Naïve Bayes

LING 572 Advanced Statistical Methods for NLP
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Adapted from F. Xia '18

Naïve Bayes Model

- Naïve Assumption
- Training & Decoding
- Variants
- Issues

ML Questions

- Modeling:
  - What is the model structure?
  - Why is it called Naïve Bayes?
  - What assumptions does it make?
  - What type of parameters are learned?
  - How many parameters?

- Training:
  - How are model parameters learned from data?

- Decoding:
  - How is model used to classify new data?

Probabilistic Model

- Given an instance x with features $f_1 \ldots f_k$,
  - Find the class with highest probability

  - Formally, $x = (f_1, f_2, \ldots, f_k)$, find $c^* = \arg\max_c p(c \mid x)$

  - Applying Bayes' Rule:
    - $$c^* = \arg\max_c \frac{p(x \mid c)p(c)}{p(x)}$$

  - Maximizing:
    - $c^* = \arg\max_c p(x \mid c)p(c)$

Naïve Bayes Model

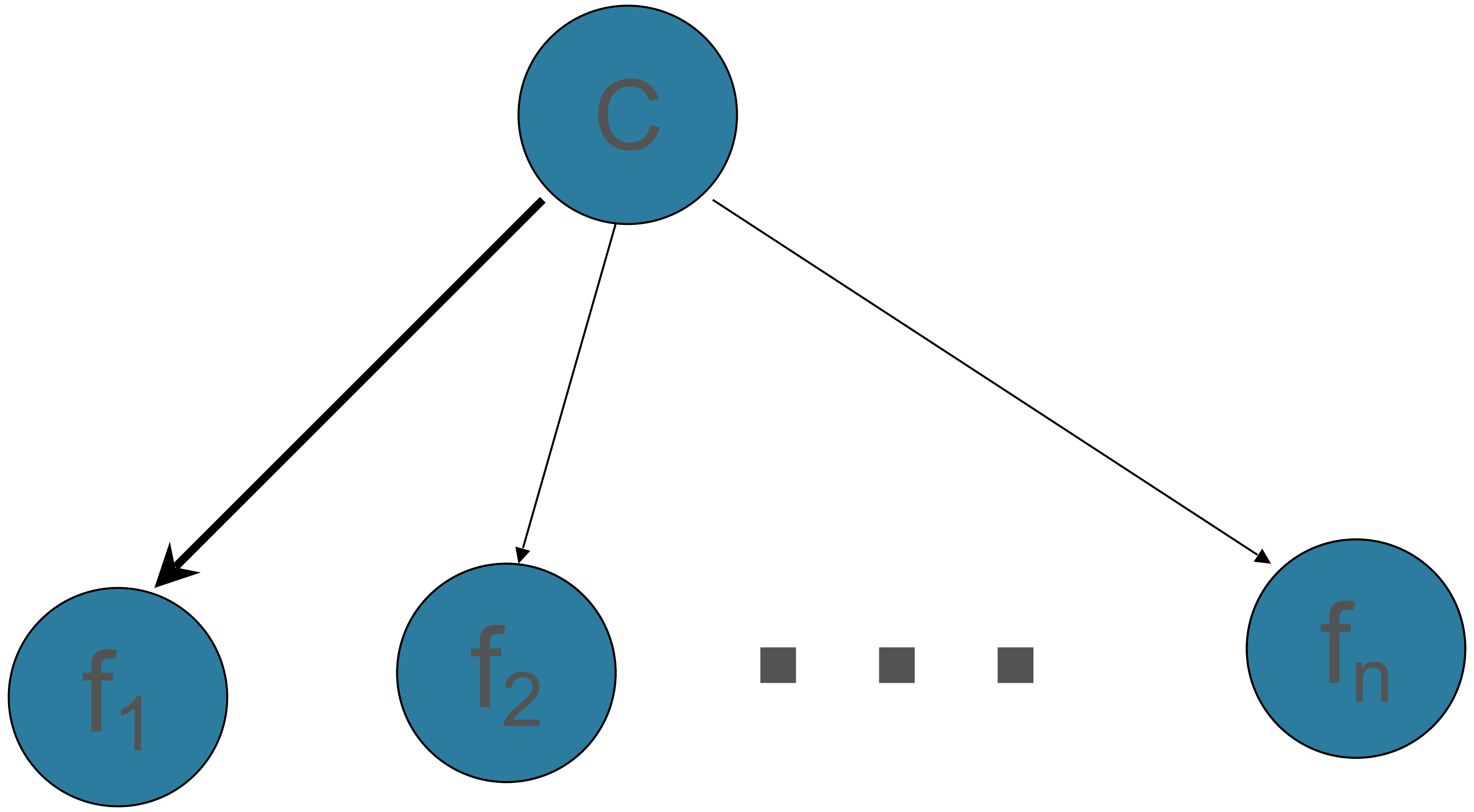
- So far just Bayes' Rule
- Key question: How do we handle/combine features?
- Consider just P(x|c)

$$p(x \mid c) = p(f_1, f_2, \ldots, f_k \mid c) = \prod_j p(f_j \mid c, f_1, \ldots, f_{j-1})$$

- Can we simplify? (Remember n-grams)
- Assume conditional independence

$$p(x \mid c) = p(f_1, f_2, \ldots, f_k \mid c) = \prod_j p(f_j \mid c)$$

Naïve Bayes Model



Assumption: each f_i is conditionally independent from f_j given C.

Model parameters

- Choose

$c^* = \arg\max_c P(c \mid x)$

$= \arg\max_c P(c) P(x \mid c) / P(x)$

$= \arg\max_c P(c) P(x \mid c)$

$= \arg\max_c P(c) \prod_k P(f_k \mid c)$

- Two types of model parameters:
  - Class prior: P(c)
  - Conditional probability: $P(f_k \mid c)$

- The number of model parameters:
  - |priors|+|conditional probabilities|
  - |C| + |F|*|C|
    - |C|+|V|*|C|, if features are words in vocabulary V

|C| is the number of classes, |F| is the number of features, |V| is the number of features.

Training stage: Estimating parameters Θ

- Maximum likelihood estimation (ML):

$$\theta^* = \arg\max_{\theta} p(\text{training data} \mid \theta)$$

- Class prior:

$$p(c_i) = \frac{\text{count}(c_i)}{\sum_j \text{count}(c_j)}$$

- Conditional prob:

$$p(f_j \mid c_i) = \frac{\text{count}(f_j, c_i)}{\text{count}(c_i)}$$

Training

- MLE issues?
  - What's the probability of a feature not seen with a c_i?
    - 0?
    - What happens then?

- Solutions?
  - Smoothing
    - Laplace smoothing, Good-Turing, Witten-Bell
    - Interpolation, Backoff….

What are Zero Counts?

- Some of those zeros are really zeros...
  - Things that really can't or shouldn't happen.

- On the other hand, some of them are just rare events.
  - If the training corpus had been a little bigger, they would have had a count (probably a count of 1!).

- Zipf's Law (long tail phenomenon):
  - A small number of events occur with high frequency
  - A large number of events occur with low frequency
  - You can quickly collect statistics on the high frequency events
  - You might have to wait an arbitrarily long time to get valid statistics on low frequency events

Laplace Smoothing (add-one smoothing)

- Pretend you saw outcome one more than you actually did.

- Suppose X has K possible outcomes, and the counts for them are $n_1, \ldots, n_K$, which sum to N.

  - Without smoothing: $P(X = i) = n_i / N$
  - With Laplace smoothing: $P(X = i) = \dfrac{n_i + 1}{N + K}$

Testing stage

- MAP (maximum a posteriori) decision rule:

- Given our model and an instance $x = < f_1, .., f_d >$

classify (x)

$= \text{classify}(f_1, .., f_d)$

$= \text{argmax}_c\ P(c|x)$

$= \text{argmax}_c\ P(x|c)\ P(c)$

$= \text{argmax}_c\ P(c) \prod_k P(f_k \mid c)$

Naïve Bayes for text classification

Features

- Features: bag of words (word order information is lost)

- Number of feature types: 1
- Number of features: |V|
- Features: w_t, where $t \in \{1,2,\ldots|V|\}$

Issues

- Is w_t a binary feature?

- Are absent features used for calculating $p(d_i \mid c_j)$?

Two Naïve Bayes Models (McCallum and Nigam, 1998)

- Multivariate Bernoulli event model
  (a.k.a. binary independence model)
  - All features are binary: the number of times a feature occurs in an instance is ignored.
  - When calculating p(d | c), all features are used, including the absent features.

- Multinomial event model: “unigram LM”

Multivariate Bernoulli event model

Bernoulli distribution

- Bernoulli trial: a statistical experiment having exactly two mutually exclusive outcomes, each with a constant probability of occurrence:
  - Ex: toss a coin

- Bernoulli distribution: has exactly two mutually exclusive outcomes: $P(X=1)=p$ and $P(X=0)=1-p$.

Multivariate Bernoulli Model

- Each document:
  - Result of |V| independent Bernoulli experiments
  - i.e., for each word in the vocabulary: does this word appear in the document?

- Another way to look at this: (to be consistent with the general NB model)
  - Each word in the voc corresponds to two features: $w_k, \overline{w_k}$
  - In any document, either w_k or $\overline{w_k}$ is present (and not both);
    - that is, it is always the case that exactly |V| features will be present in any document.

Training stage

MLE estimate:

$$P(w_t \mid c_i) = \frac{\text{count}(w_t, c_i)}{\text{count}(c_i)}$$

$$P(c_i) = \frac{\text{count}(c_i)}{\sum_j \text{count}(c_j)}$$

With add-one smoothing:

$$P(w_t \mid c_i) = \frac{1 + \text{count}(w_t, c_i)}{2 + \text{count}(c_i)}$$

$$P(c_i) = \frac{1 + \text{count}(c_i)}{|C| + \sum_j \text{count}(c_j)}$$

Notation used in the paper

$$P(w_t \mid c_j) = \frac{1 + \text{count}(w_t, c_j)}{2 + \text{count}(c_j)}$$

$$B_{it} = \begin{cases} 1 & w_t \text{ appears in } d_i \\ 0 & \text{otherwise} \end{cases}$$

$$P(c_j \mid d_i) = \begin{cases} 1 & d_i \text{ has label } c_j \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{\theta}_{w_t|c_j} = P(w_t \mid c_j; \theta) = \frac{1 + \sum_{i=1}^{|D|} B_{it} P(c_j \mid d_i)}{2 + \sum_{i=1}^{|D|} P(c_j \mid d_i)}$$

Testing stage

$$\text{classify}(d_i) = \arg\max_c P(c)P(d_i \mid c)$$

$$P(d_i \mid c) = \prod_k P(f_k \mid c)$$

$$= \prod_{w_k \in d_i} P(w_k \mid c) \prod_{w_k \notin d_i} P(\overline{w_k} \mid c)$$

$$= \prod_{w_k \in d_i} P(w_k \mid c) \prod_{w_k \notin d_i} 1 - P(w_k \mid c)$$

Multinomial event model

Multinomial distribution

- Possible outcomes = $\{w_1, w_2, \ldots, w_{|v|}\}$
- A trial for each word position: $P(\text{CurWord} = w_i) = p_i$ (with $\sum_i p_i = 1$)
- Let X_i be the number of times that w_i is observed in the document.

$$P(X_1 = x_1, \ldots, X_v = x_v) = p_1^{x_1} \cdots p_v^{x_v} \frac{n!}{x_1! \cdots x_v!}$$

$$= n! \prod_k \frac{p_k^{x_k}}{x_k!}$$

An example

- Suppose
  - the voc, V, contains only three words: a, b, and c.
  - a document, d_i, contains only 2 word tokens
  - For each position, P(w=a)=p1, P(w=b)=p2 and P(w=c)=p3.

- What is the prob that we see “a” once and “b” once in d_i?

An example (cont)

- 9 possible sequences: aa, ab, ac, ba, bb, bc, ca, cb, cc.
- The number of sequences with one "a" and one "b" (ab and ba): $n!/(x_1!...x_v!)$
- The prob of the sequence "ab" is p_1*p_2,

  so is the prob of the sequence "ba".

- So the prob of seeing "a" once and "b" once is: $$n!\prod_k \frac{p_k^{x_k}}{x_k!} = 2p_1p_2$$

Multinomial event model

- A document is seen as a sequence of word events, drawn from the vocabulary V.
- N_{it}: the number of times that w_t appears in d_i
- Modeling: multinomial distribution:

$$P(d_i \mid c_j) = P(|d_i|)\,|d_i|!\prod_{t=1}^{|V|}\frac{P(w_t \mid c_j)^{N_{it}}}{N_{it}!}$$

Training stage for multinomial model

$$P(c_j \mid d_i) = \begin{cases} 1 & d_i \text{ has label } c_j \\ 0 & \text{otherwise} \end{cases}$$

$$P(w_t \mid c_j) = \frac{1 + \sum_{i=1}^{|D|} N_{it} P(c_j \mid d_i)}{|V| + \sum_{s=1}^{|V|} \sum_{i=1}^{|D|} N_{is} P(c_j \mid d_i)}$$

Testing stage

$$\text{classify}(d_i) = \arg\max_c P(c)P(d_i \mid c)$$

$$P(d_i \mid c) = P(|d_i|)\,|d_i|!\prod_{k=1}^{|V|} \frac{P(w_k \mid c)^{N_{ik}}}{N_{ik}!}$$

$$\text{classify}(d_i) = \arg\max_c P(c)\prod_{k=1}^{|V|} P(w_k \mid c)^{N_{ik}}$$

Two models

- Multi-variate Bernoulli event model: treat features as binary; each trial corresponds to a word in the voc.

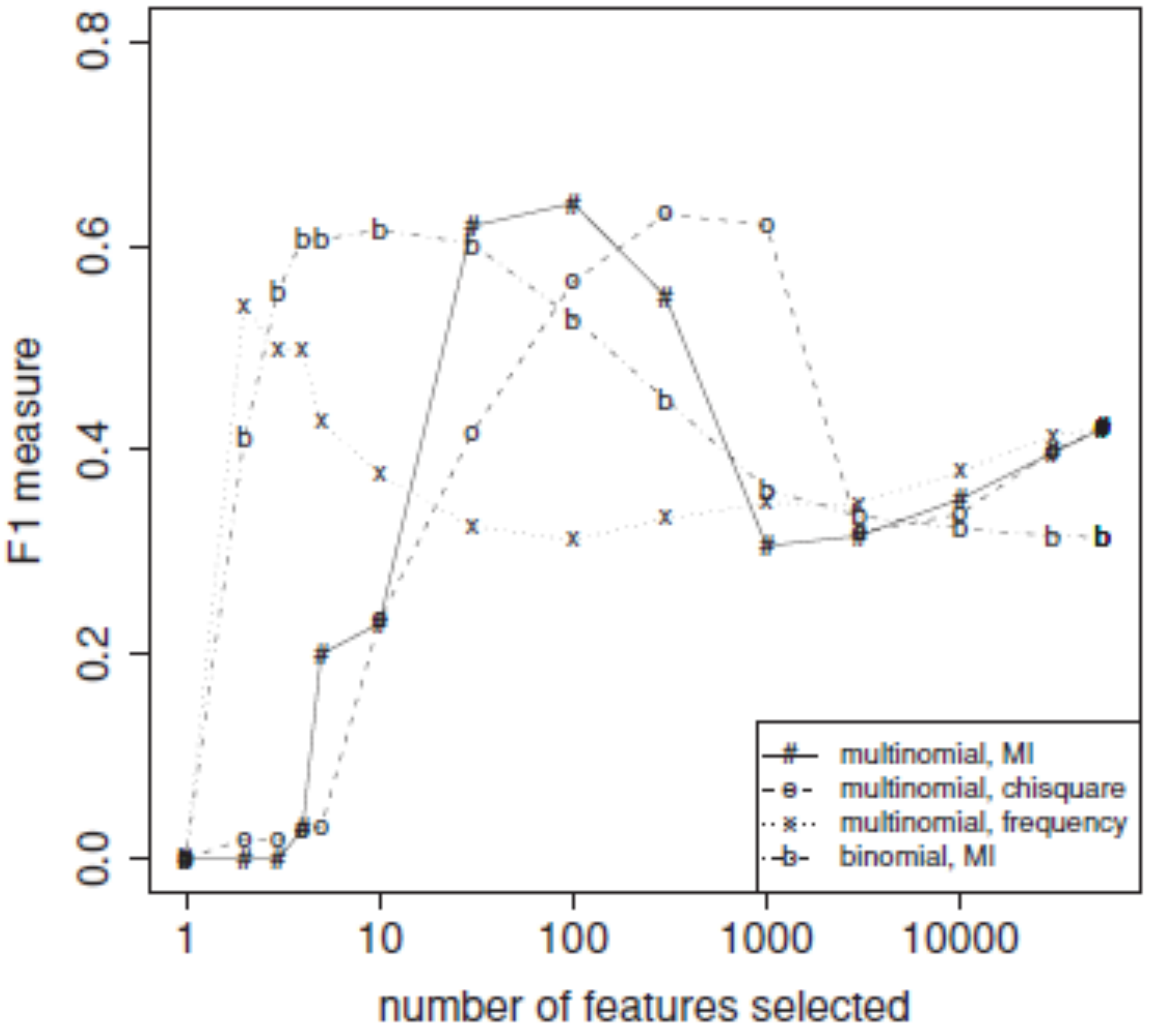
- Multinomial event model: treat features as non-binary; each trial corresponds to a word position in the document.

Which model is better?

- (McCallum and Nigam, 1998): Multinomial event model usually beats the Bernoulli event model

- Chapter 13 in (Manning et al., 2008): The Bernoulli model
  - is particularly robust w.r.t. concept shift
  - is more sensitive to noisy features (requiring feature selection)
  - peaks early for feature selection (see fig)
  - works well for shorter documents

From (Manning et al., 2008)



► **Figure 13.8** Effect of feature set size on accuracy for multinomial and Bernoulli models.

Two models (cont)

	Multivariate Bernoulli	Multinomial
Features	Binary: present or absent	Real-valued: the occurrence
Each trial	Each word in the voc	Each word position in the doc
$P(c_i)$	$\frac{1 + \text{count}(c_i)}{\lvert C \rvert + \sum_j \text{count}(c_j)}$	$\frac{1 + \text{count}(c_i)}{\lvert C \rvert + \sum_j \text{count}(c_j)}$
$P(w_t \lvert c_j)$	$P(w_t \lvert c_j) = \frac{1 + \text{count}(w_t, c_j)}{2 + \text{count}(c_j)}$	$\frac{1 + \sum_{i=1}^{\lvert D \rvert} N_{it} P(c_j \lvert d_i)}{\lvert V \rvert + \sum_{s=1}^{\lvert V \rvert} \sum_{i=1}^{\lvert D \rvert} N_{is} P(c_j \lvert d_i)}$
classify(d_i)	$\arg\max_c \prod_{w_k \in d_i} P(w_k \lvert c) \prod_{w_k \notin d_i} 1 - P(w_k \lvert c)$	$\arg\max_c P(c) \prod_{k=1}^{\lvert V \rvert} P(w_k \lvert c)^{N_{ik}}$

Summary of Naïve Bayes

- It makes a strong independence assumption: all the features are conditionally independent given the class.
- It generally works well despite the strong assumption. Why?
- “Correct estimation implies accurate prediction, but accurate prediction does not imply correct estimation.”
- Both training and testing are simple and fast.

Summary of Naïve Bayes (cont)

- Strengths:
  - Simplicity (conceptual)
  - Efficiency at training
  - Efficiency at testing time
  - Handling multi-class
  - Scalability
  - Output topN

- Weaknesses:
  - Theoretical validity: the independence assumption
  - Prediction accuracy: might not be as good as MaxEnt etc.

Additional slides

Spam detection

- Task: Given an email message, classify it as ‘spam’ or ‘not spam’
  - Form of automatic text categorization

- Features?

Doc1

Western Union Money Transfer office29@yahoo.com.ph

One Bishops Square Akpakpa E1 6AO, Cotonou

Benin Republic

Website: http://www.westernunion.com/ info/selectCountry.asP

Phone: +229 99388639

Attention Beneficiary,

This to inform you that the federal ministry of finance Benin Republic has started releasing scam victim compensation fund mandated by United Nation Organization through our office.

I am contacting you because our agent have sent you the first payment of $5,000 for your compensation funds total amount of $500 000 USD (Five hundred thousand united state dollar)

We need your urgent response so that we shall release your payment information to you.

You can call our office hot line for urgent attention(+22999388639)

Doc2

Hello! my dear. How are you today and your family? I hope all is good, kindly pay Attention and understand my aim of communicating you today through this Letter, My names is Saif al-Islam al-Gaddafi the Son of former Libyan President. i was born on 1972 in Tripoli Libya, By Gaddafi’s second wive.

I want you to help me clear this fund in your name which i deposited in Europe please i would like this money to be transferred into your account before they find it. the amount is 20.300,000 million GBP British Pounds sterling through a …

What are good features?

- Words:
  - E.g., account, money, urgent

- Particular types of words/phrases:
  - large amount of money
  - Foreign/fake address/email/phone number

- Errors:
  - Spelling, grammatical errors

- …