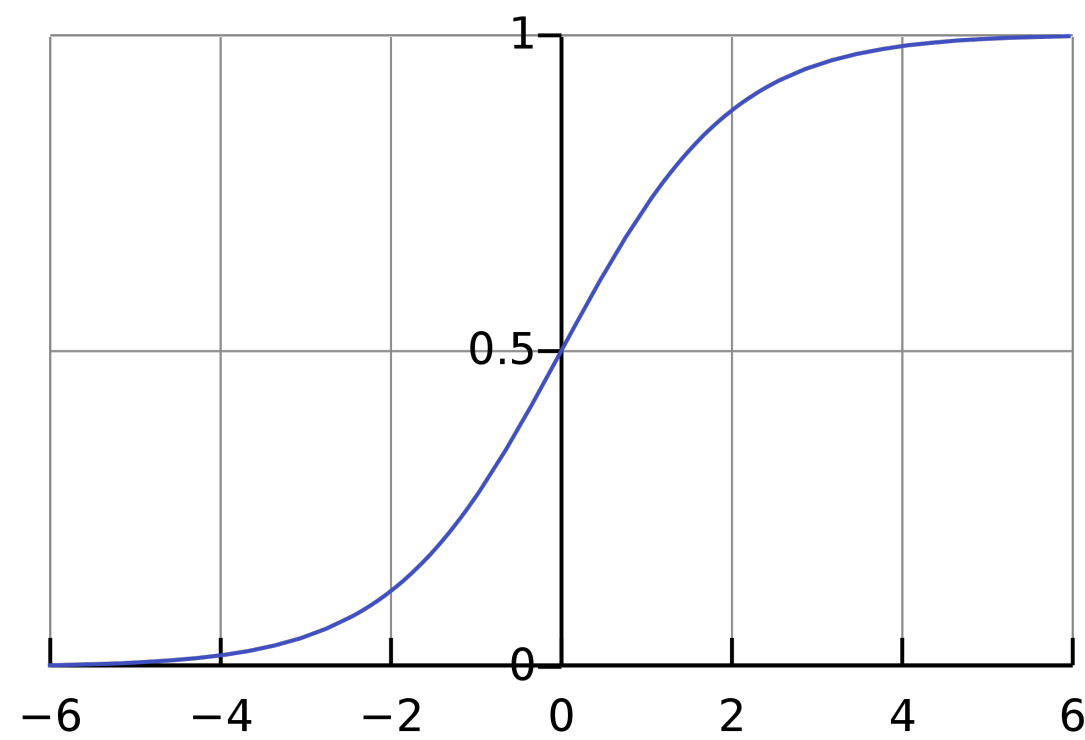
sigmoid



$$\sigma(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{e^x + 1}$$

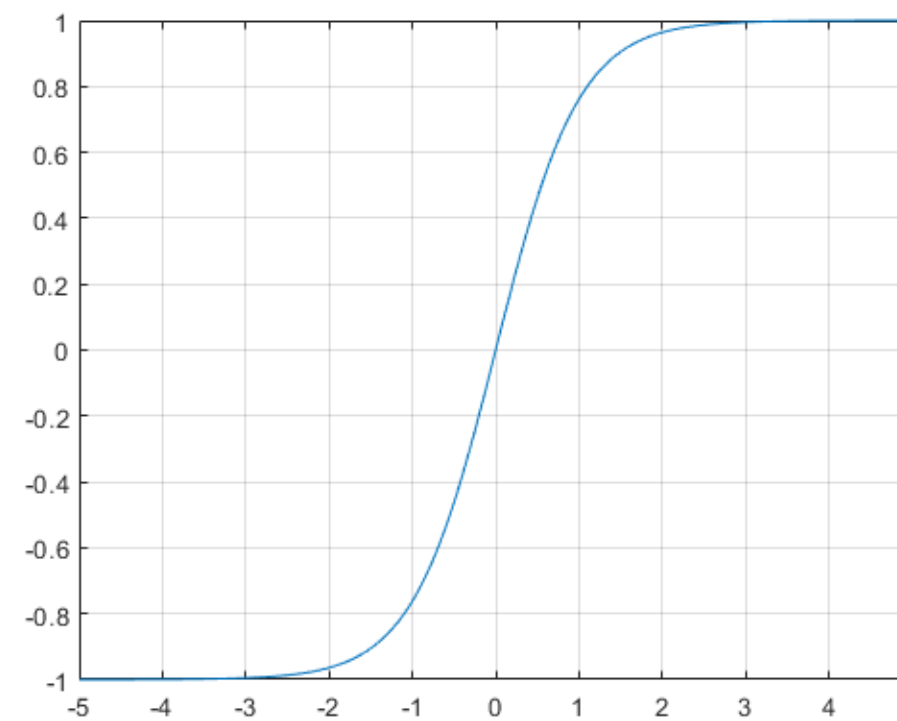
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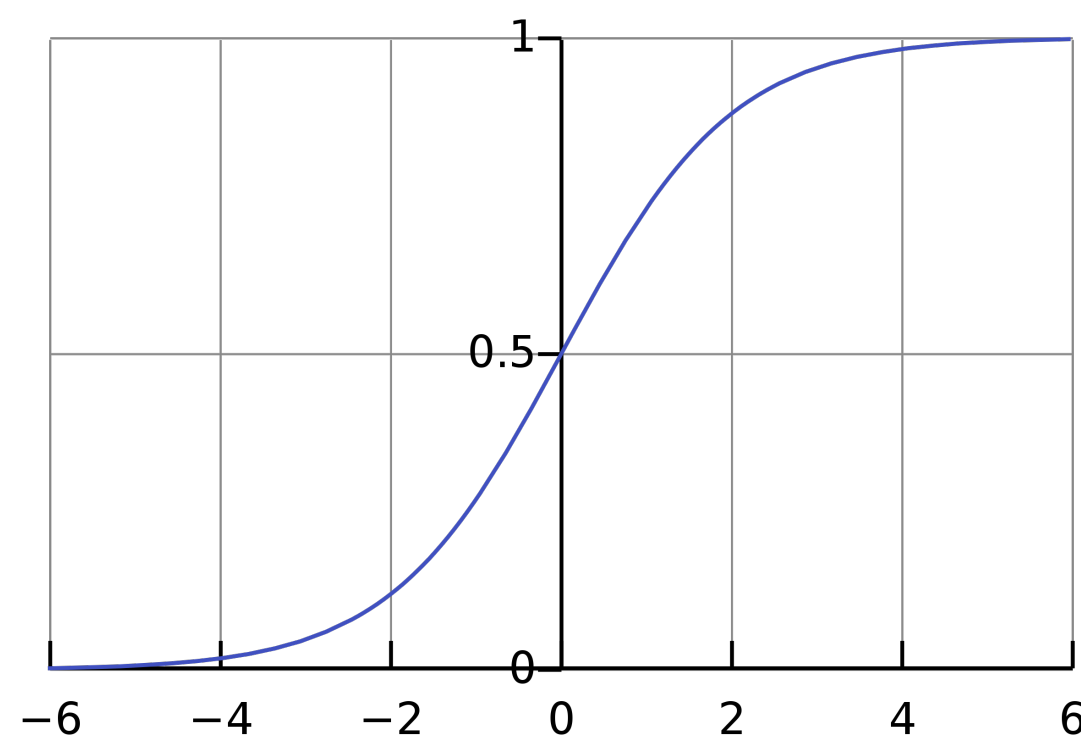
tanh



$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = 2\sigma(2x) - 1$$

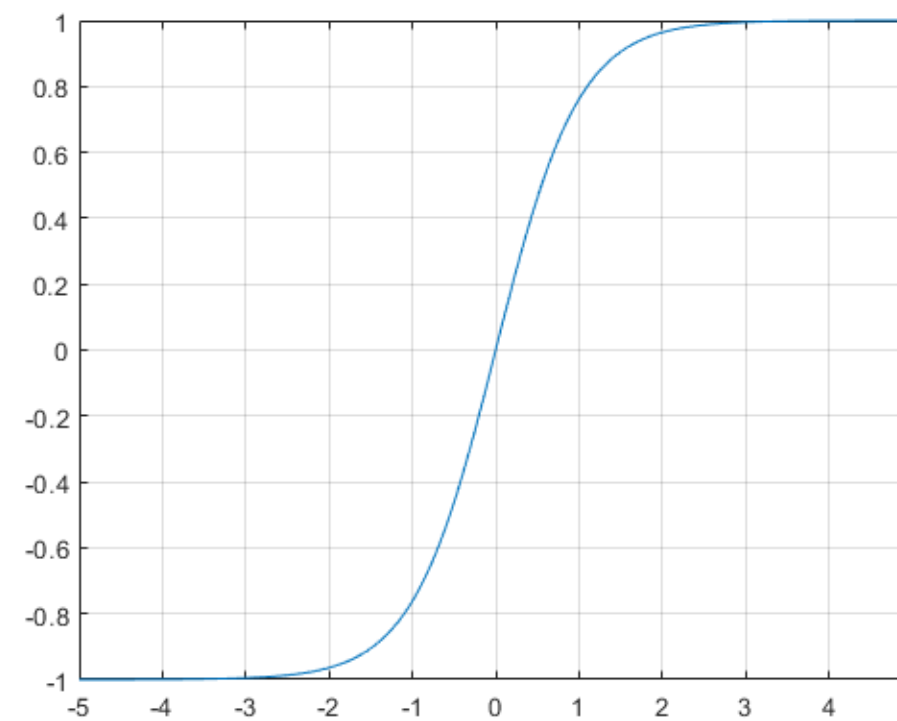
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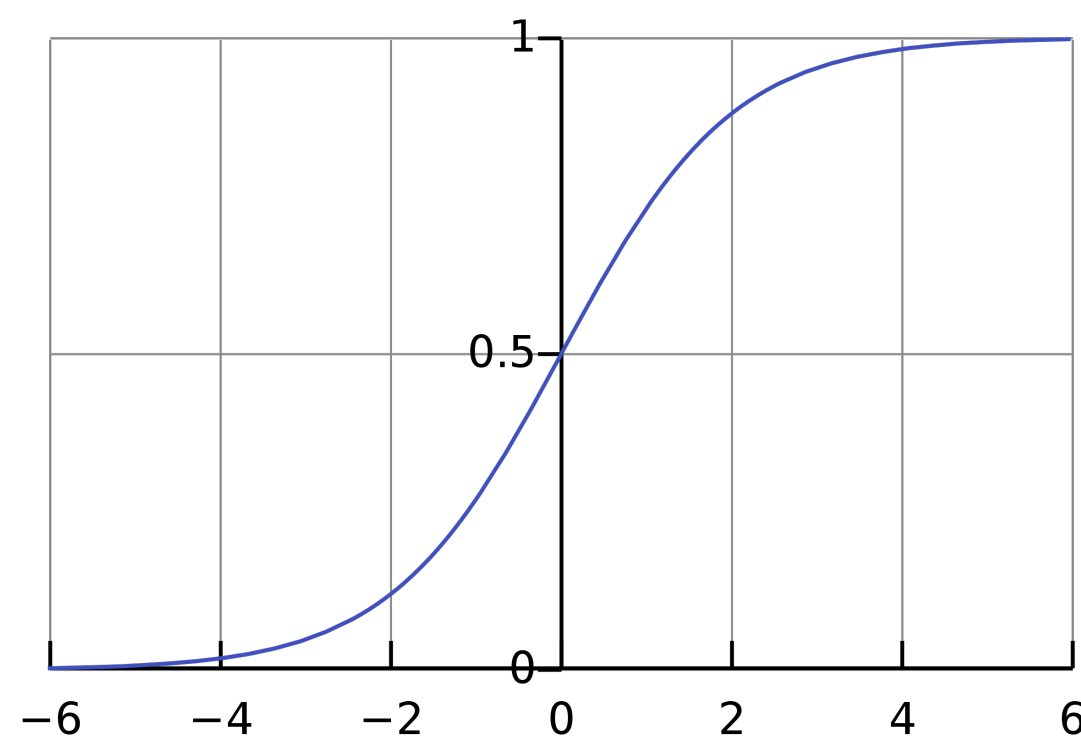


$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = 2\sigma(2x) - 1$$

Problem: derivative “saturates” (nearly 0)
everywhere except near origin

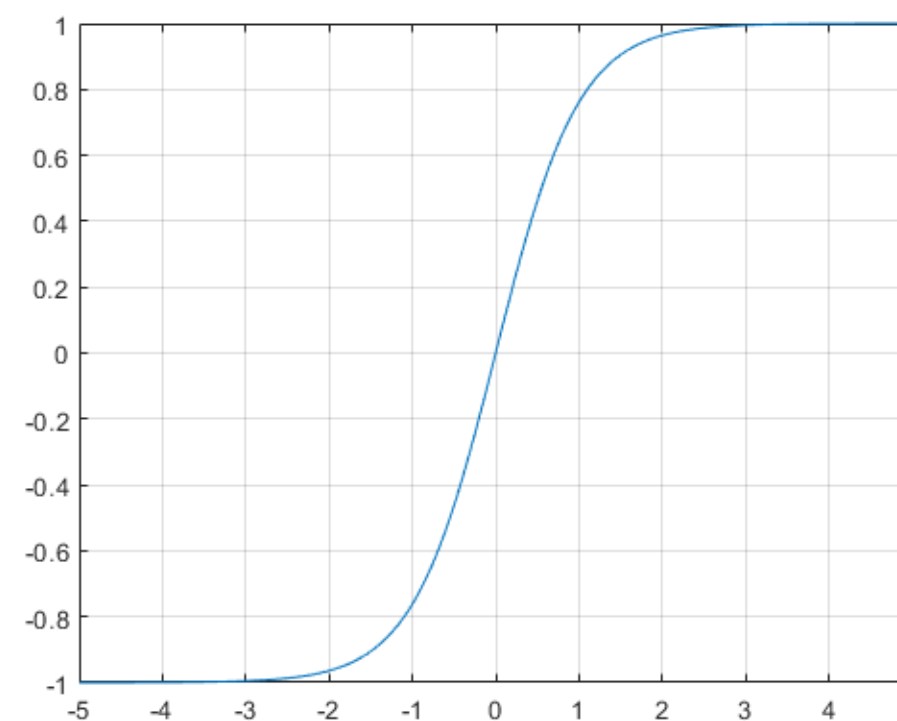
Activation Functions: Hidden Layer

sigmoid



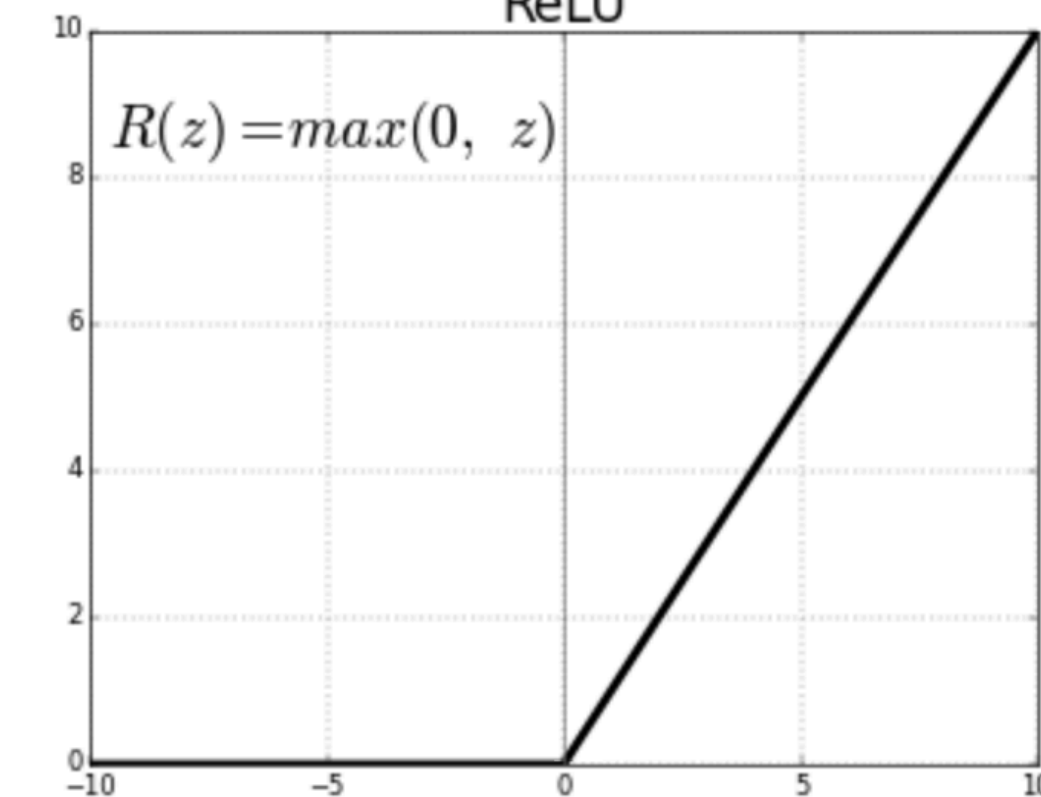
$$\sigma(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{e^x + 1}$$

tanh



$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = 2\sigma(2x) - 1$$

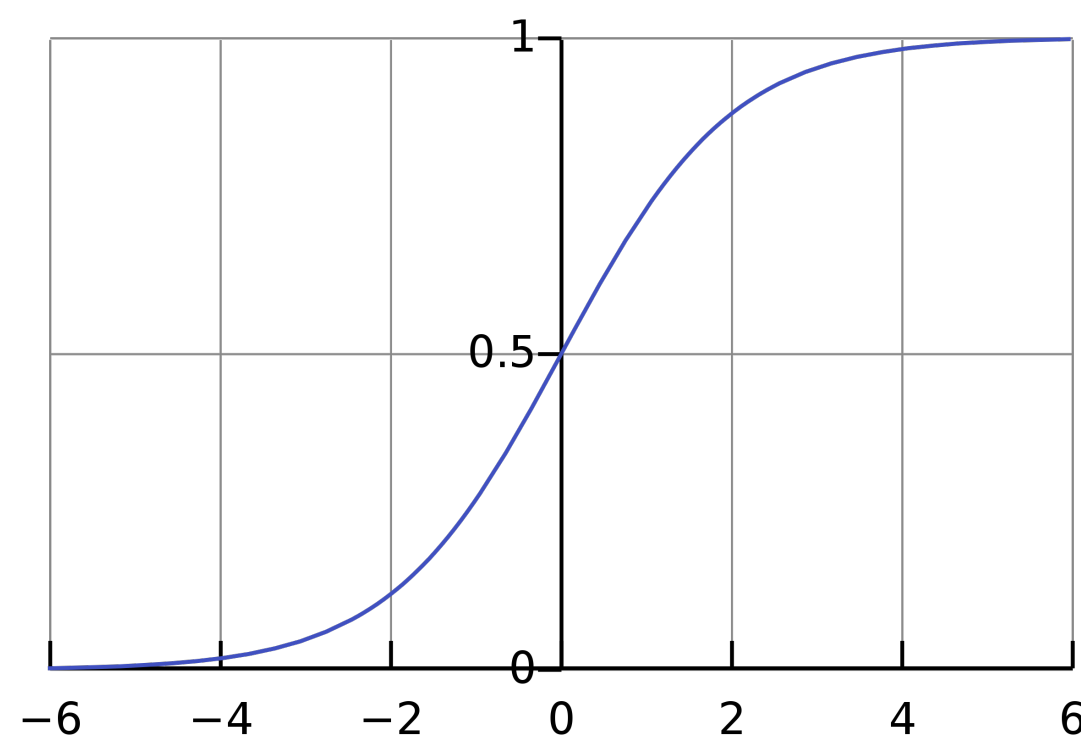
ReLU



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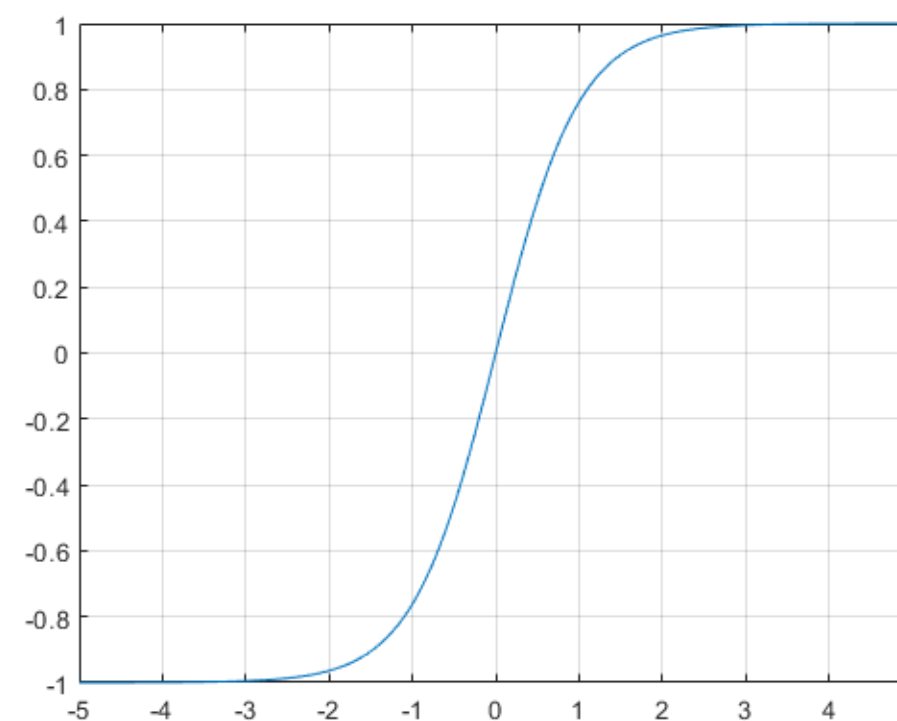
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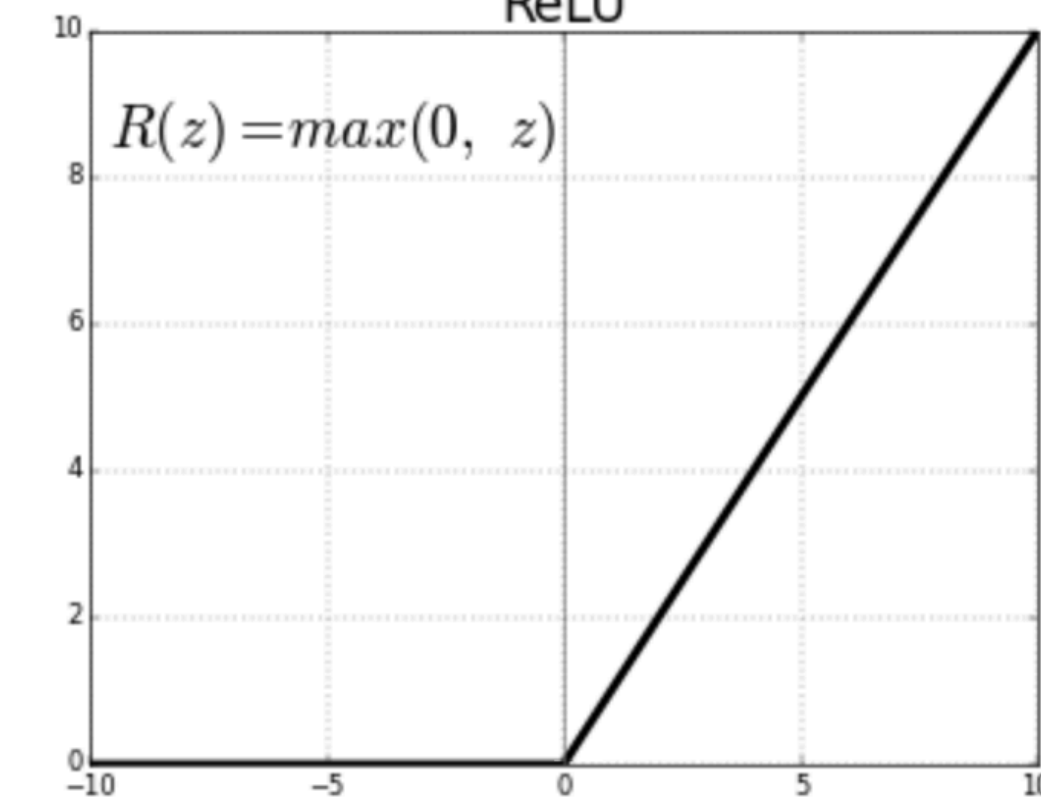
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ReLU



- Use ReLU by default
- Generalizations:
 - Leaky
 - ELU
 - Softplus
 - ...

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Activation Functions: Output Layer

- Depends on the task!
- Regression (continuous output(s)): none!
 - Just use final linear transformation
- Binary classification: sigmoid
 - Also for *multi-label* classification
- Multi-class classification: softmax
 - Terminology: the inputs to a softmax are called *logits*
 - [there are sometimes other uses of the term, so beware]

$$\text{softmax}(x)_i = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

Mini-batch computation

Computing with a Single Input

$$\hat{y} = f_n \left(f_{n-1} \left(\cdots f_2 \left(f_1 (xW^1 + b^1) W^2 + b^2 \right) \cdots \right) W^n + b^n \right)$$

$$x = [x_0 \quad x_1 \quad \cdots \quad x_{n_0}]$$

Shape: $(1, n_0)$

$$b^1 = [b_0^1 \quad b_1^1 \quad \cdots \quad b_{n_1}^1]$$

Shape: $(1, n_1)$

$$W^1 = \begin{bmatrix} w_{00}^1 & w_{10}^1 & \cdots & w_{0n_1}^1 \\ w_{10}^1 & w_{11}^1 & \cdots & w_{1n_1}^1 \\ \vdots & \vdots & \ddots & \vdots \\ w_{n_00}^1 & w_{n_01}^1 & \cdots & w_{n_0n_1}^1 \end{bmatrix}$$

Shape: (n_0, n_1)

n_0 : number of neurons in layer 0 (input)

n_1 : number of neurons in layer 1

Stochastic Gradient Descent

```
initialize parameters / build model
```

```
for each epoch:
```

```
    data = shuffle(data)
```

```
    batches = make_batches(data)
```

```
    for each batch in batches:
```

```
        outputs = model(batch)
```

```
        loss = loss_fn(outputs, true_outputs)
```

```
        compute gradients
```

```
        update parameters
```

Computing with Mini-batches

- Bad idea:

```
for each batch in batches:
    for each datum in batch:
        outputs = model(datum)
        loss = loss_fn(outputs, true_outputs)
        compute gradients
    update parameters
```

Computing with a Batch of Inputs

Computing with a Batch of Inputs

$$\hat{y} = f_n \left(f_{n-1} \left(\cdots f_2 \left(f_1 (XW^1 + b^1) W^2 + b^2 \right) \cdots \right) W^n + b^n \right)$$

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$$X = \begin{bmatrix} x_0^0 & x_1^0 & \cdots & x_{n_0}^0 \\ x_1^0 & x_1^1 & \cdots & x_{n_0}^1 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^n & x_1^n & \cdots & x_{n_0}^n \end{bmatrix}$$

Shape: (n, n_0)

n : batch_size

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Shape: $(1, n_1)$
 Added to each row of XW^1

Note on mini-batches and shape

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- Most modern neural net libraries (e.g. PyTorch) expect the *first* dimension of matrices/tensors to be a batch size
 - Produce a sequence of representations, *for each item* in the batch
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- Two comments:
 - In your code, **annotate every tensor** with a comment saying intended shape
 - When debugging, look at shapes early on!!

Homework 2

Learning Goals

- Understand skip-gram with negative sampling in more detail
 - Compute various derivatives in order to get the gradient of the loss with respect to the parameters
- Learn how to translate math into code, for
 - The model forward pass
 - Gradient computations

Understanding Word2Vec

- Count parameters
- Understand sigmoid, and the role it plays in SGNS
 - Compute its derivative
- Compute the gradient of L_{CE} with respect to parameters
 - Done in stages
 - Uses:
 - Logarithm rules
 - Derivative of logarithm
 - Addition / product / chain rule for derivatives

Implementing Word2Vec

- SGNS will be implemented in raw numpy
- We provide the entire training loop, but various methods that are called need to be filled in
 - Data processing: generating positive and negative samples
 - Model computation: implement the $P(1 | w, c; \theta)$ computation
 - Gradient computation: compute ∇L_{CE} w/r/t each of the relevant parameters

Training Word Vectors

- Finally, you will train word vectors by iterating through the SST training set
- Plot the vectors of a list of words, using PCA for dimensionality reduction
 - We provide all of this code!
- Describe any trends you see in the embeddings

Next Time

- Further abstraction: *computation graph*
- Backpropagation algorithm for computing gradients
 - Using forward/backward API for nodes in a comp graph