CKY Parsing

LING 571 — Deep Processing Techniques for NLP

Shane Steinert-Threlkeld

Announcements

- HW1 grades coming soon
- Reference code in hw1/ref on dropbox
  - Will try to give reference implementations for many assignments (not all, where it is less applicable)
  - Please do not distribute them!
- HW2: only need the three operations we discussed on the slides
  - NB: order of application matters!

Type Hinting in Python

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- Supported in ≥3.6

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from nltk.grammar import Production

def fix_hybrid_production(hybrid_prod: Production) -> list[Production]:
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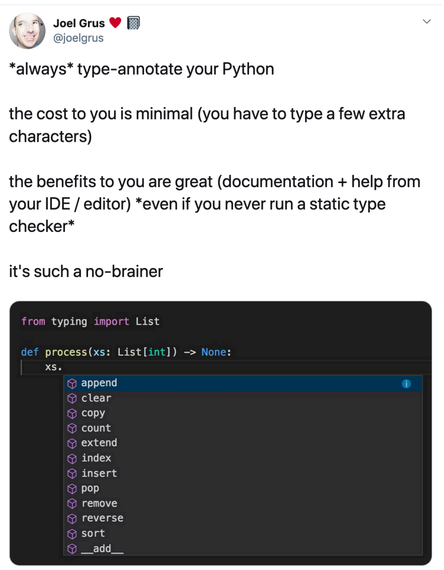
- https://peps.python.org/pep-0483/
- Tools for your editor:
  - Mypy
  - Pyright: a great default to use!



Roadmap

- CKY Parsing Algorithm
  - Dynamic programming structure
  - Algorithm and example
  - Analysis & Discussion

- Next time: Probabilistic CFGs
  - Motivation: Ambiguity
  - Augmenting CFGs with probabilities

CKY Parsing

- (Relatively) efficient parsing algorithm
- Based on tabulating substring parses to avoid repeat work
- Approach:
  - Use CNF Grammar
  - Build an $(n + 1) \times (n + 1)$ matrix to store subtrees
    - Upper triangular portion
  - Incrementally build parse spanning whole input string

CKY

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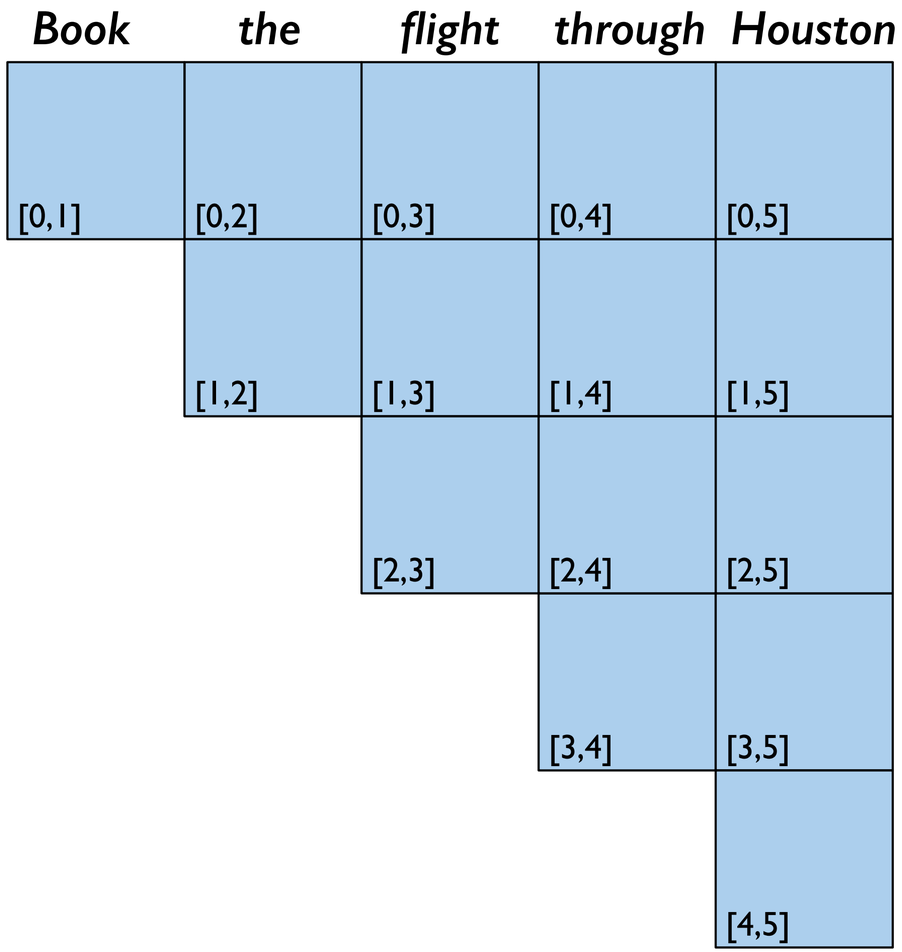
  - if [0, *n*] contains the start symbol, the sentence is ***accepted***

CKY Matrix


Book
the
flight
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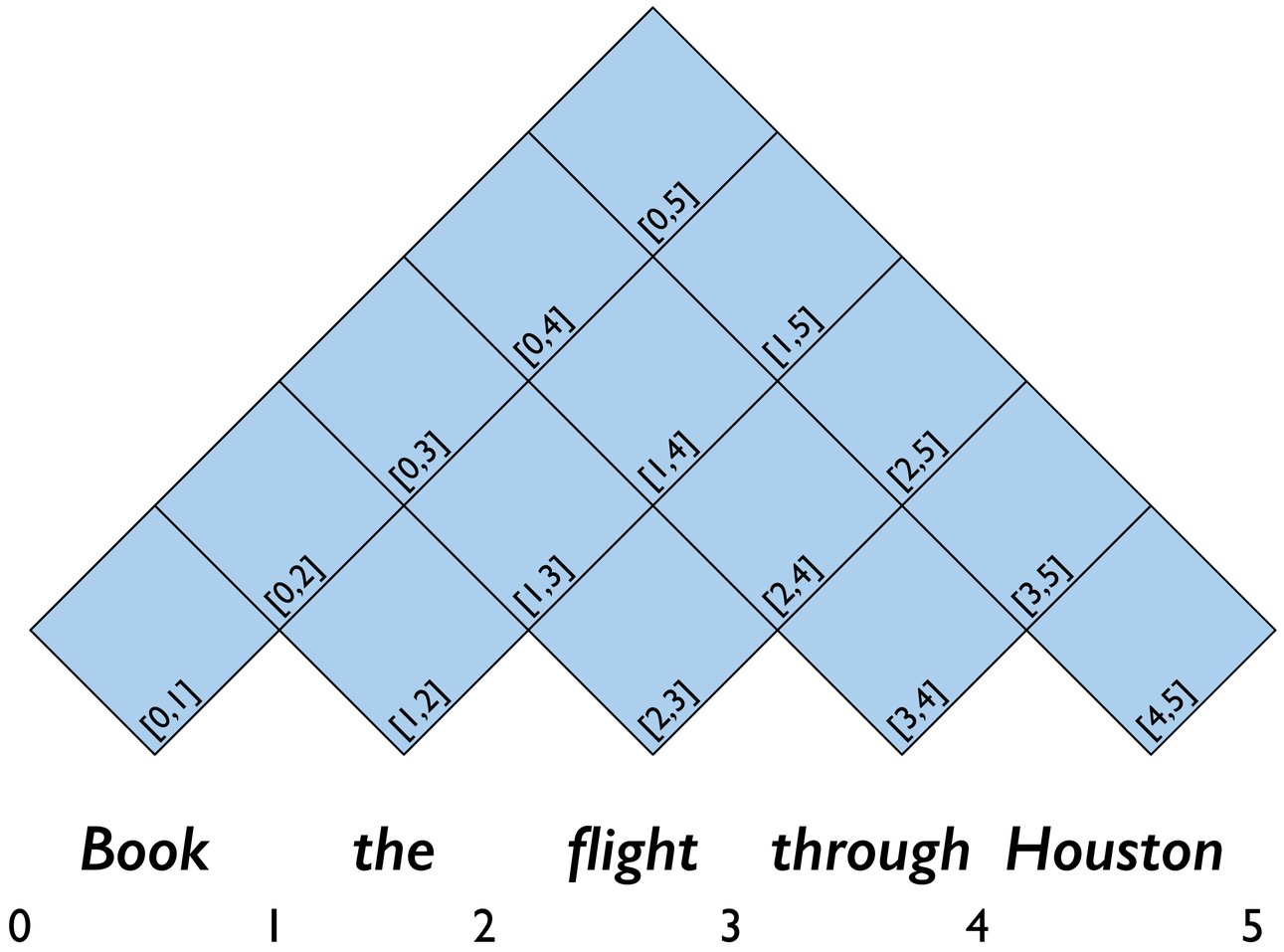
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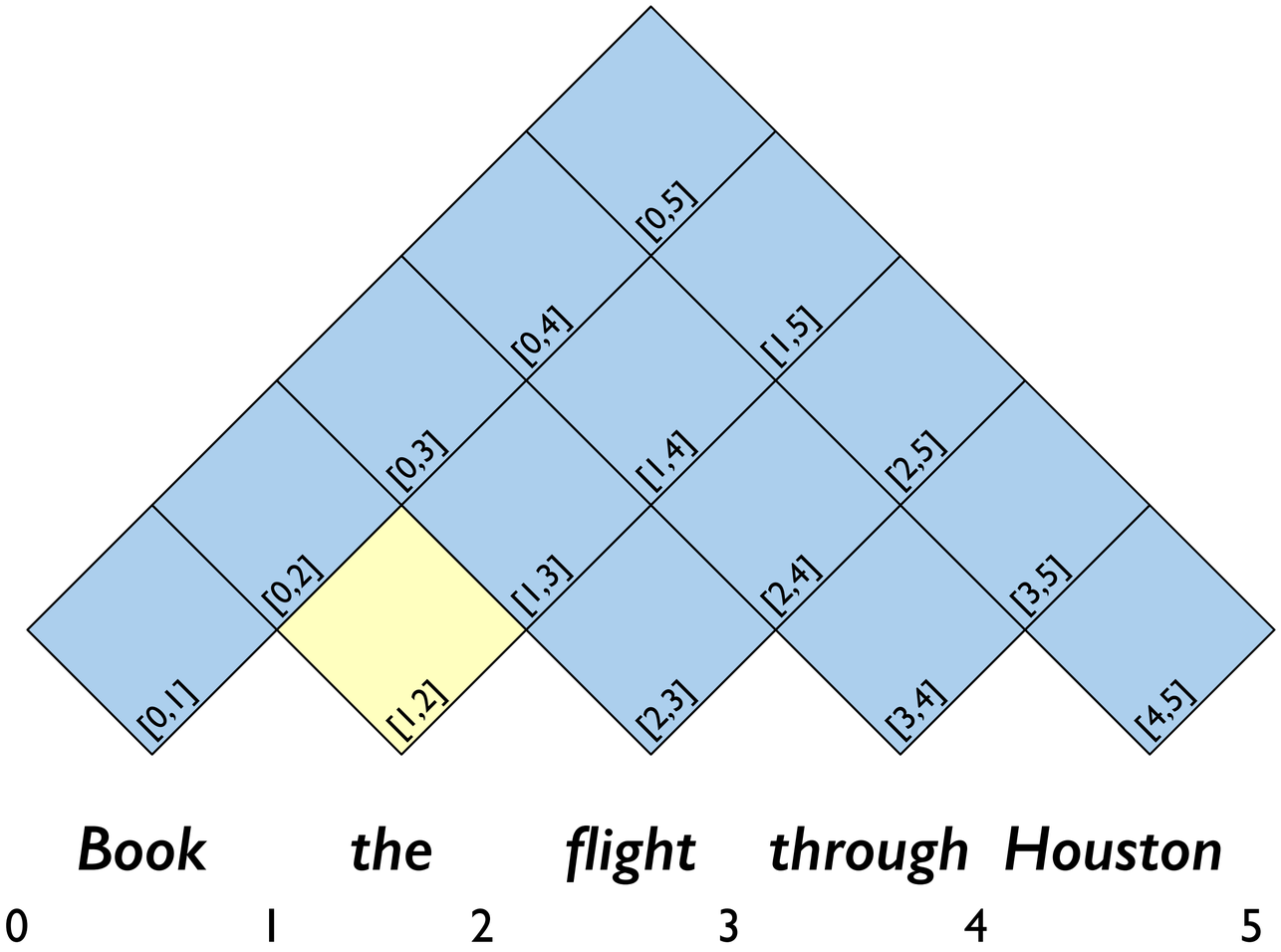
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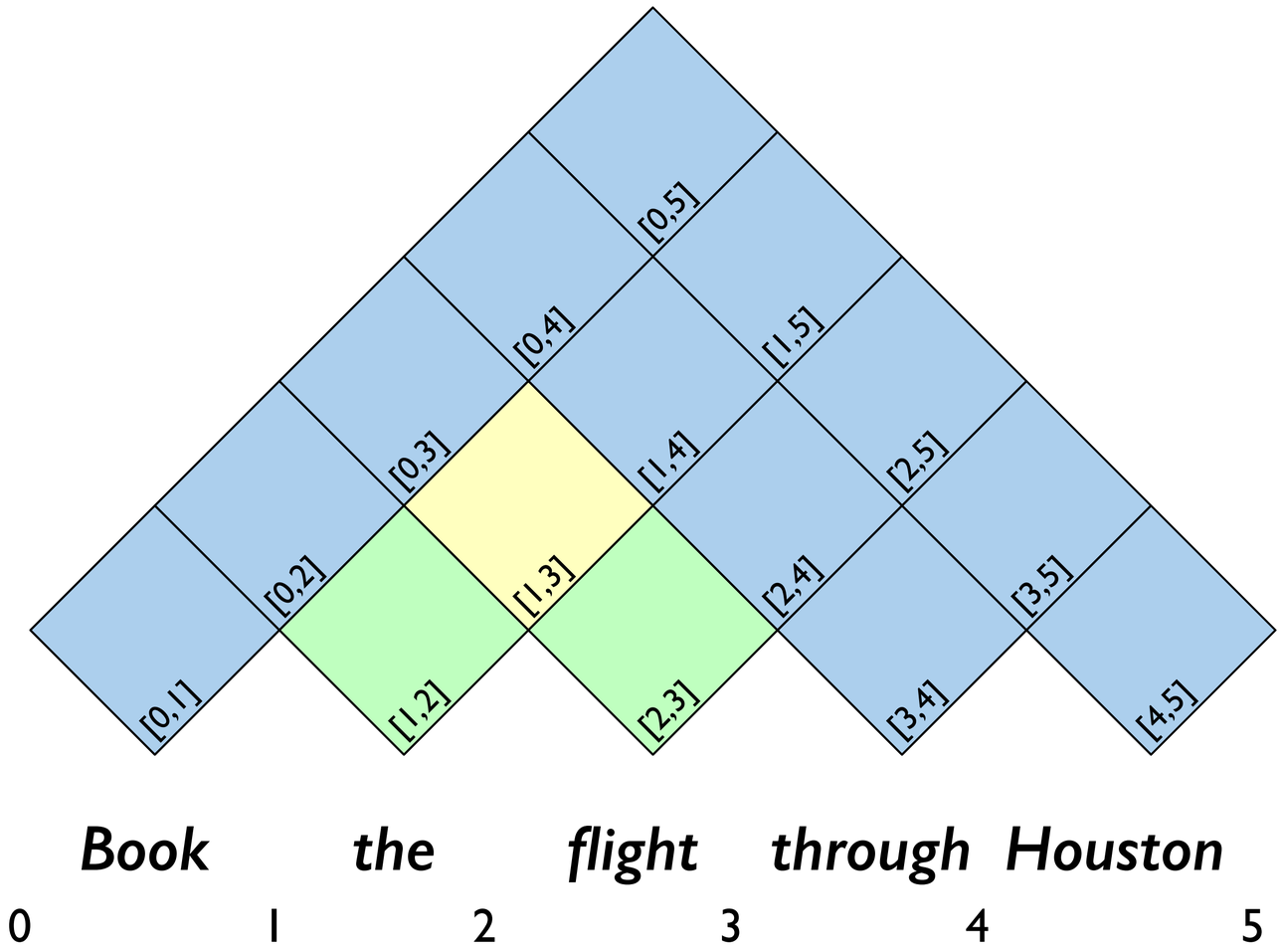
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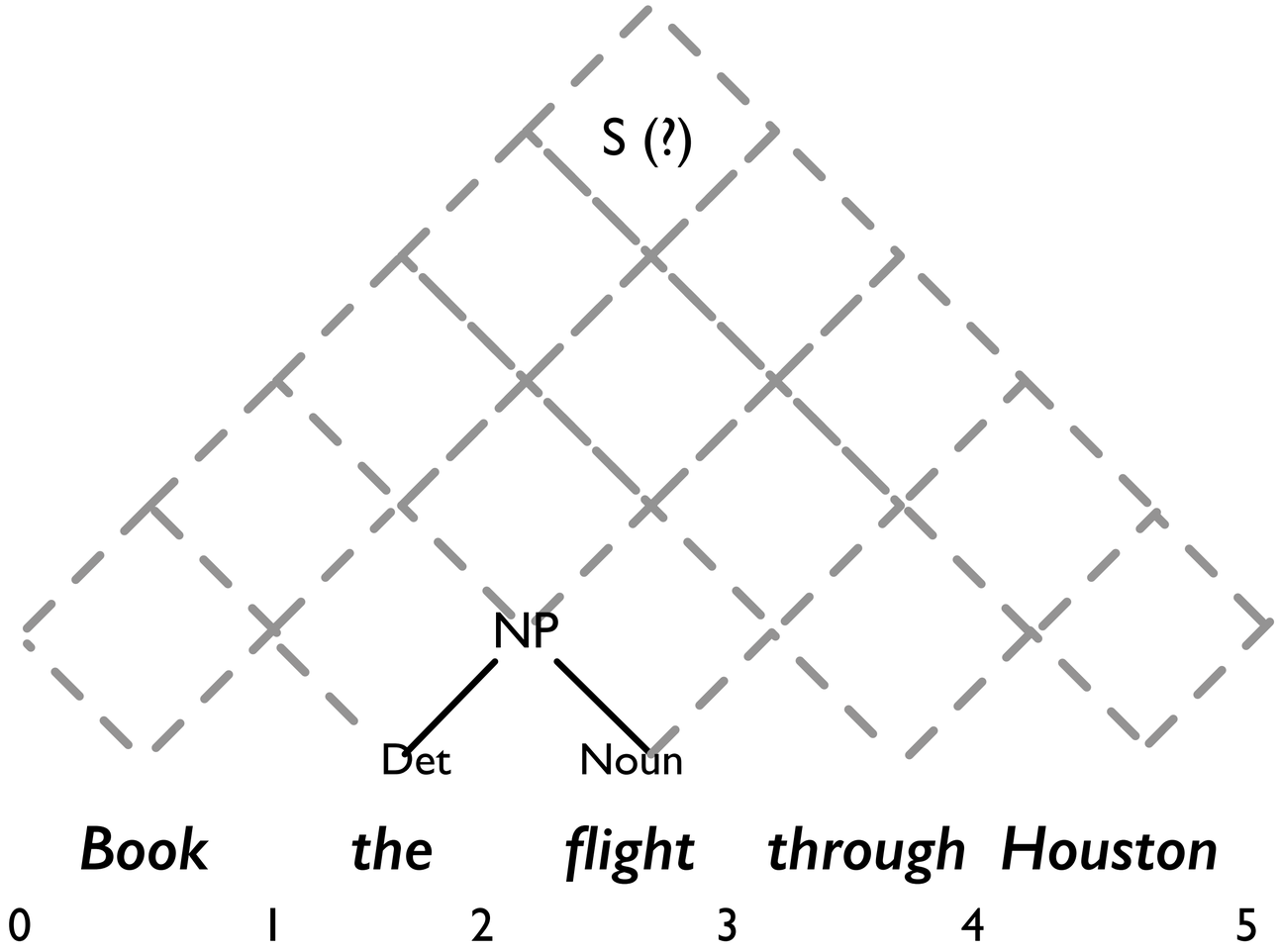
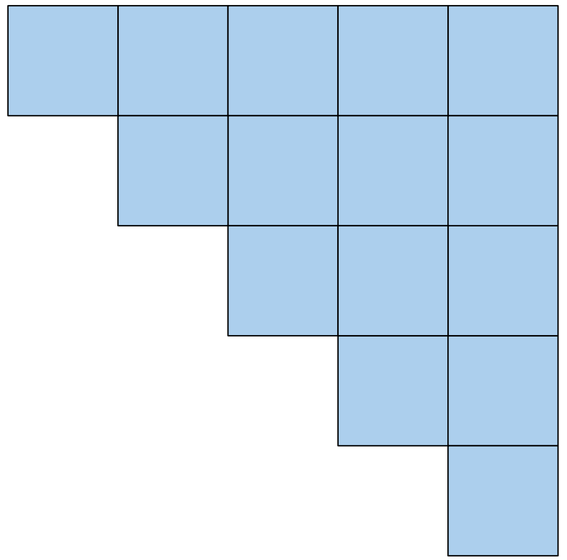


Dynamic Programming in CKY

- Key idea:
  - for $i < k < j$
  - …and a parse spanning substring [i, j]
  - There is a k such that there are parses spanning [i, k] and [k, j]
  - We can construct parses for whole sentences by building from these partial parses
- So to have a rule $A \rightarrow B\ C$ in [i, j]
  - Must have B in [i, k] and C in [k, j] for some $i < k < j$
  - CNF forces this for all $j > i + 1$

CKY

- Table fills:



CKY

- Table fills:
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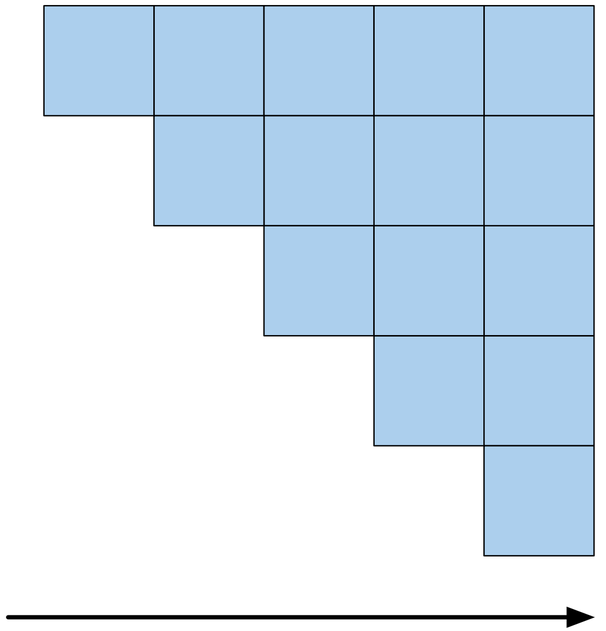

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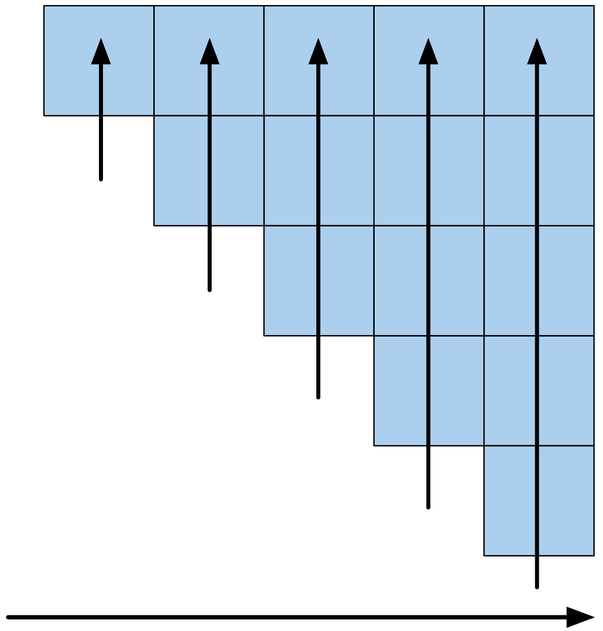

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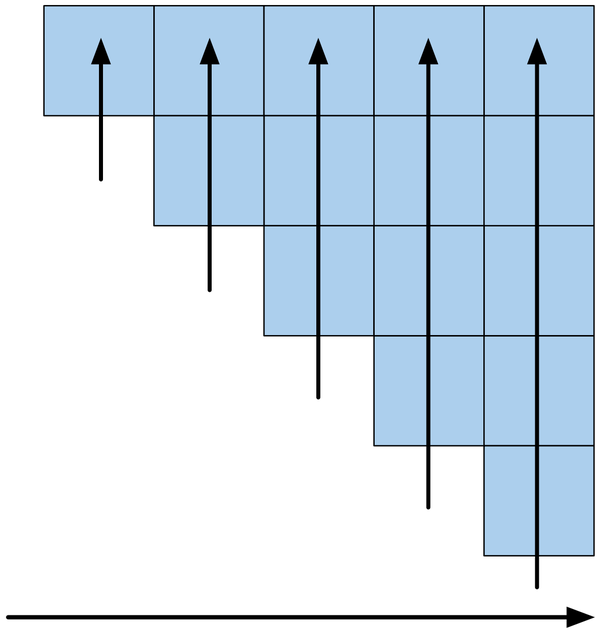

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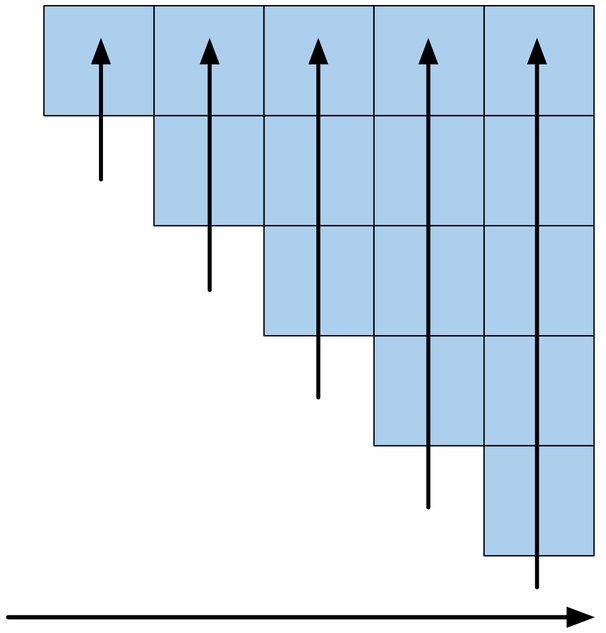

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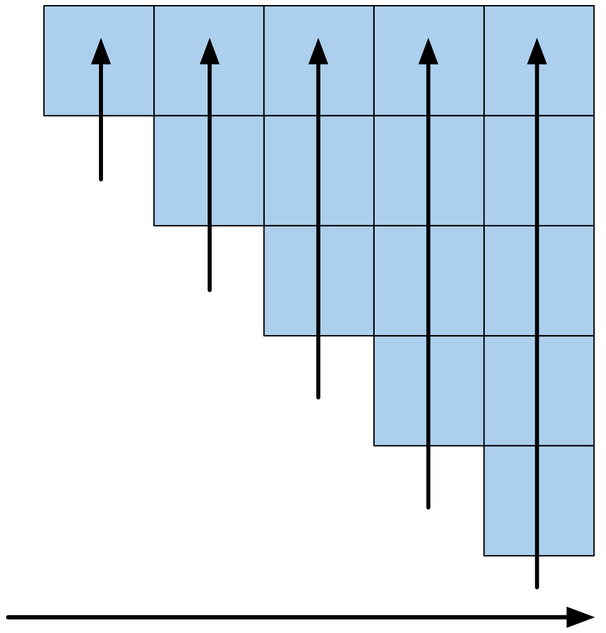

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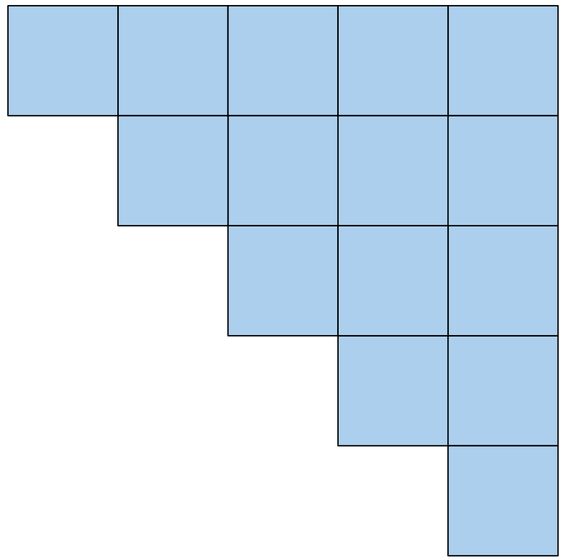

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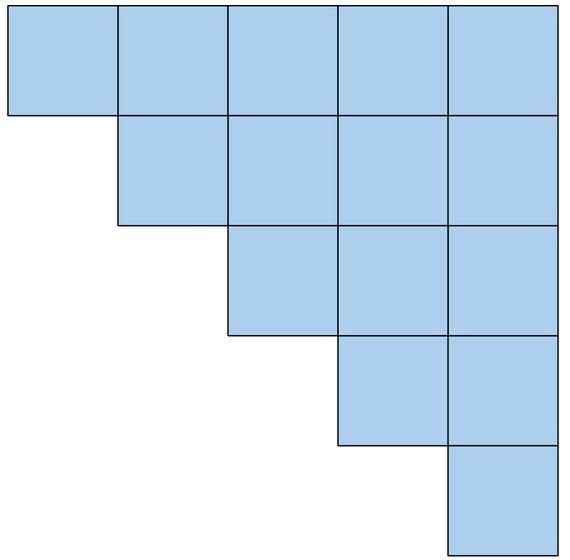

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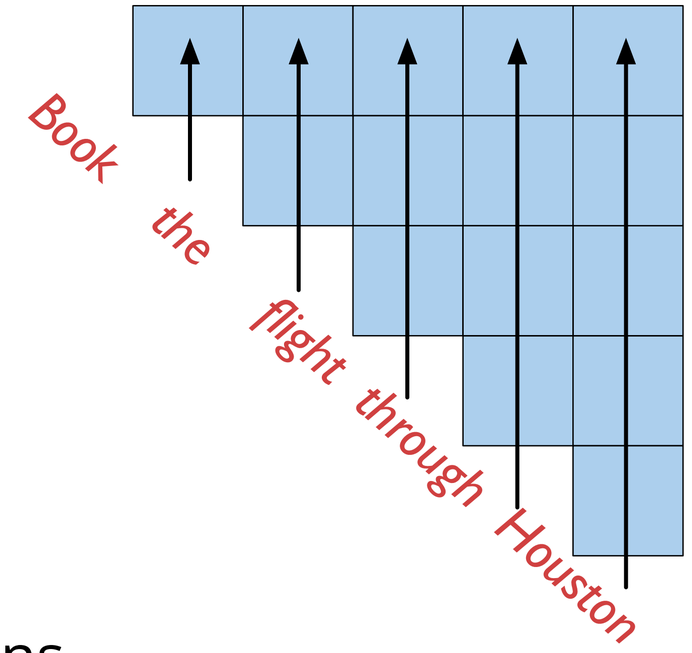

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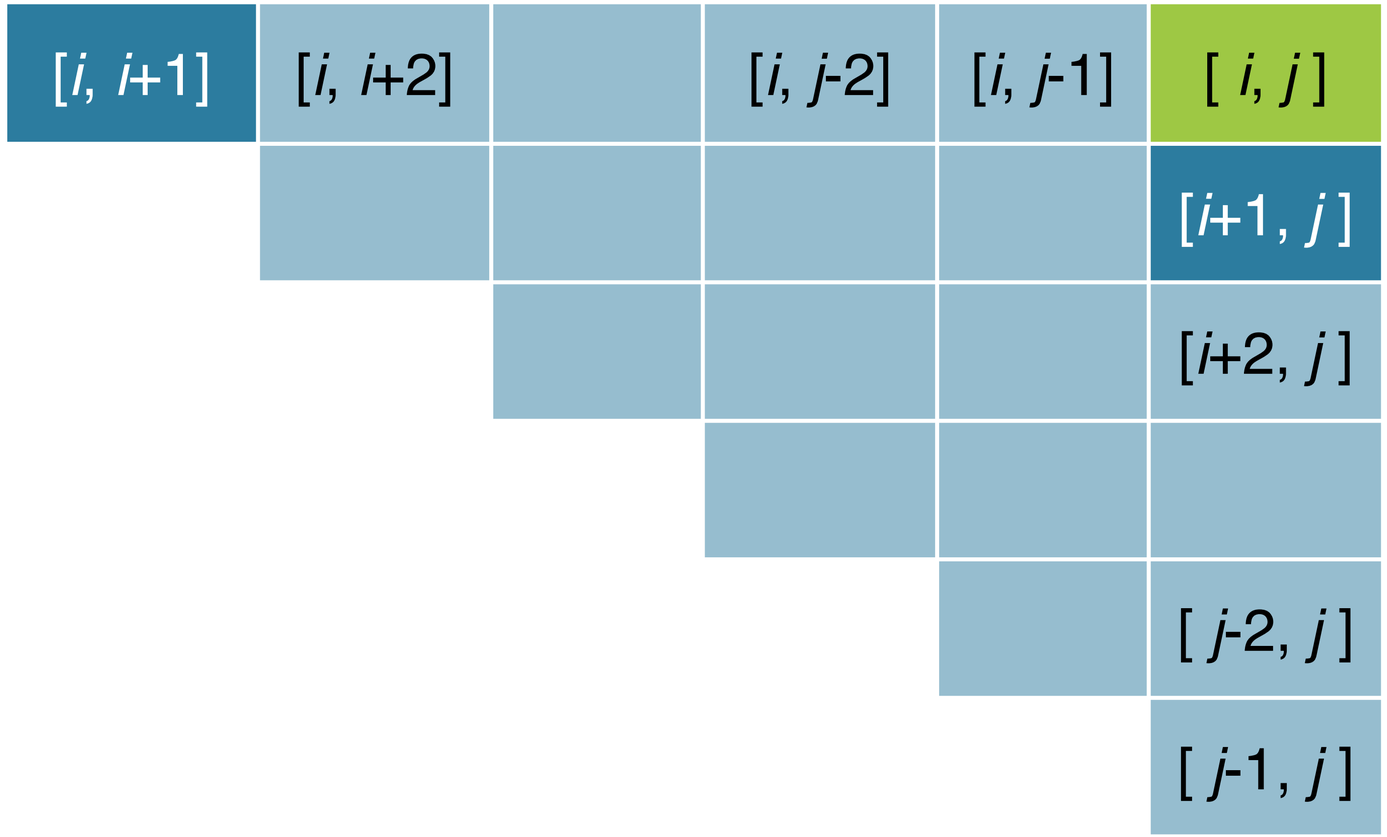
Left-to-right, bottom-to-top

Sliding k as the pivot for child nodes

Filling the cell with set of applicable rules

Filling Cell [i, j]



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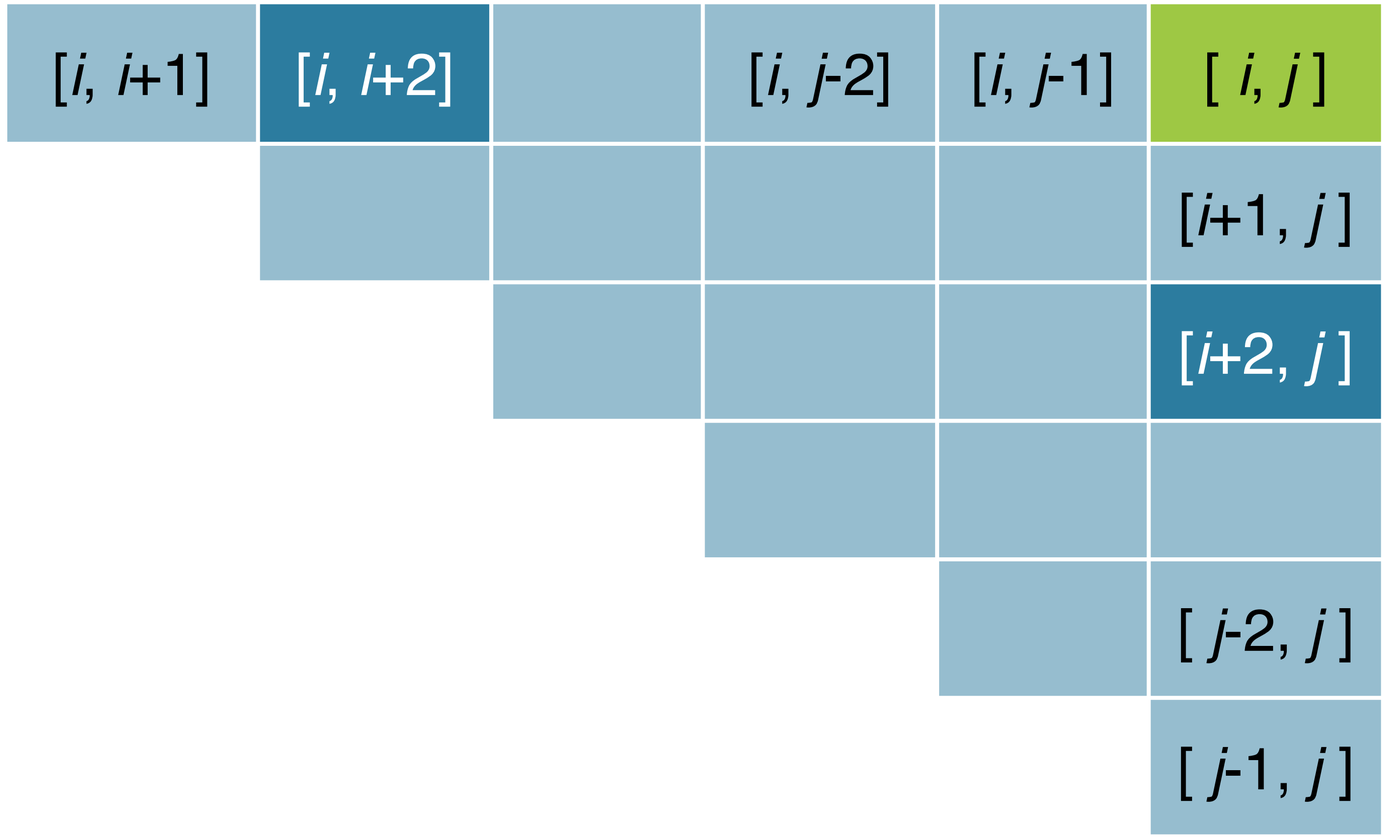

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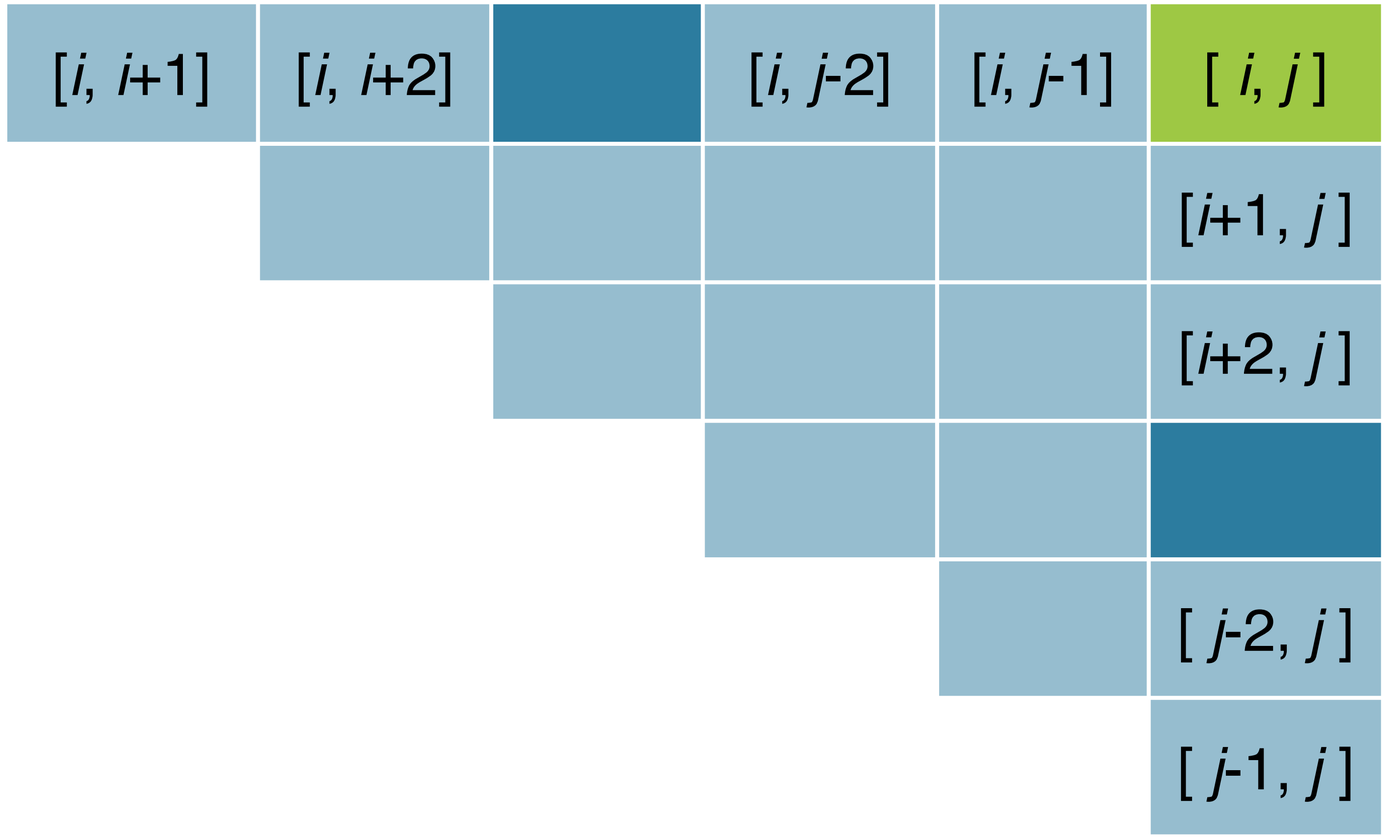

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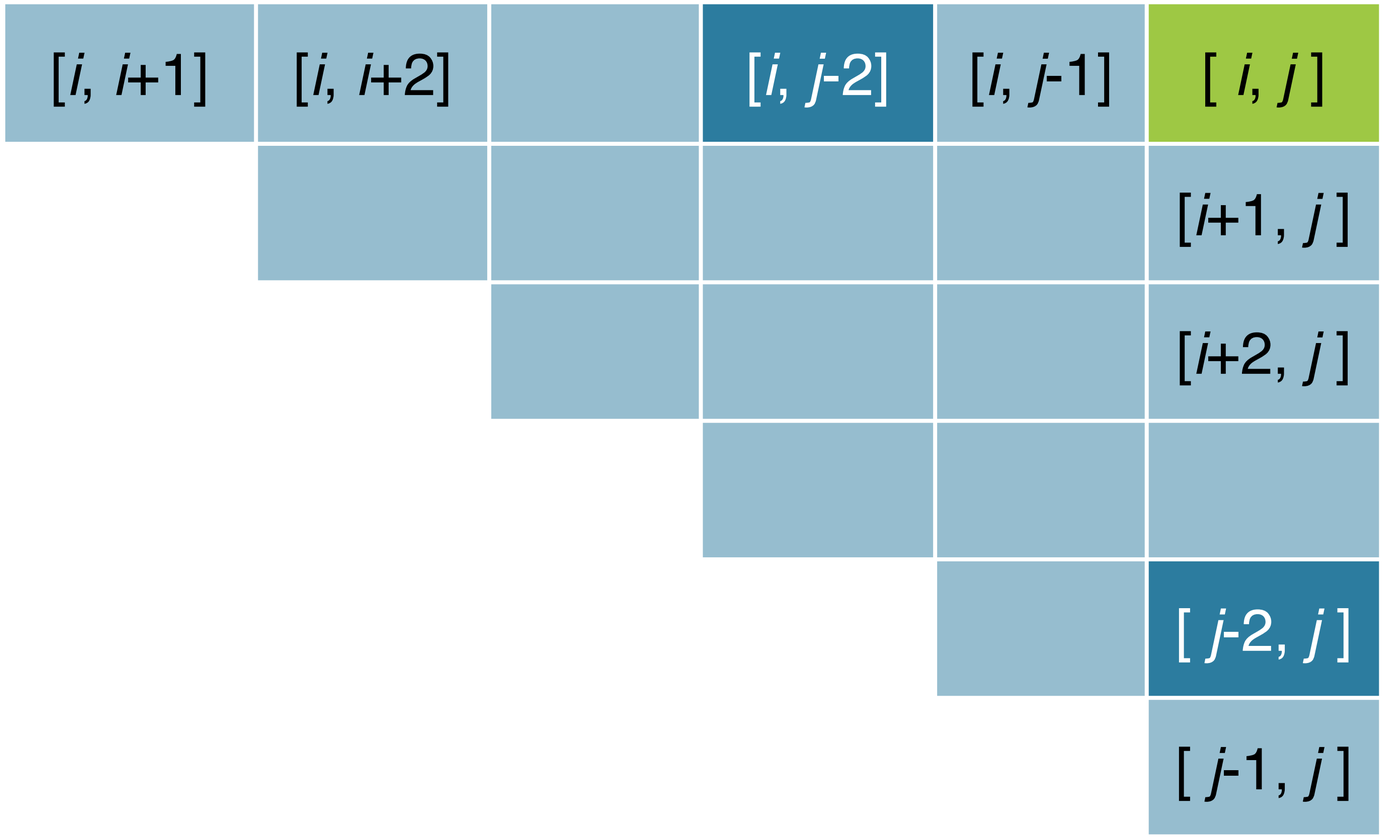

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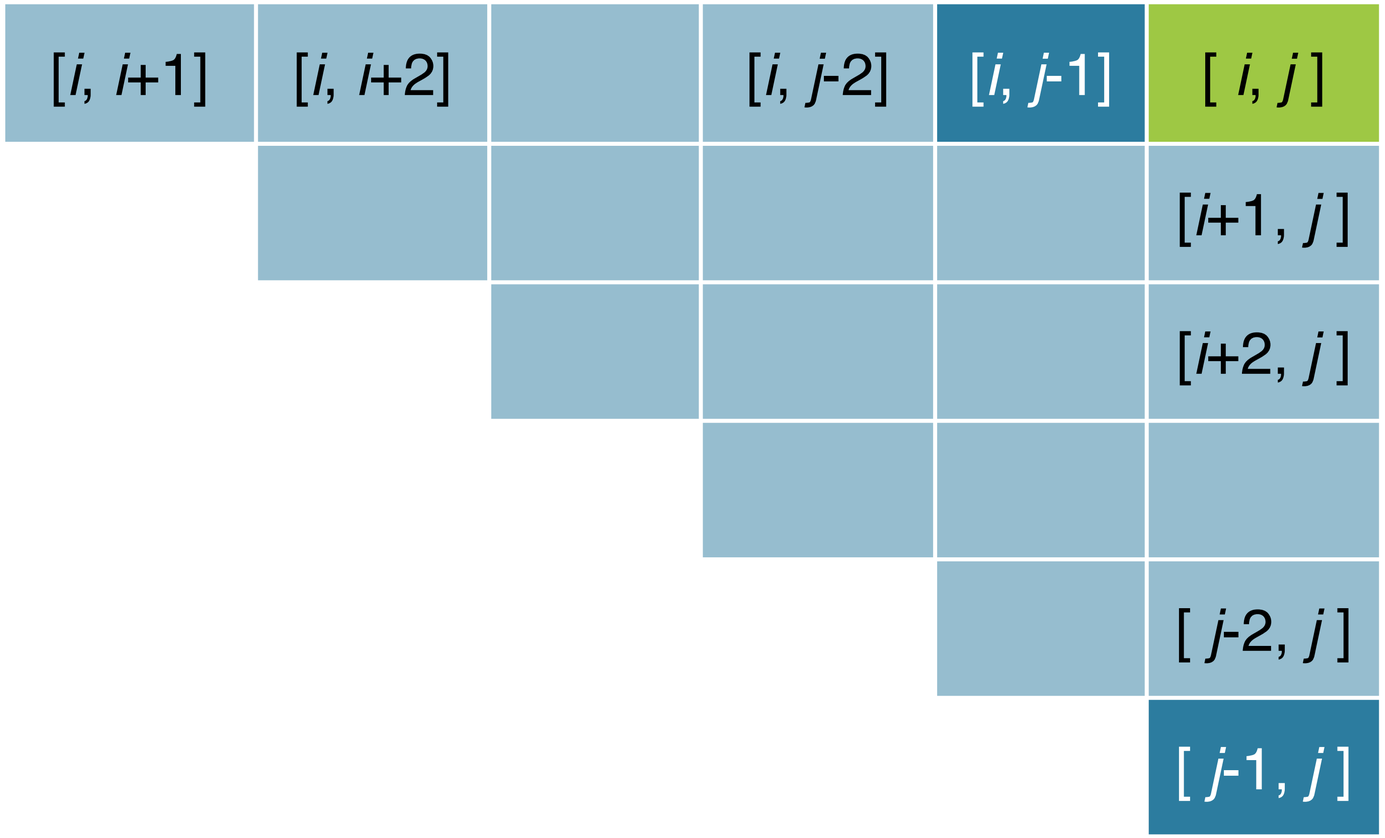

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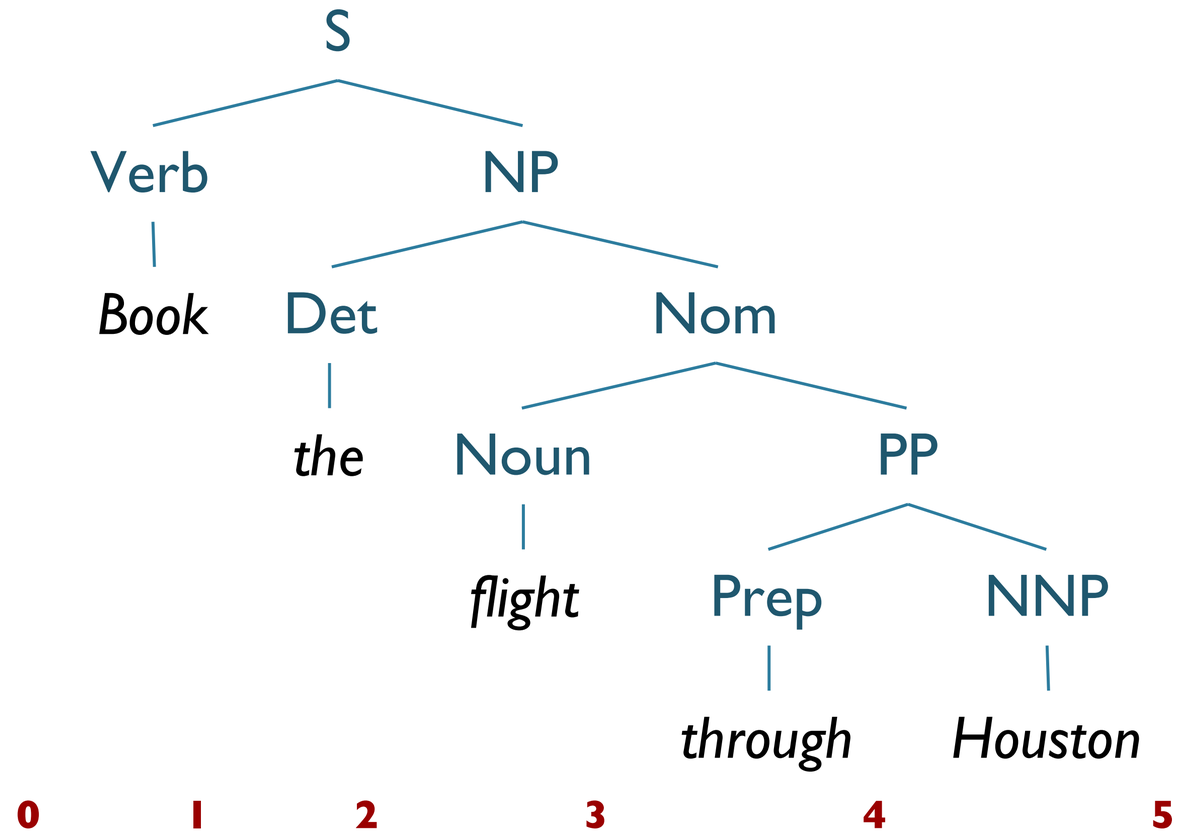

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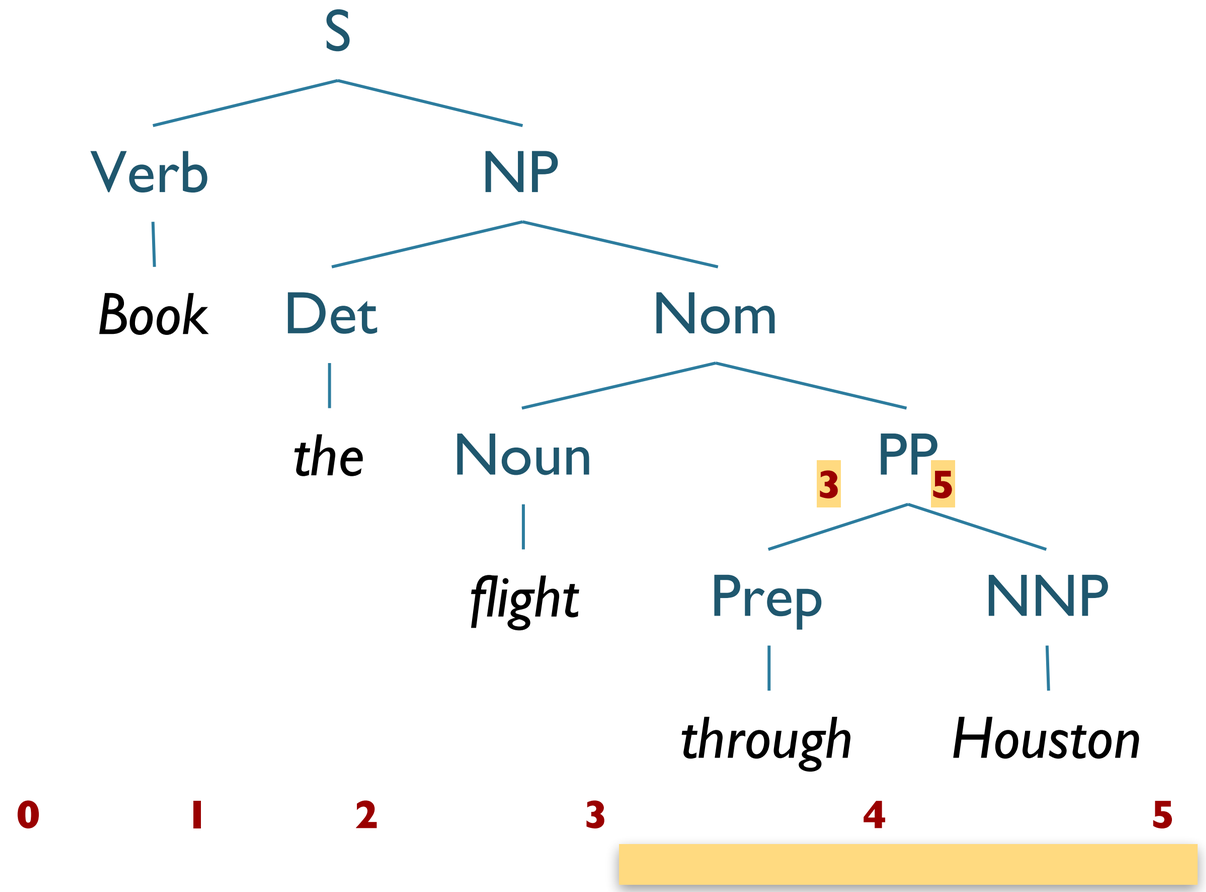

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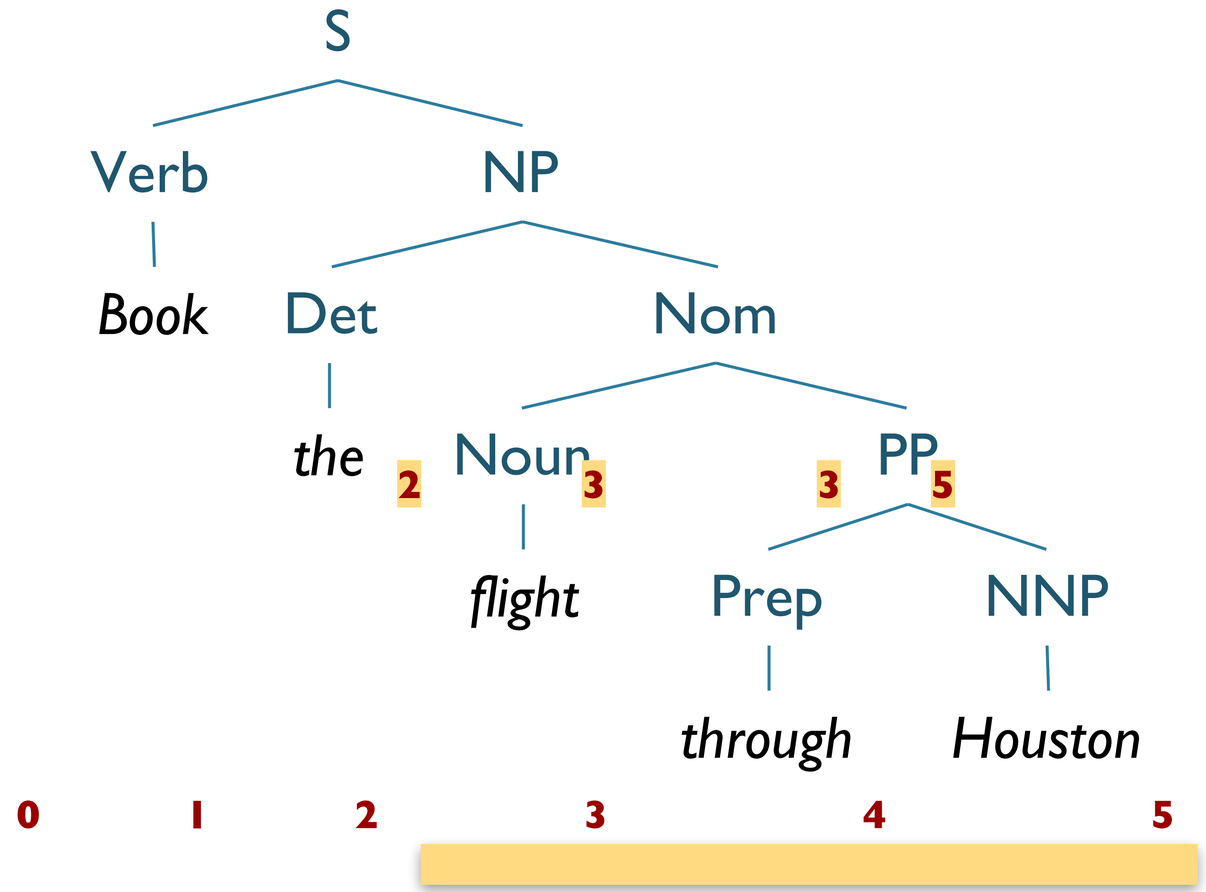

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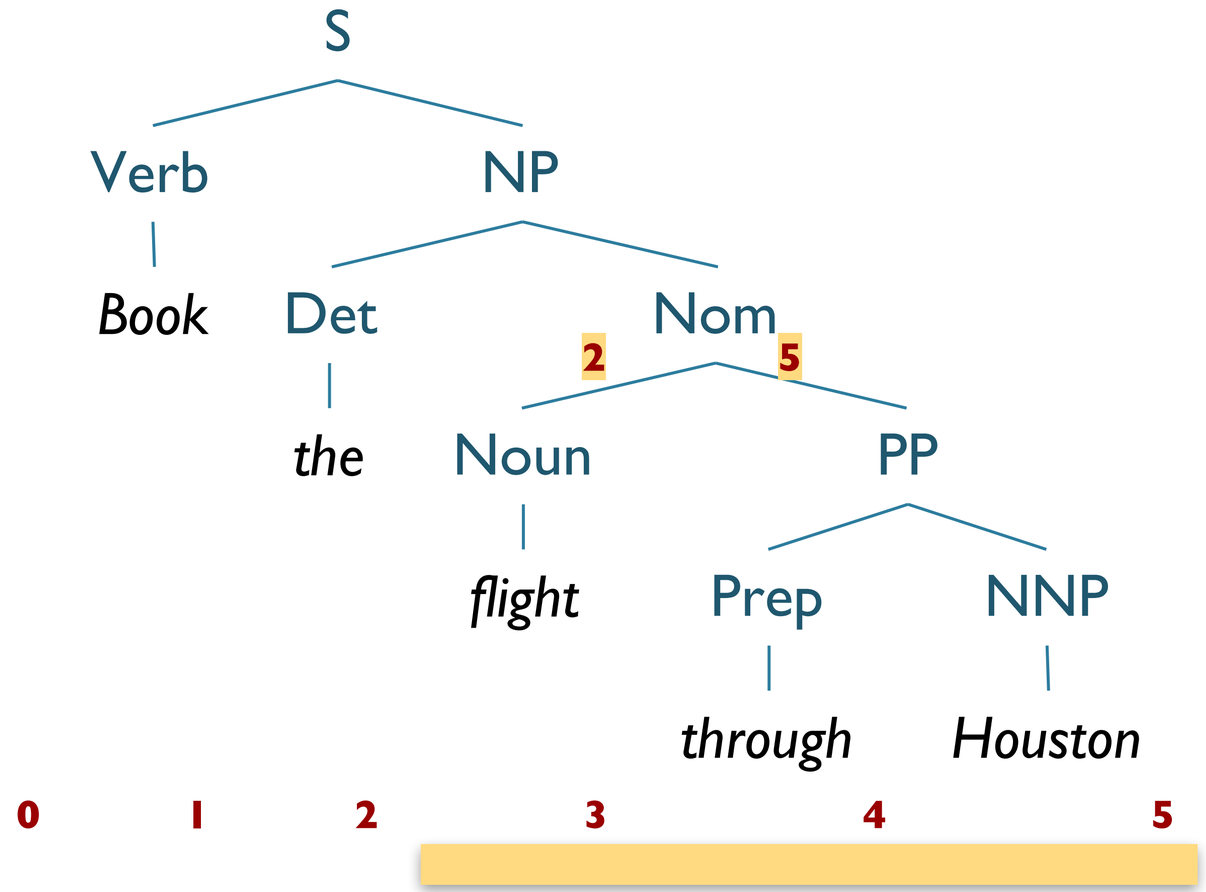

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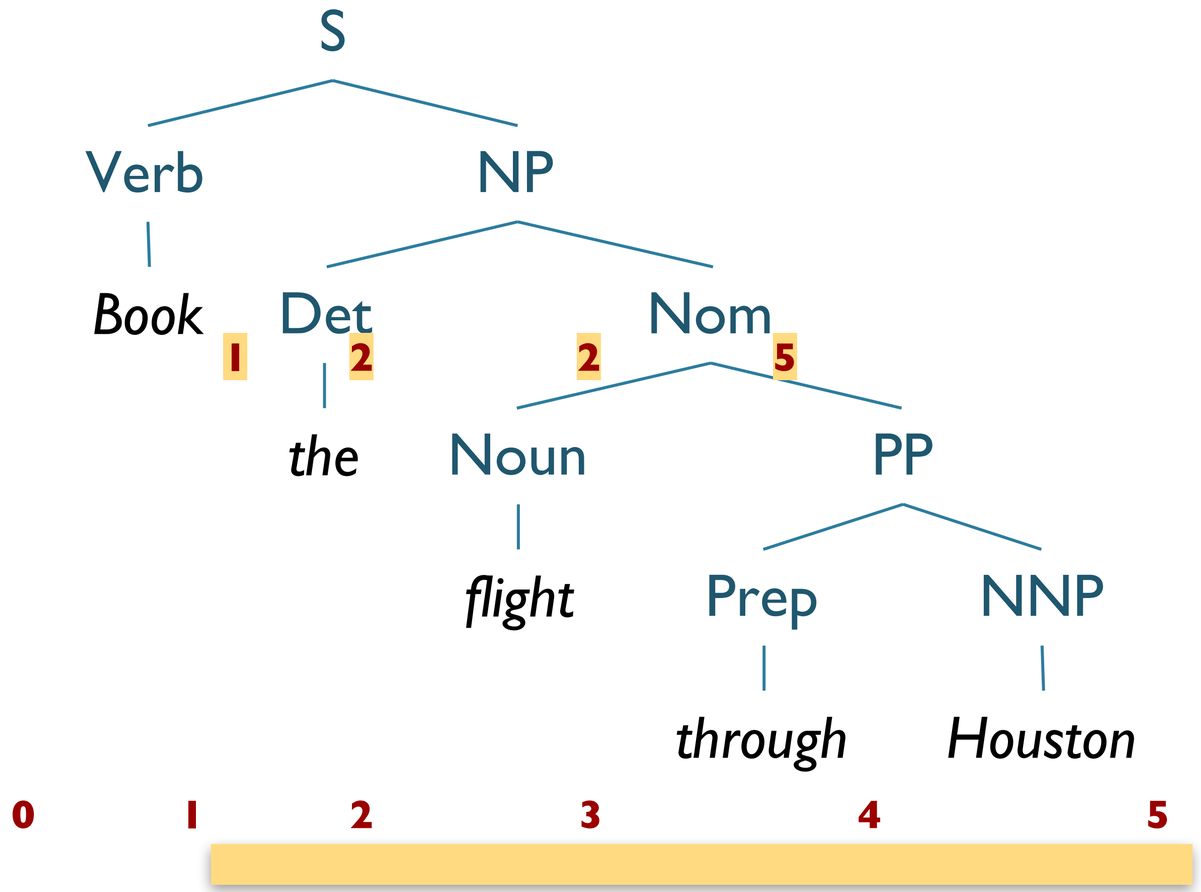

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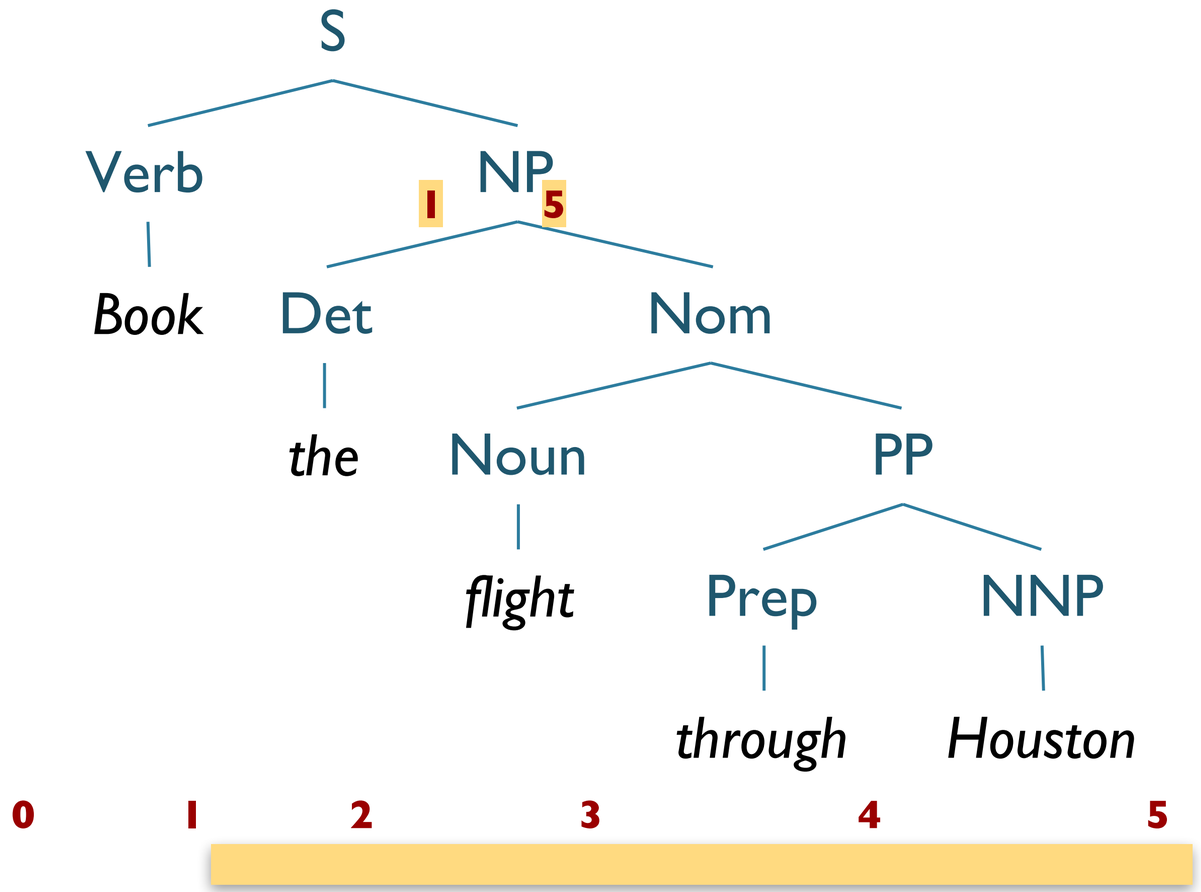

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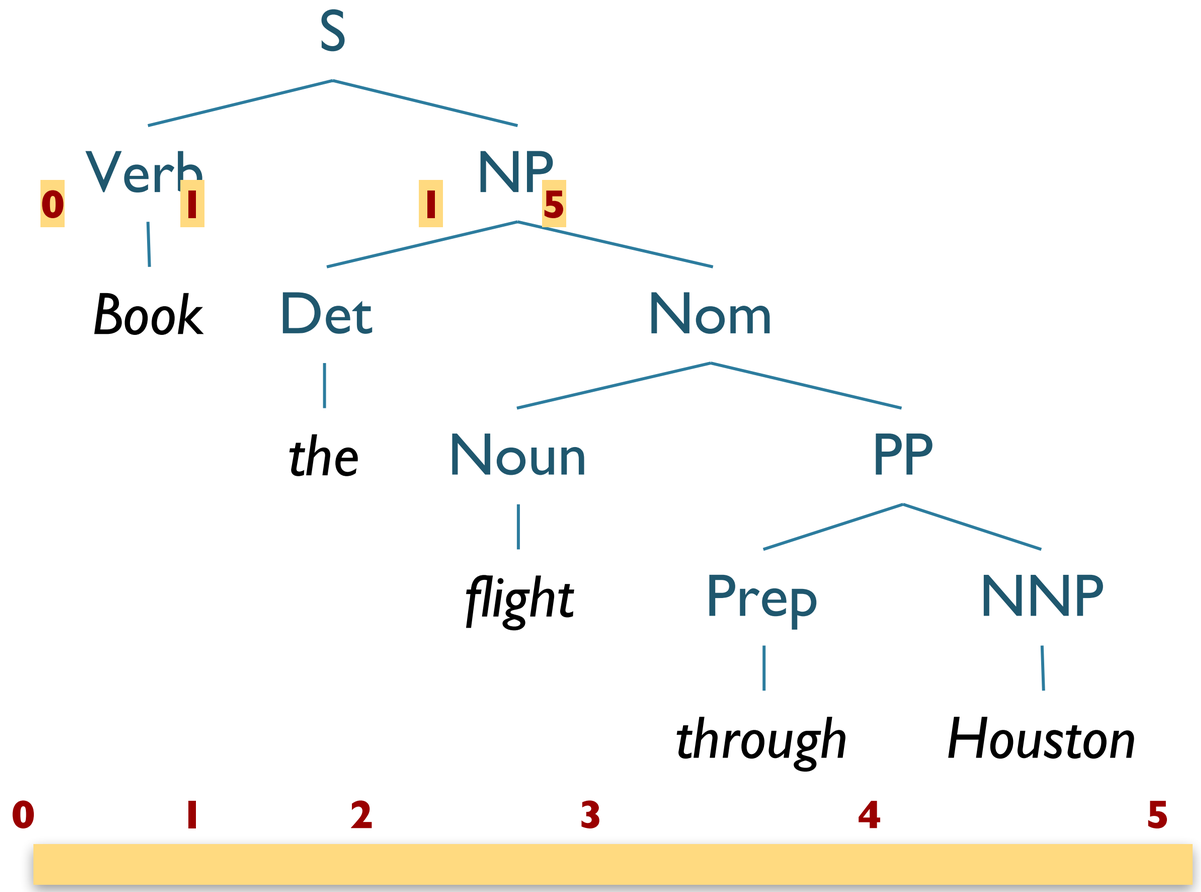

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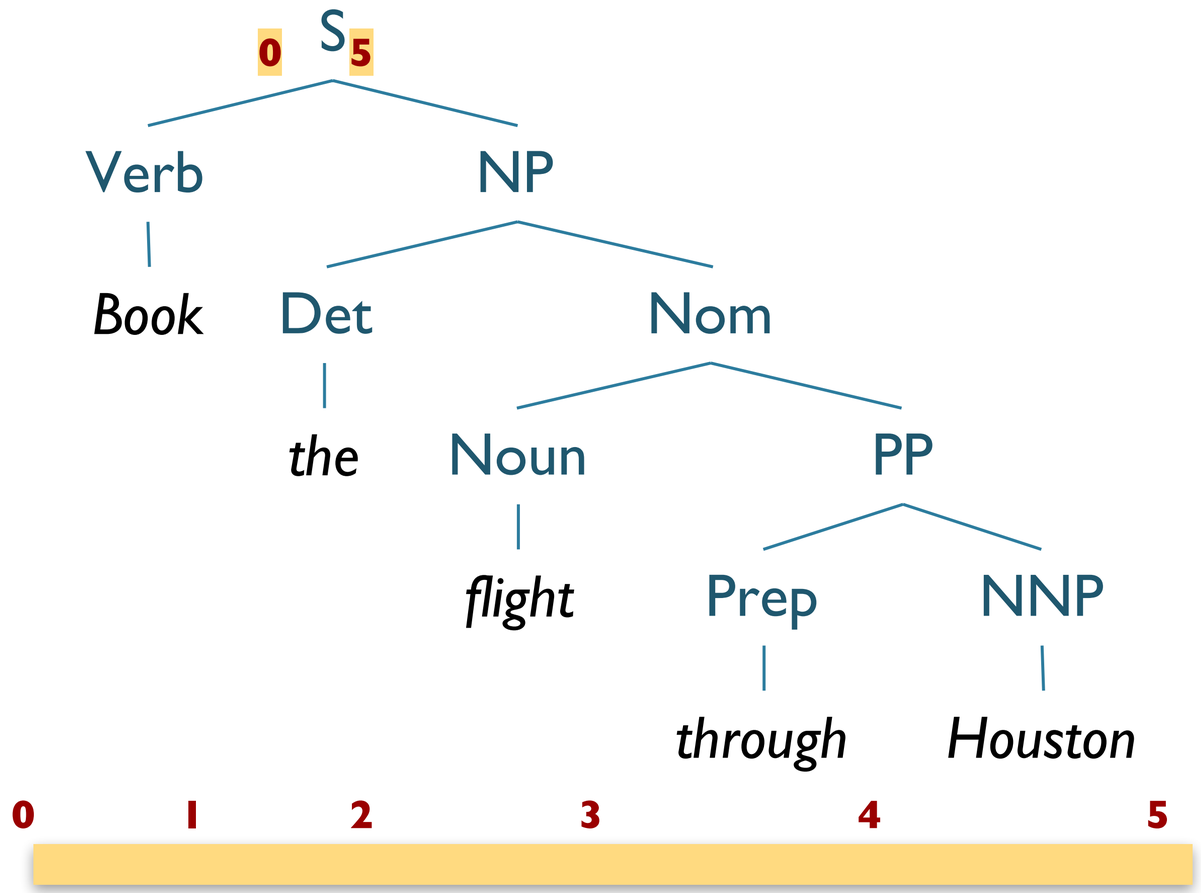

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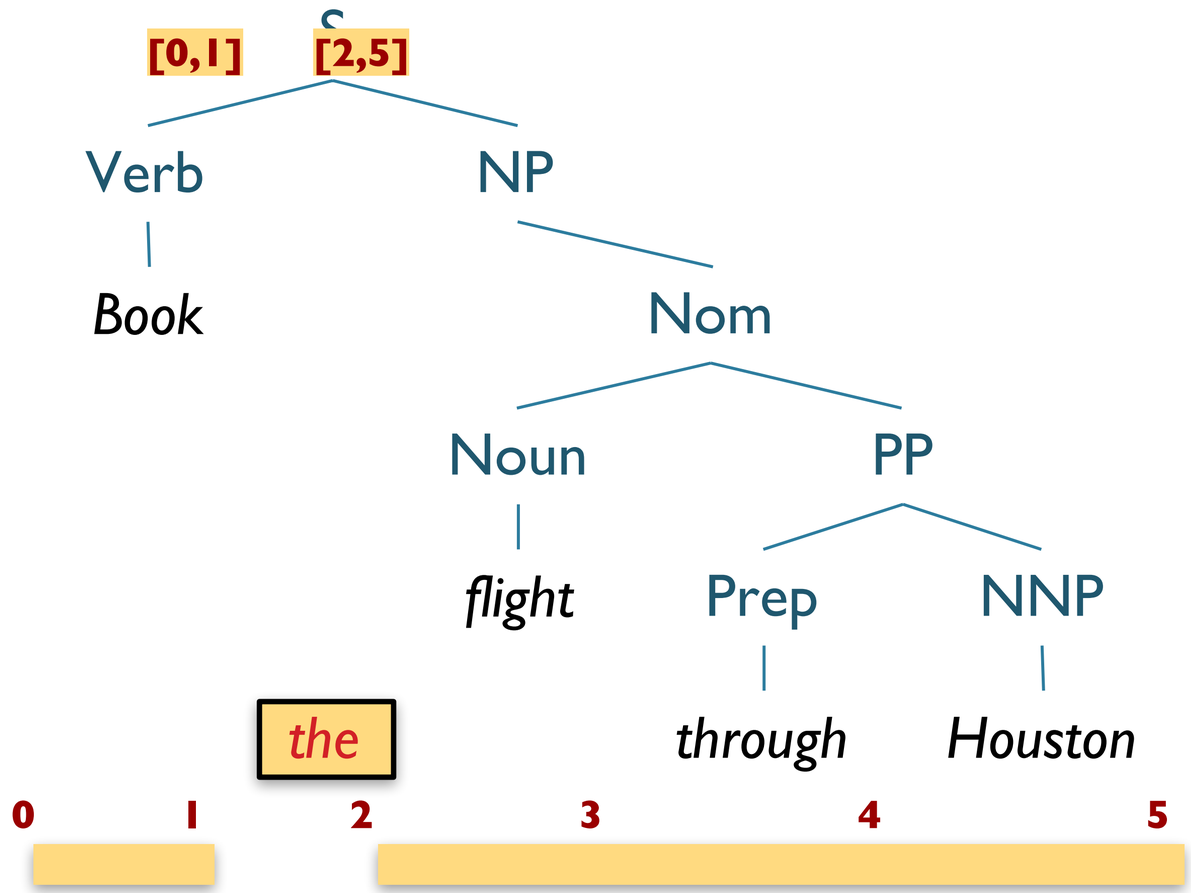
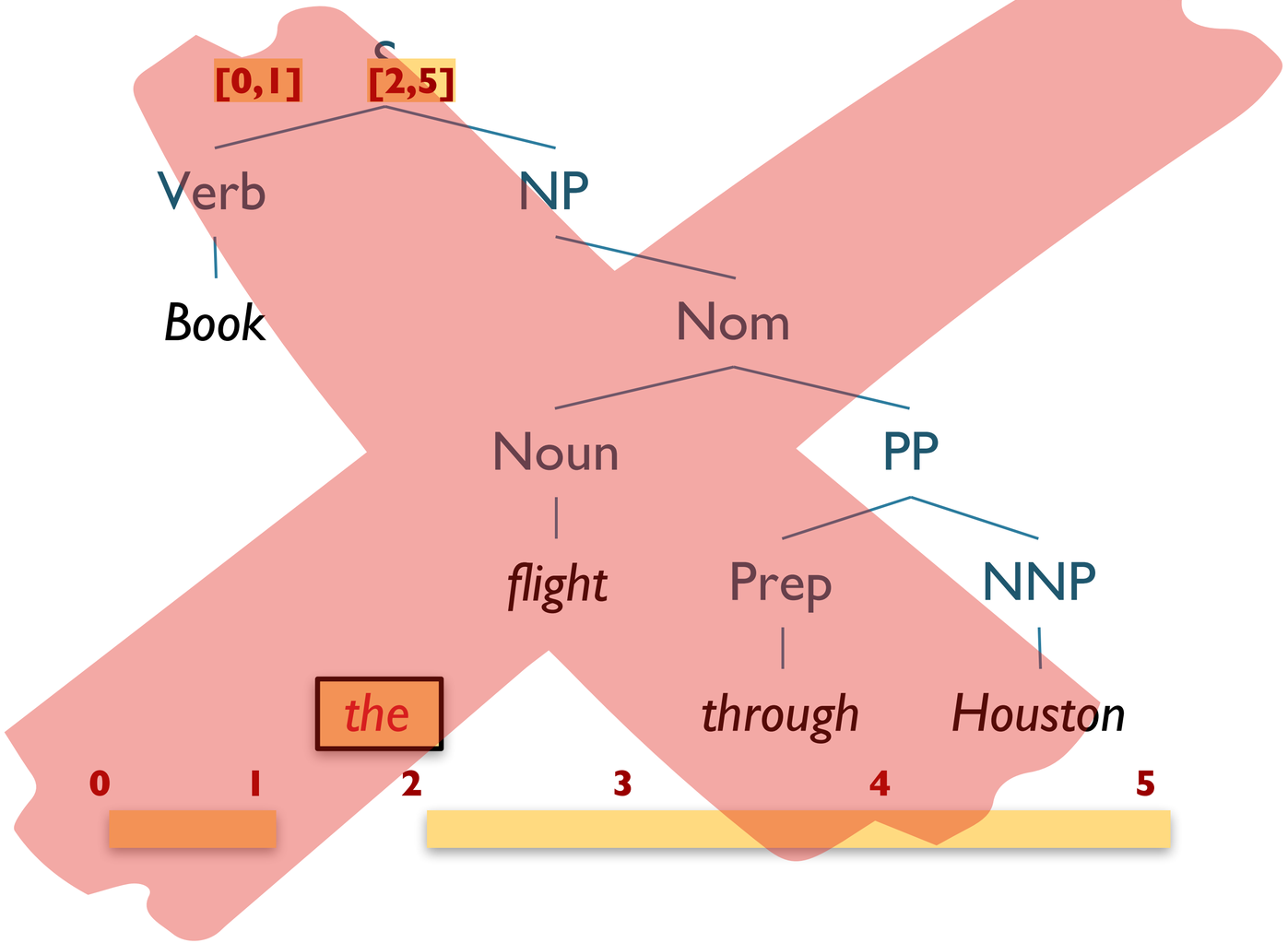


Why fill cells this way?

- For cell [*i*, *j*], must be able to access all cells [*i*, *k*] and [*k*, *j*] where $i < k < j$



Would you say CKY is a top-down or a bottom-up parsing algorithm?

0

Top-down

0

Bottom-up

0

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CKY Parsing Example

$\mathcal{L}_1$ Grammar

S → NP VP
S → X1 VP
X1 → Aux NP
S → book | include | prefer
S → Verb NP
S → X2 PP
S → Verb PP
S → VP PP

NP → I | she | me
NP → TWA | Houston
NP → Det Nominal

Nominal → book | flight | meal | money
Nominal → Nominal Noun
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VP → book | include | prefer
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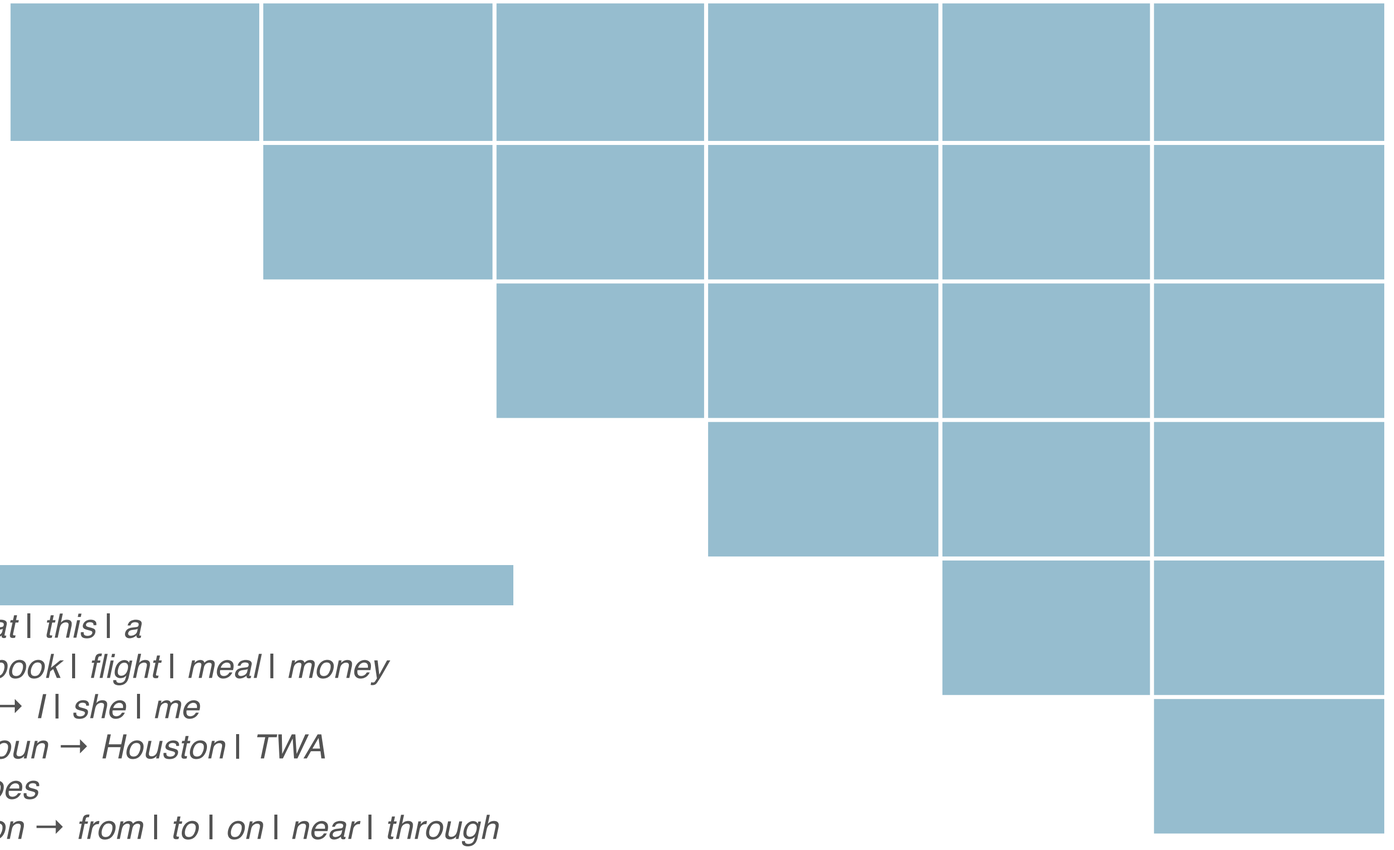
PP → Preposition NP



Lexicon

Det → that | this | a
Noun → book | flight | meal | money
Pronoun → I | she | me
Proper-Noun → Houston | TWA
Aux → does
Preposition → from | to | on | near | through
Verb → book | include | prefer

I *prefer* *a* *flight* *on* *TWA*

0 1 2 3 4 5 6

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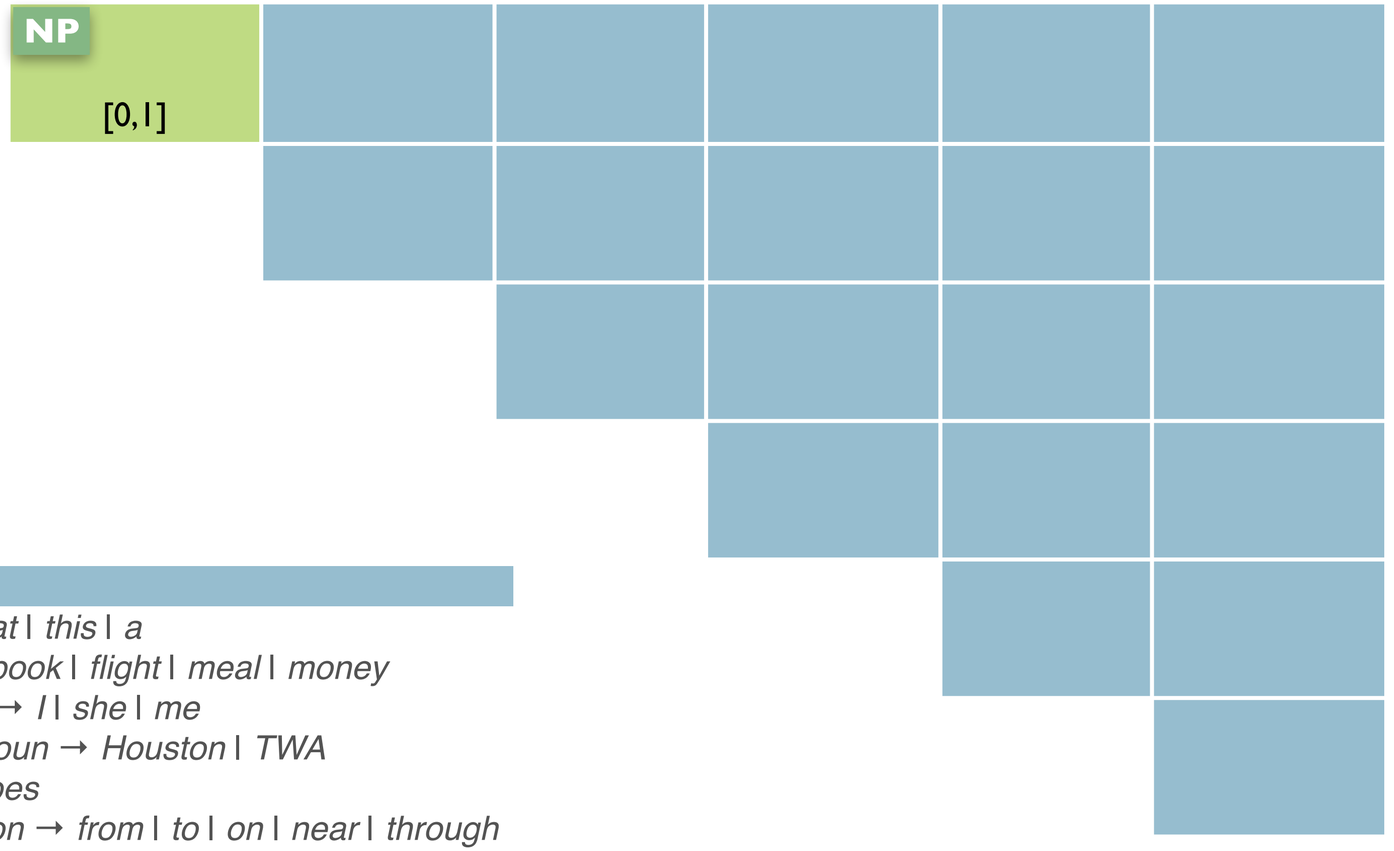

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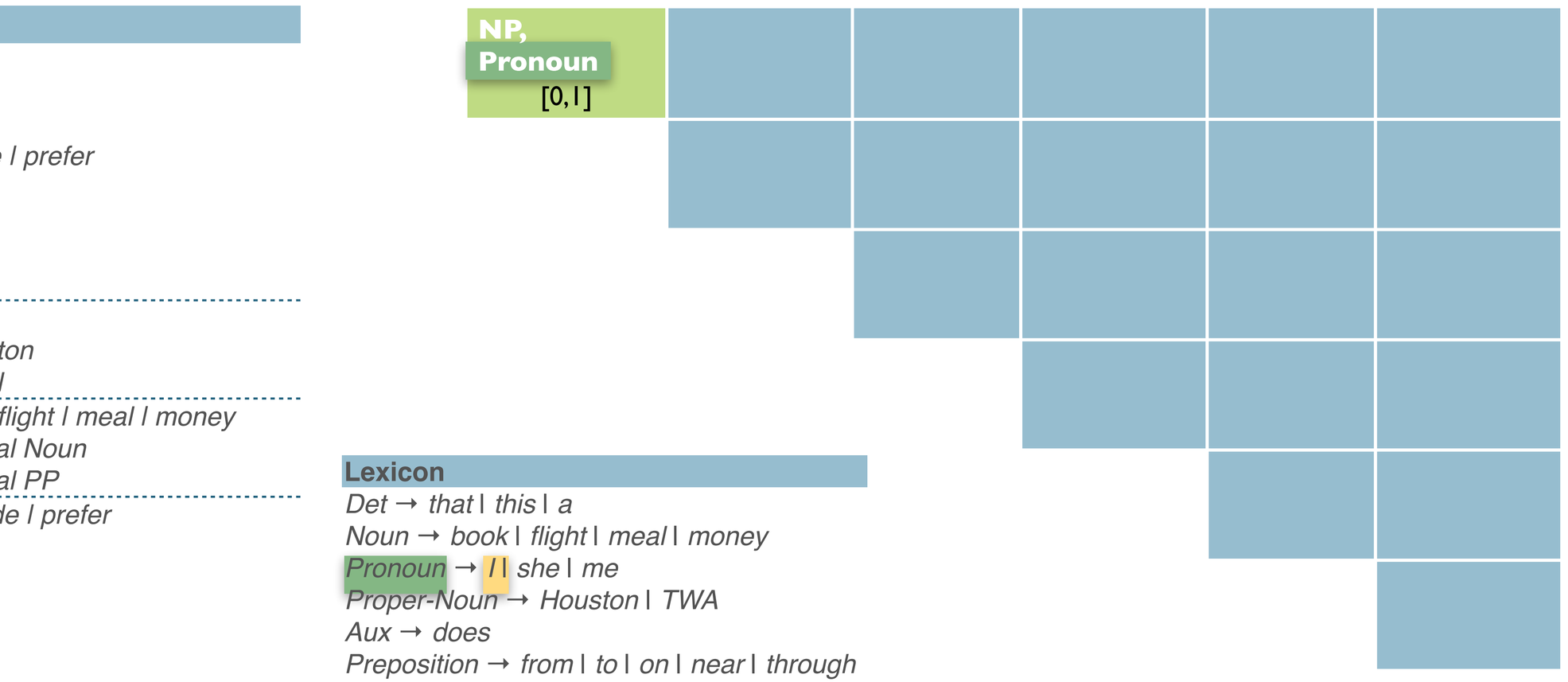

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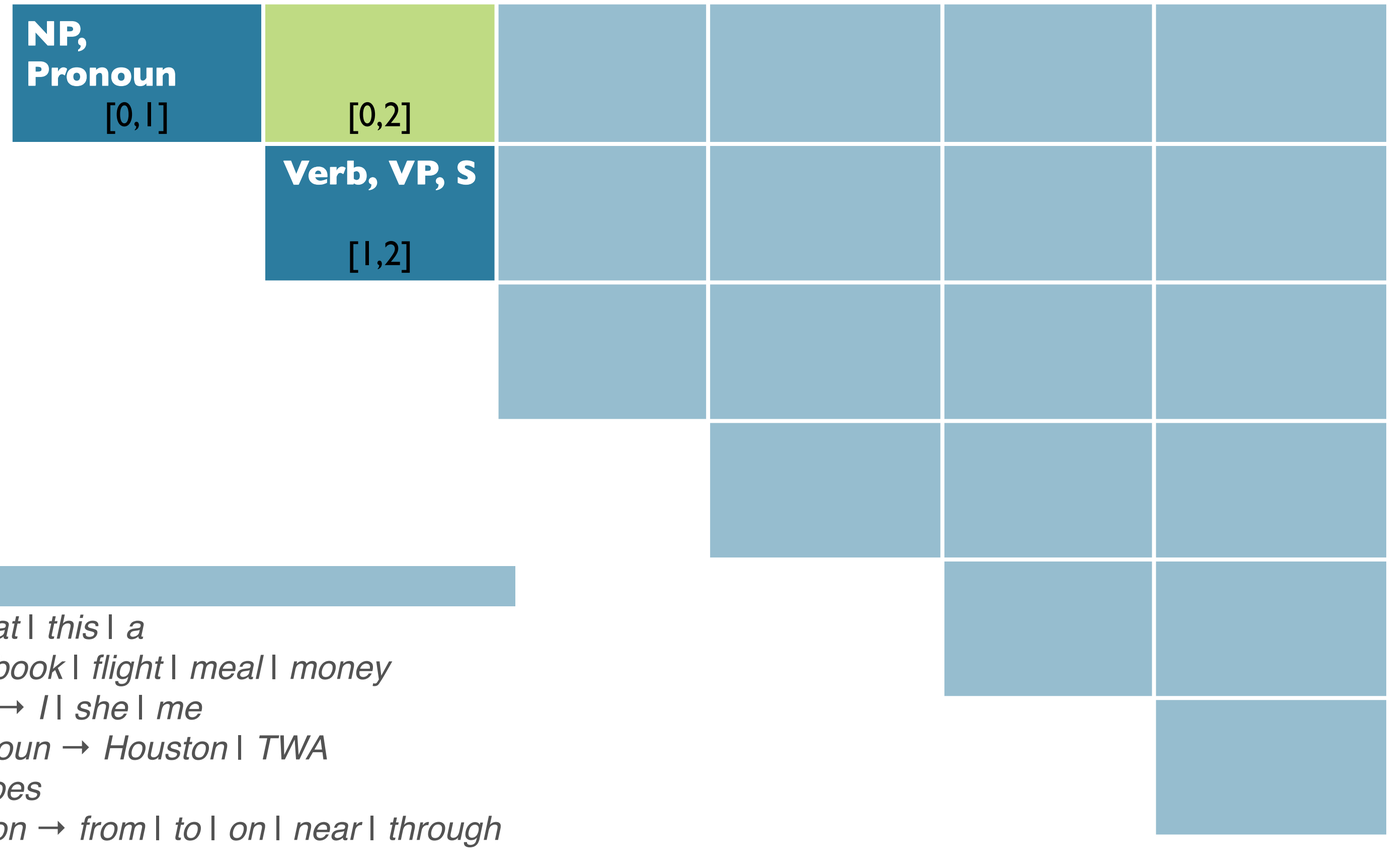

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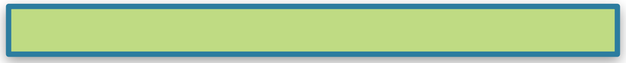

I prefer a flight on TWA

0 1 2 3 4 5 6

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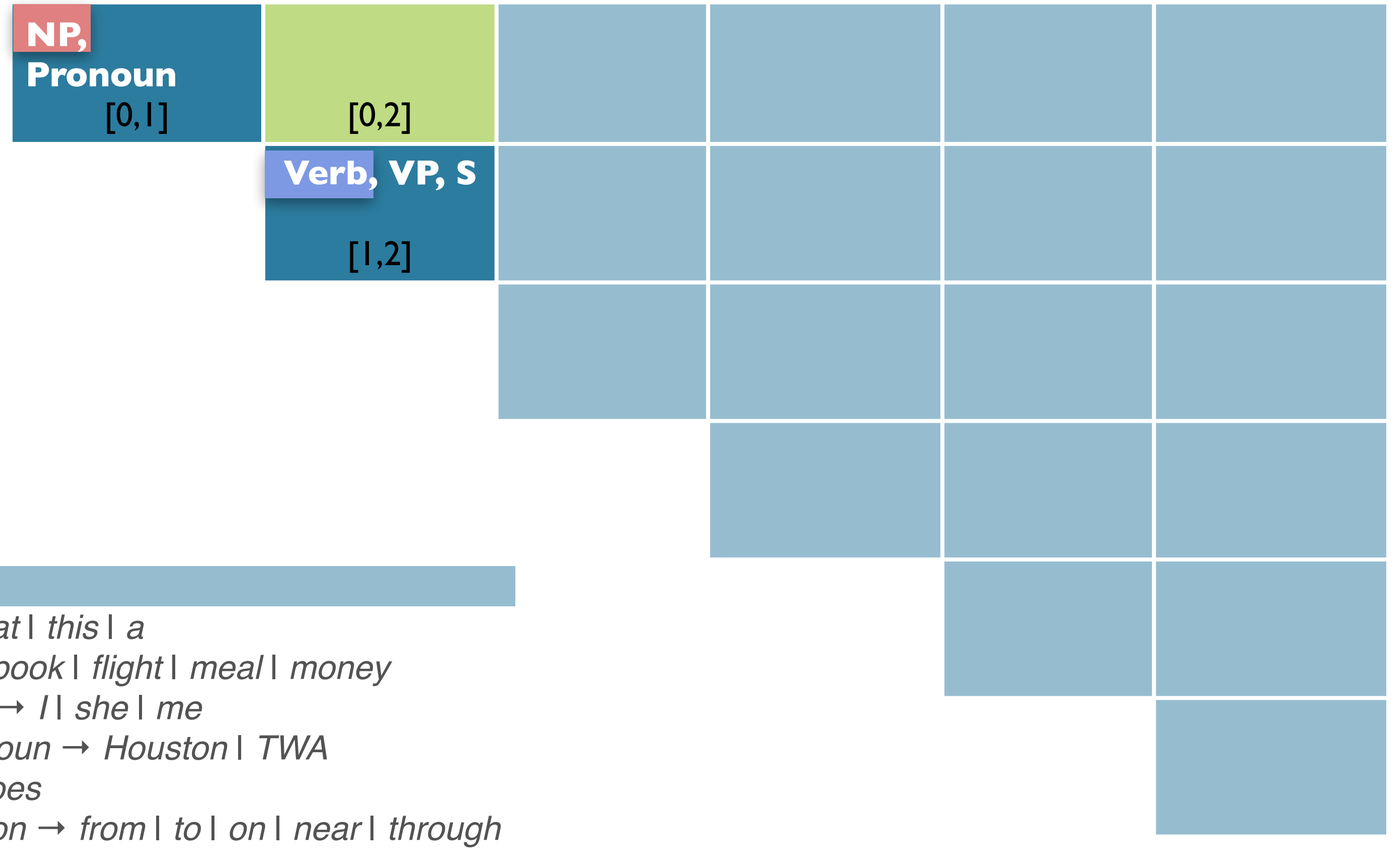

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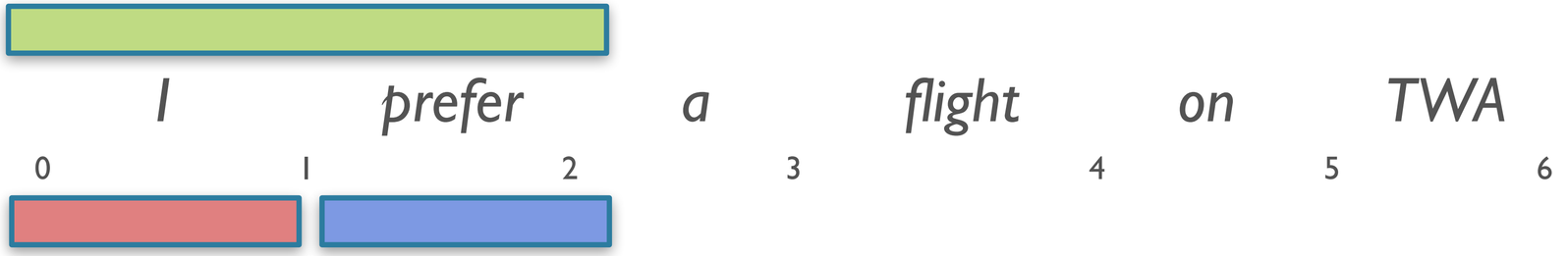

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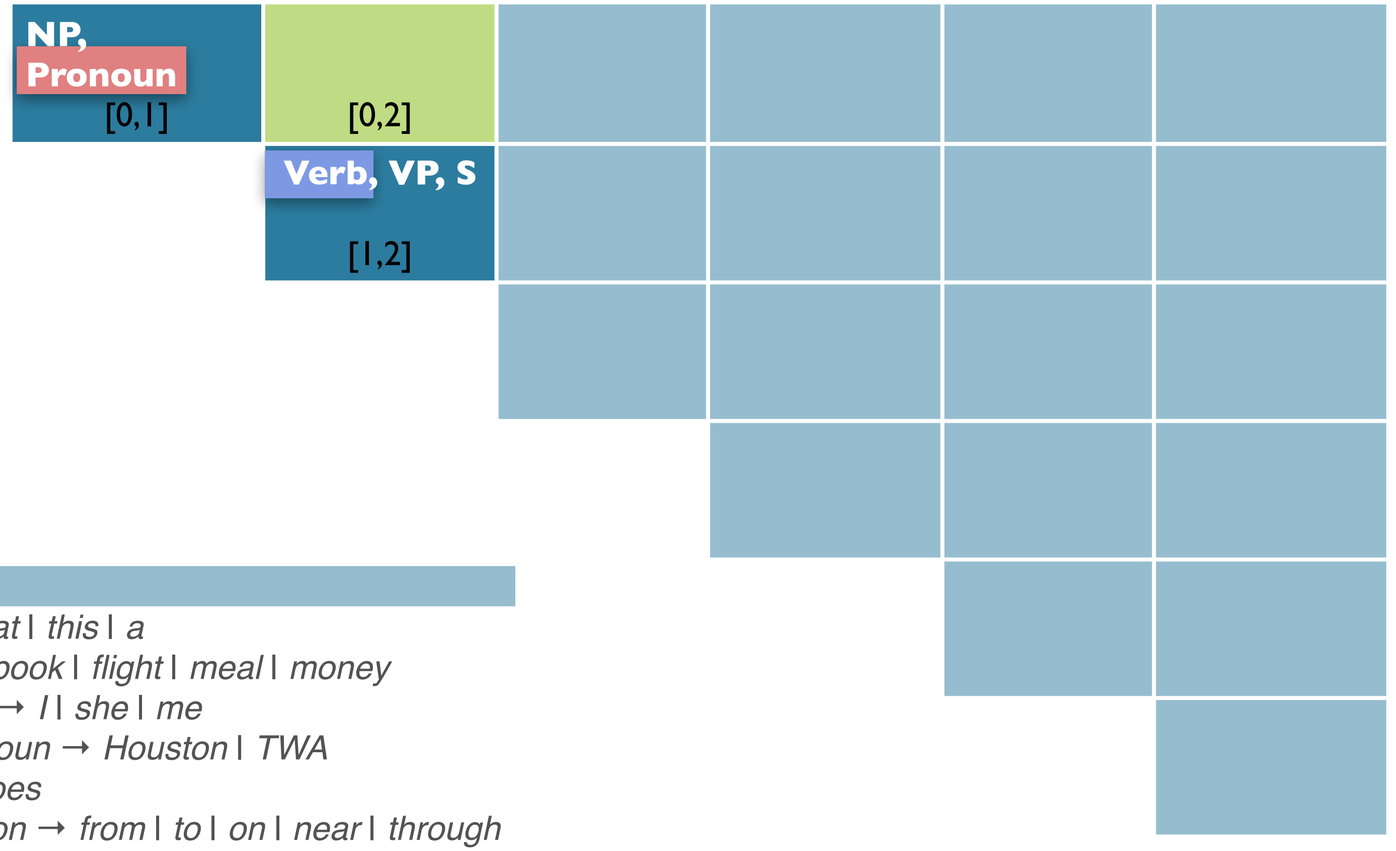

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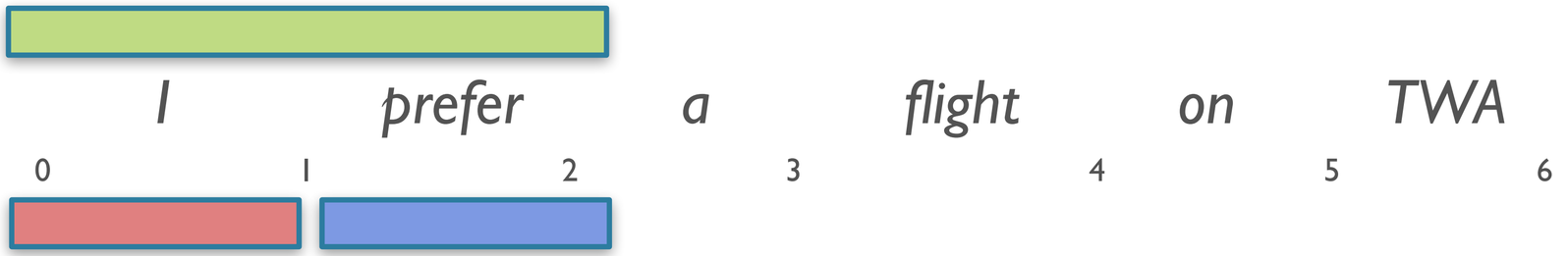

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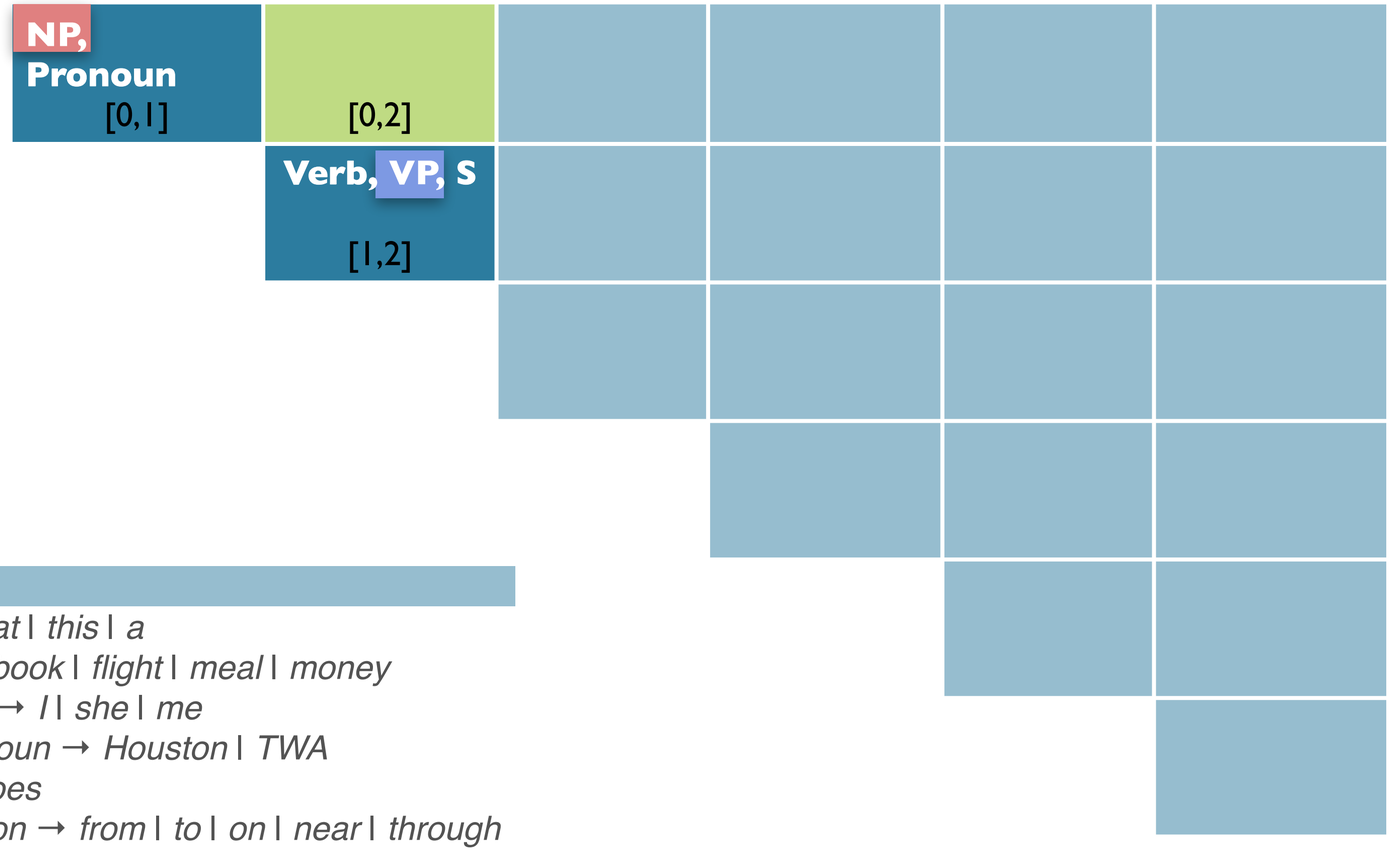

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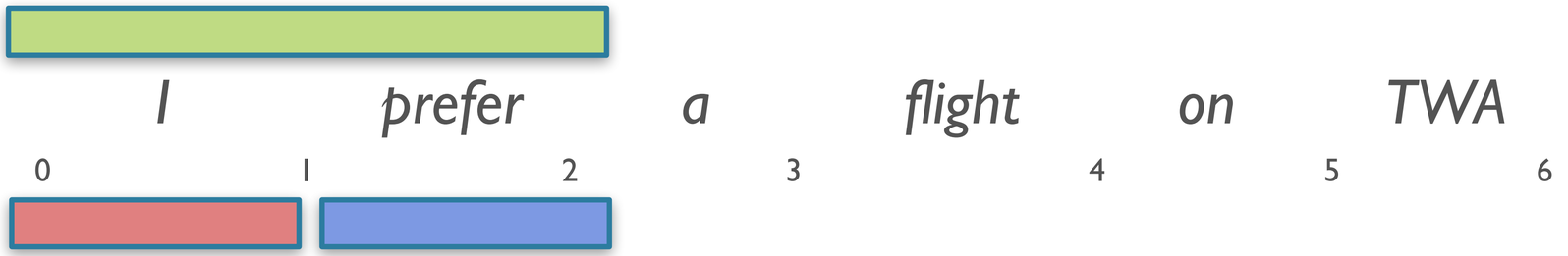

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NP, Pronoun [0,1]	S [0,2]			
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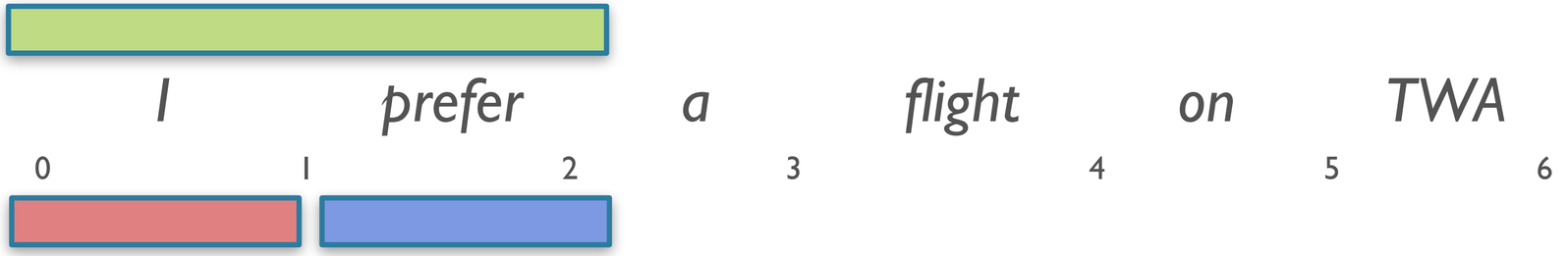

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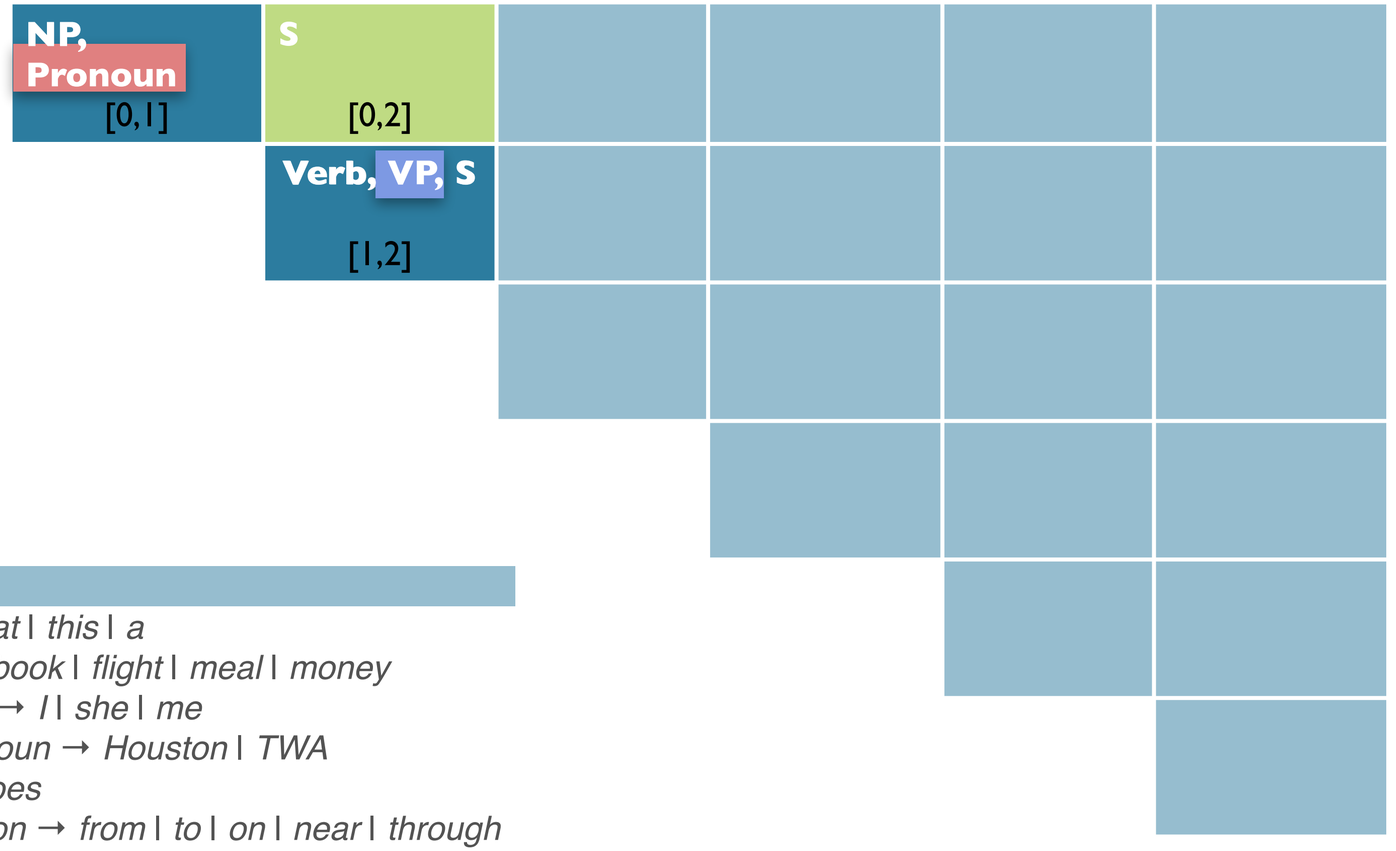

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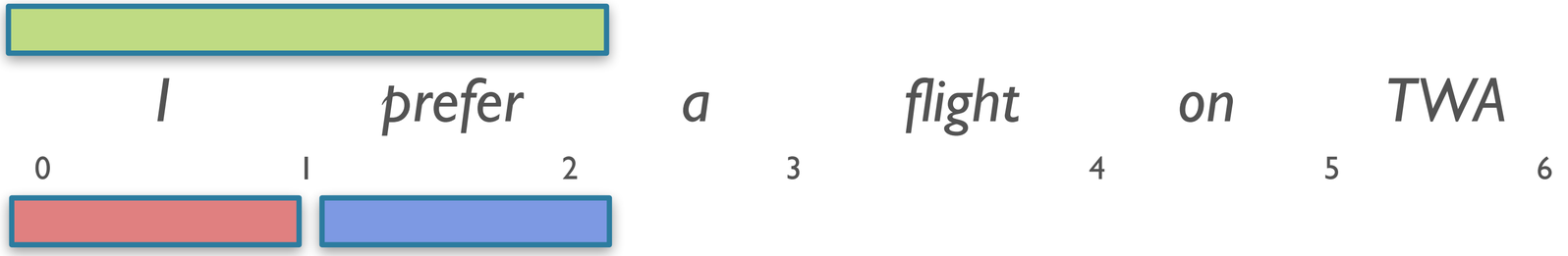

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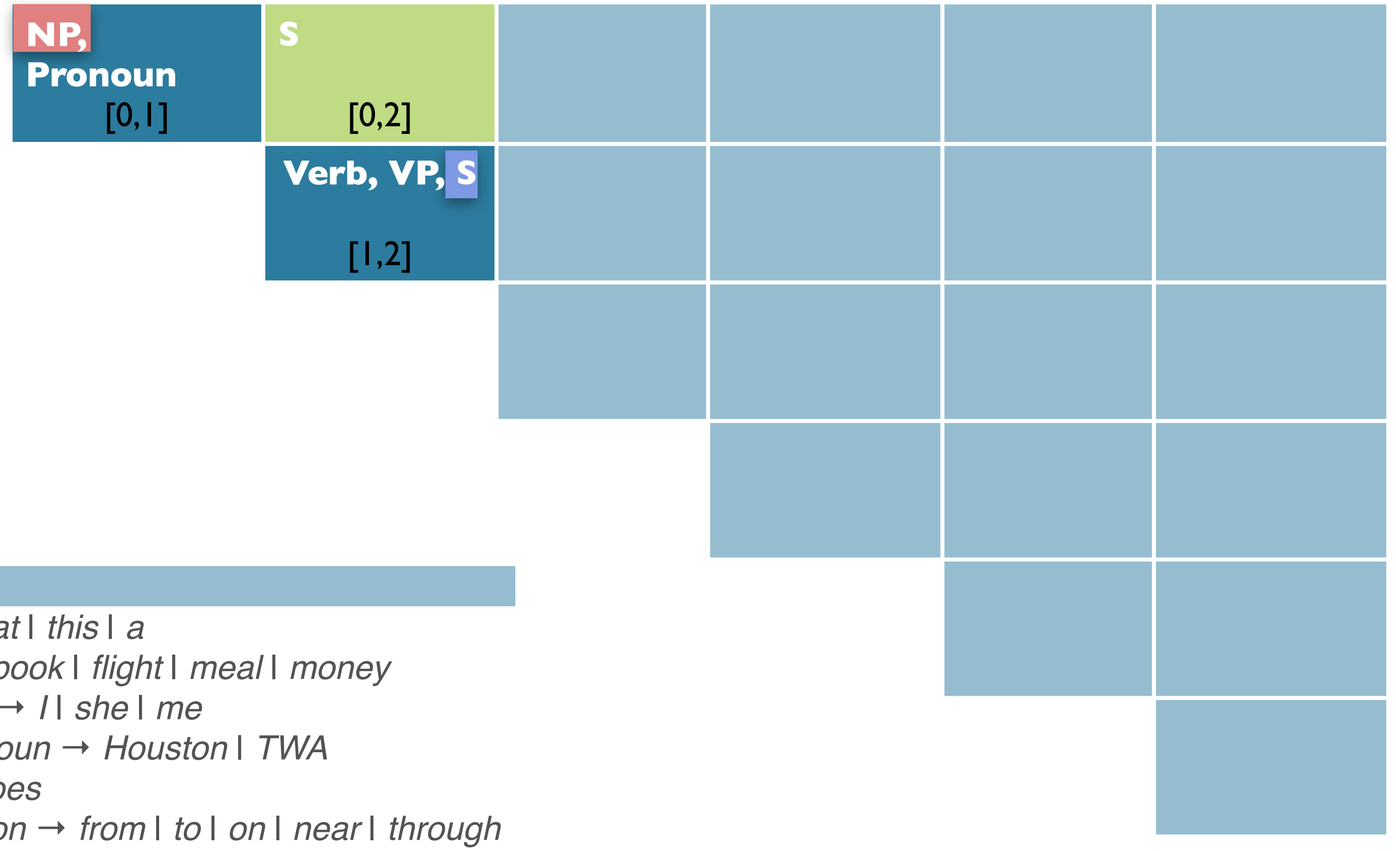

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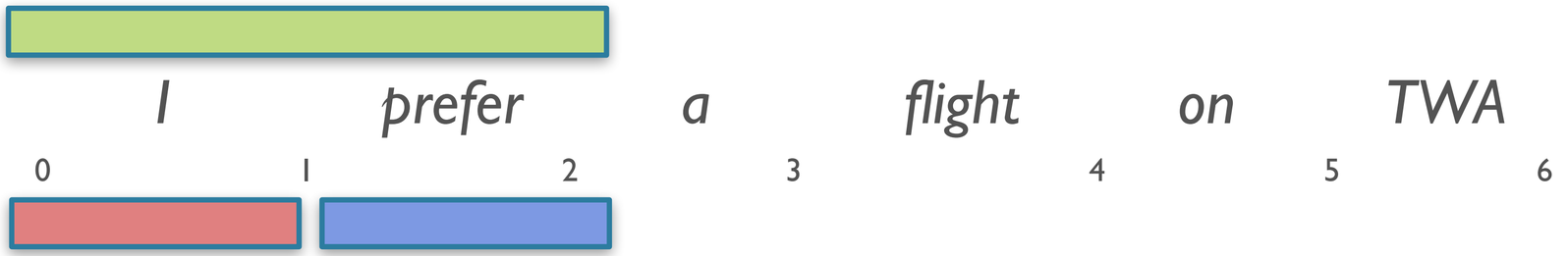

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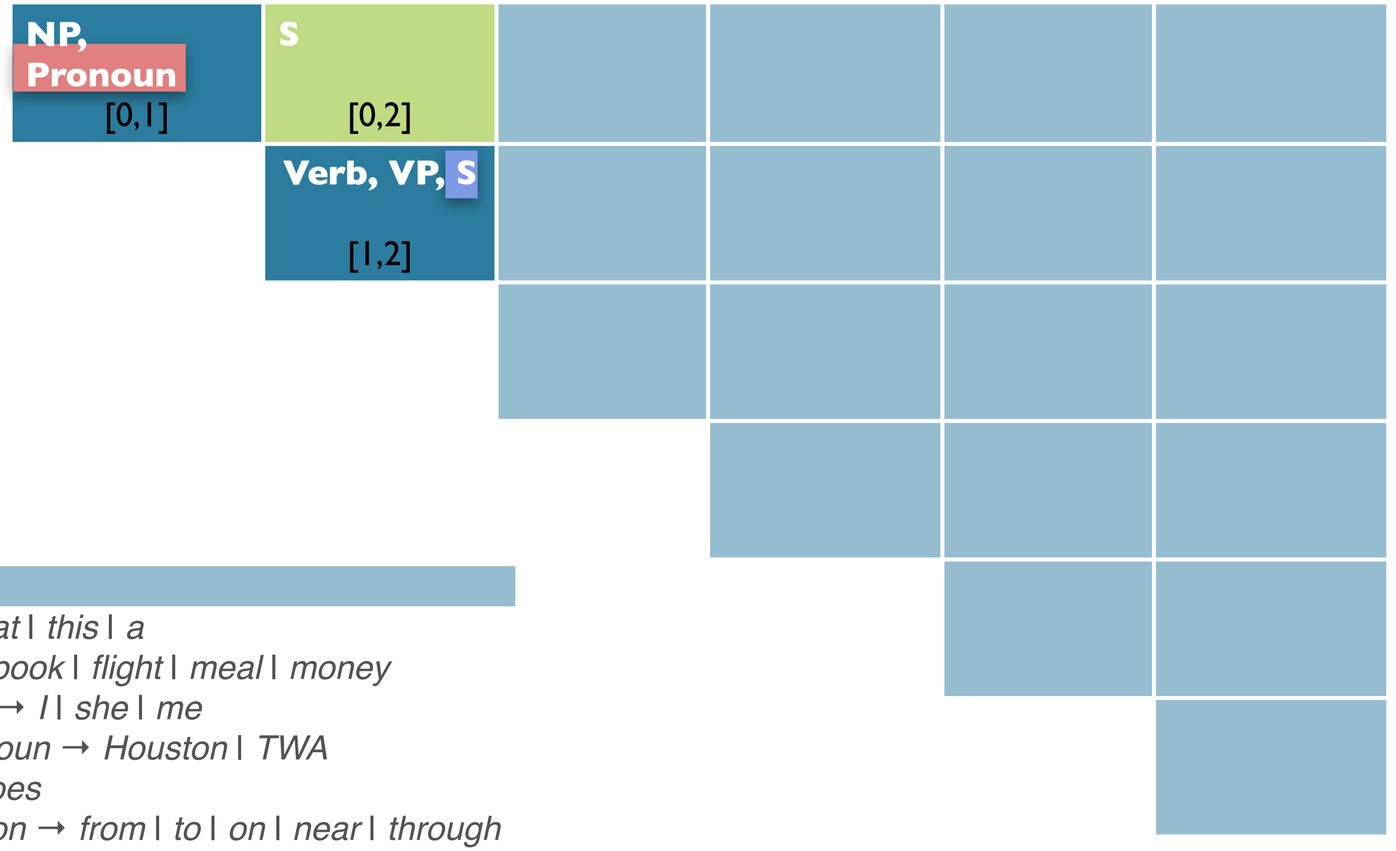

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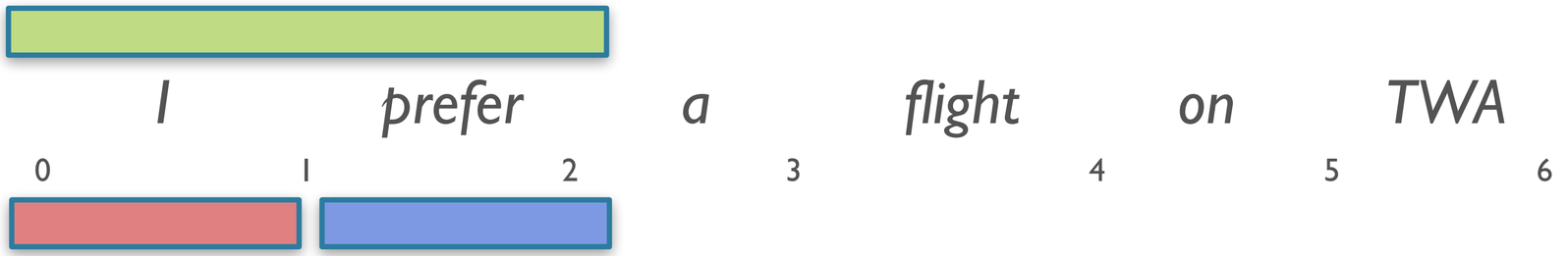

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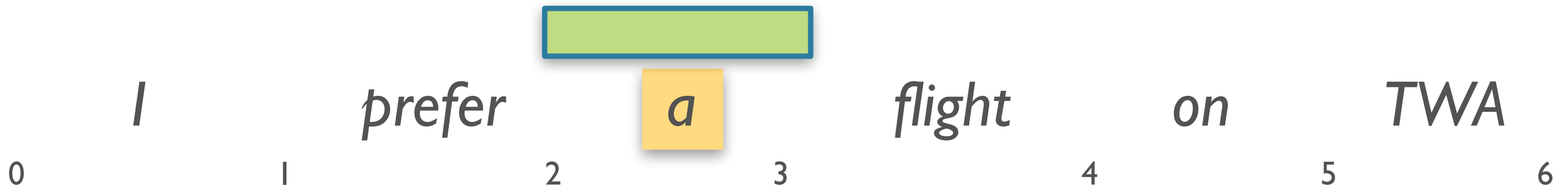

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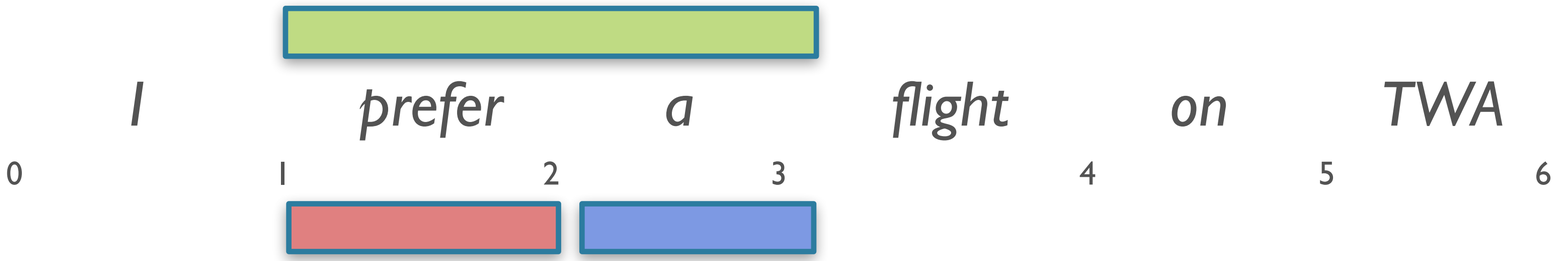

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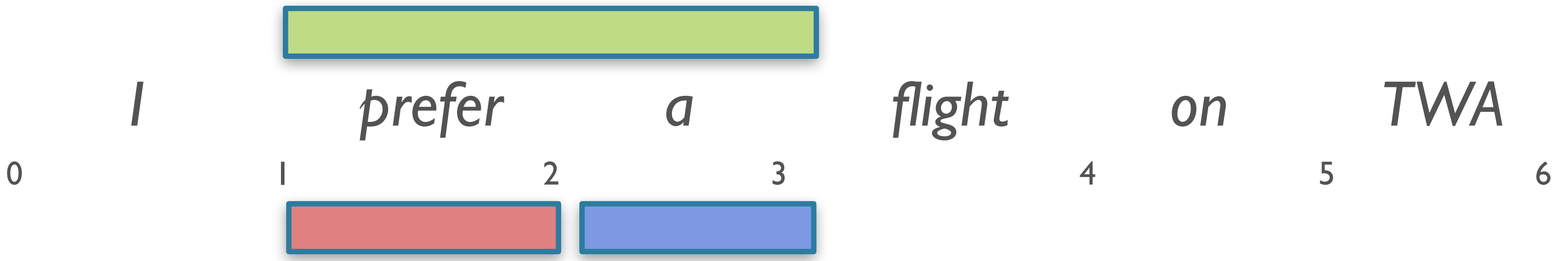

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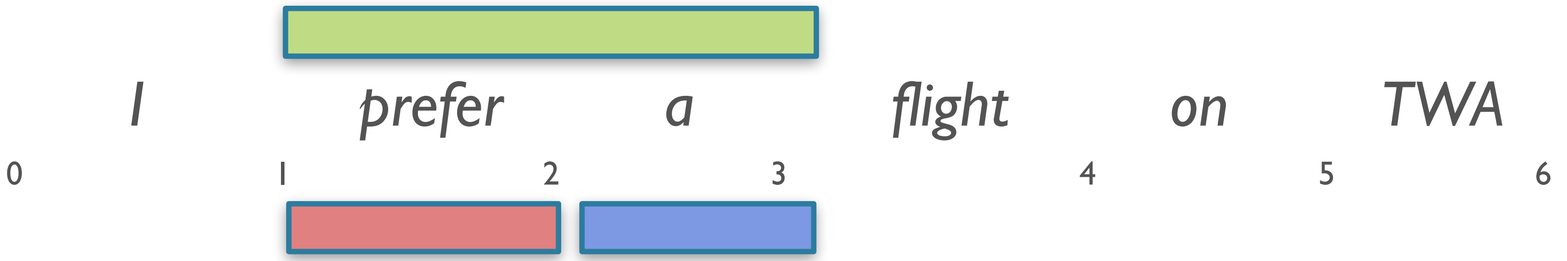

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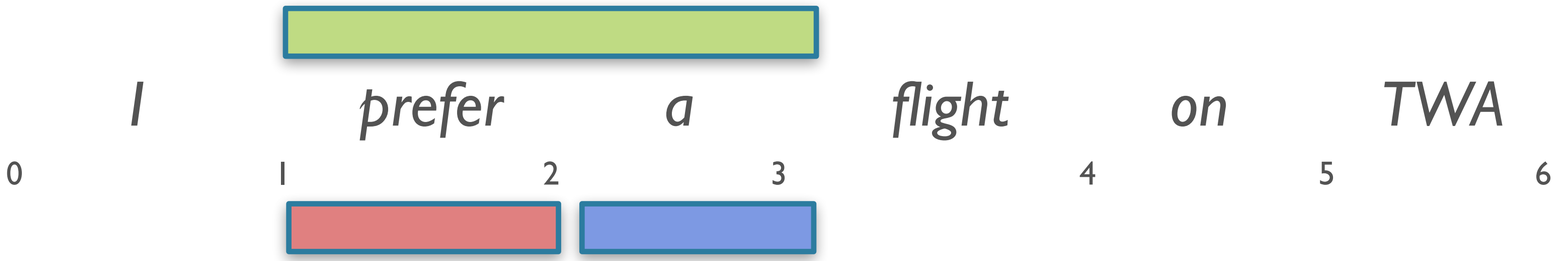

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Nominal → book | flight | meal | money
Nominal → Nominal Noun
Nominal → Nominal PP
VP → book | include | prefer
VP → Verb NP
VP → X2 PP
X2 → Verb NP
VP → Verb PP
VP → VP PP
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NP, Pronoun [0,1]	**S** [0,2]				
	Verb, VP, S [1,2]	[1,3]			
		Det [2,3]			

Lexicon

Det → that | this | a
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Pronoun → I | she | me
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Aux → does
Preposition → from | to | on | near | through
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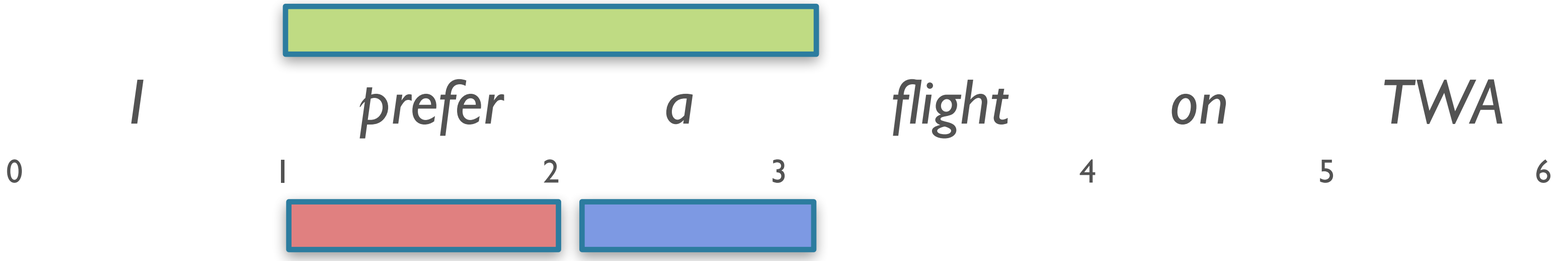

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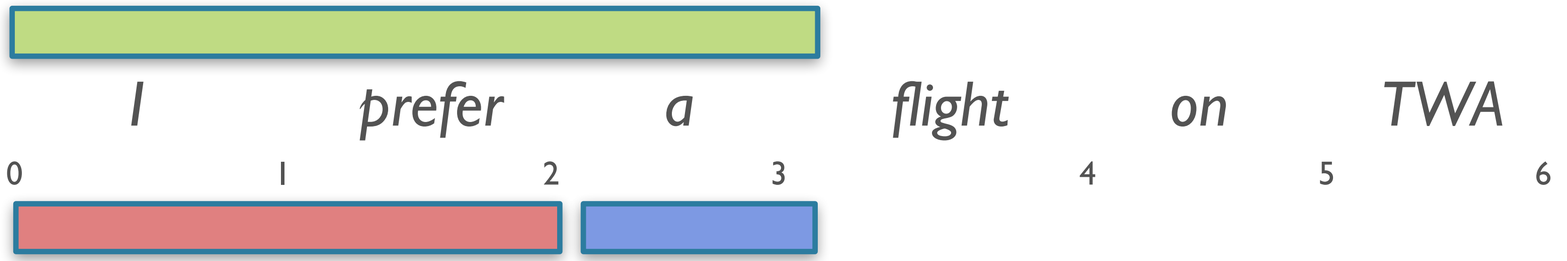

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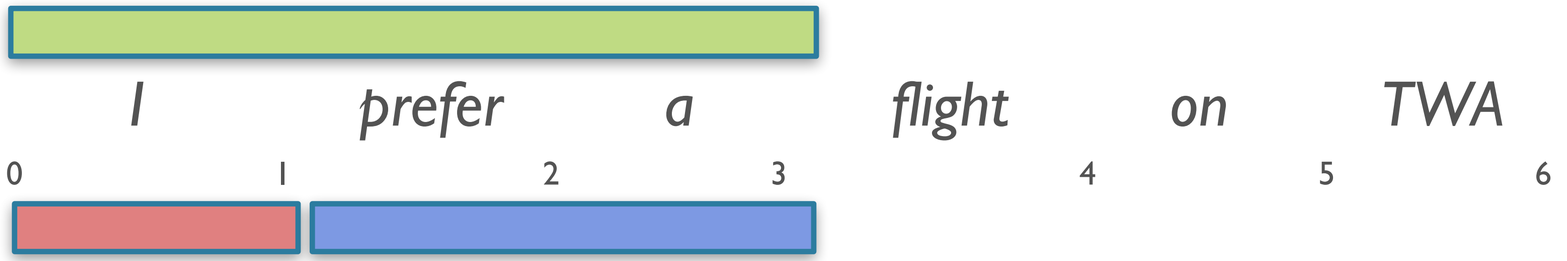

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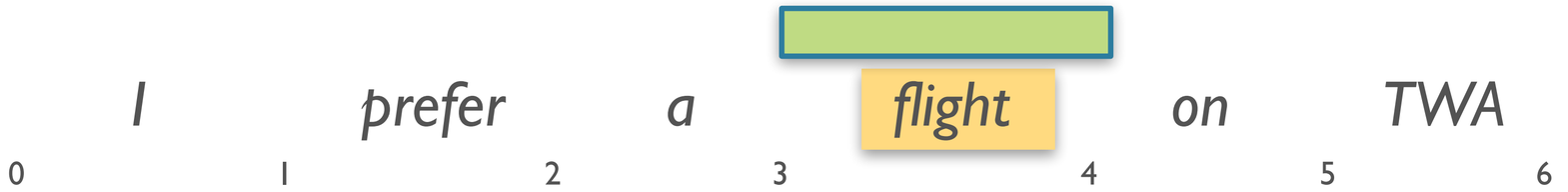

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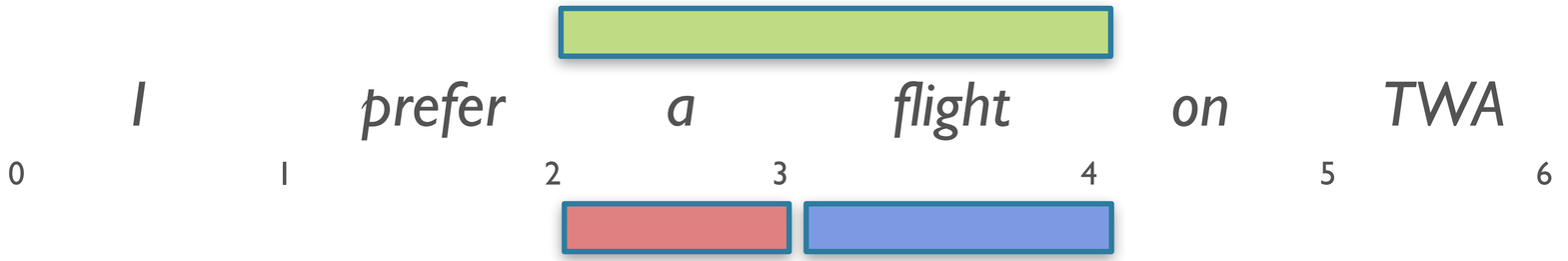

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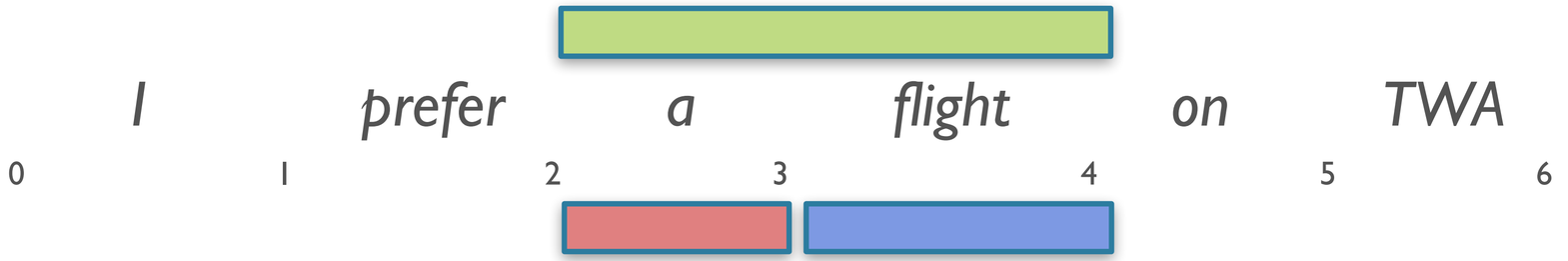

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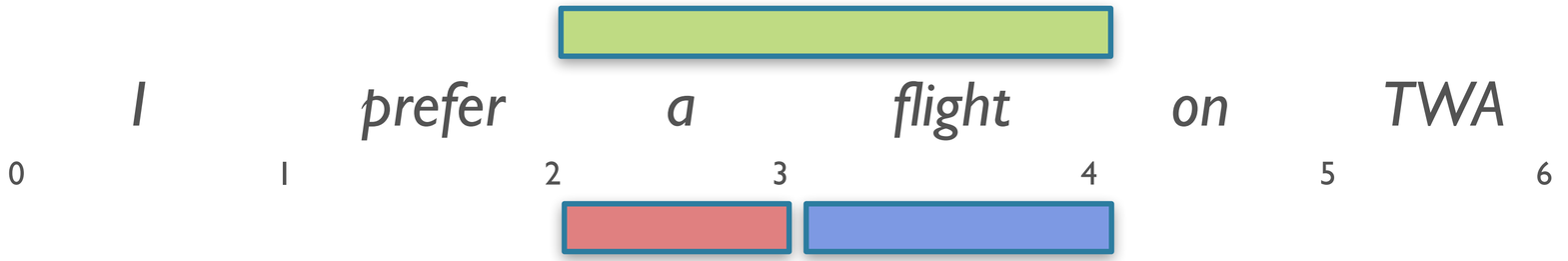

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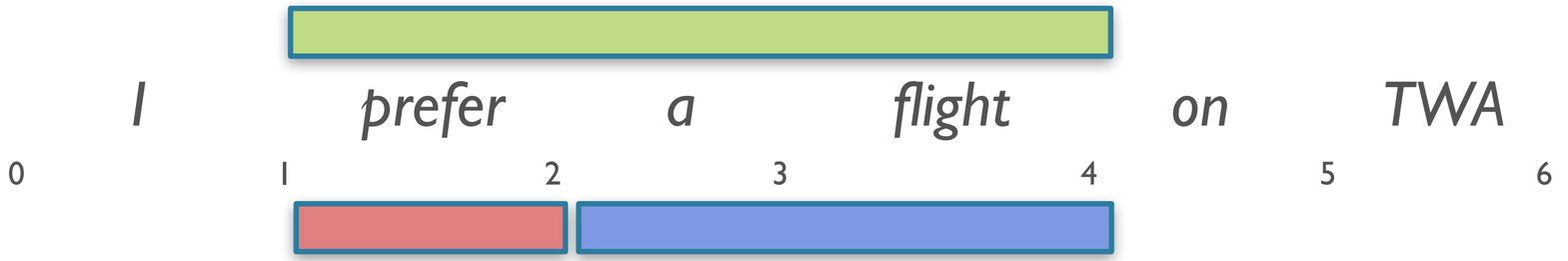

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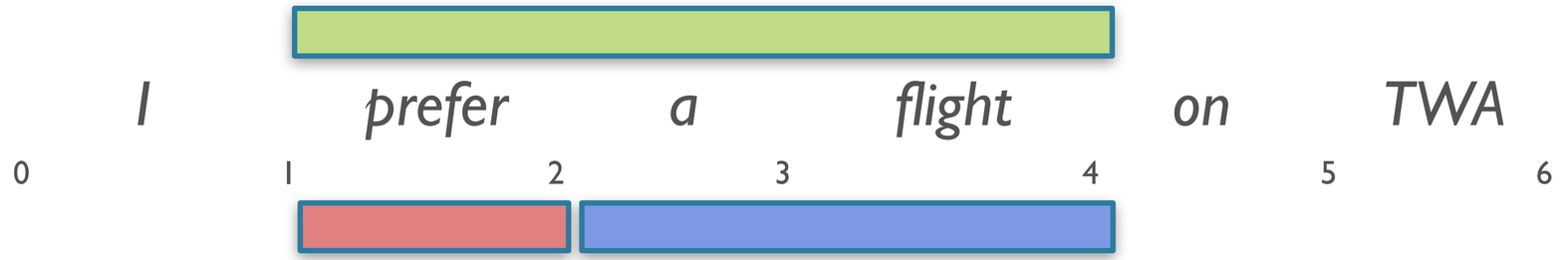

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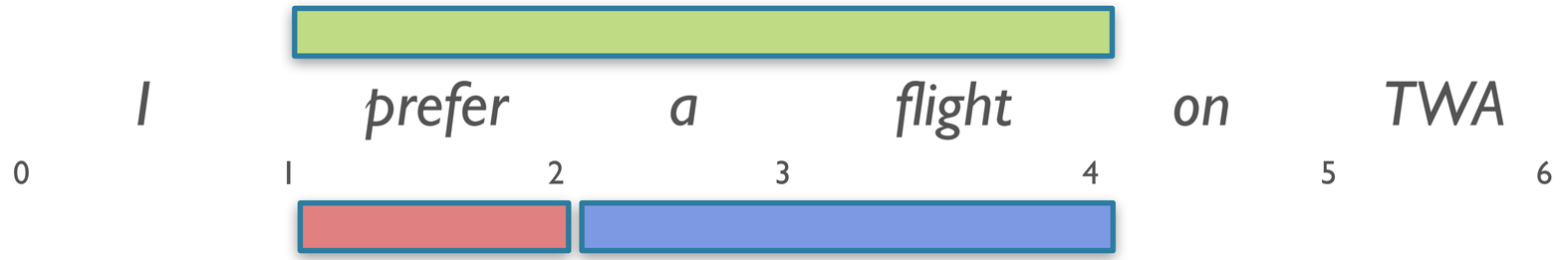

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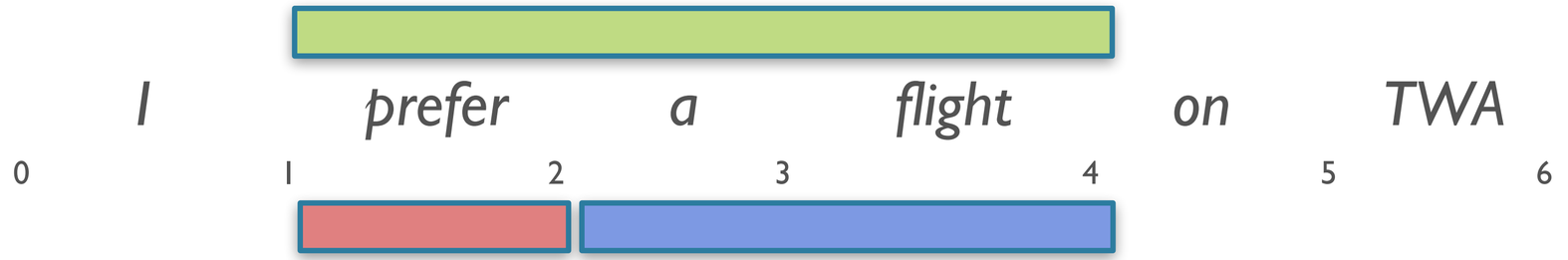

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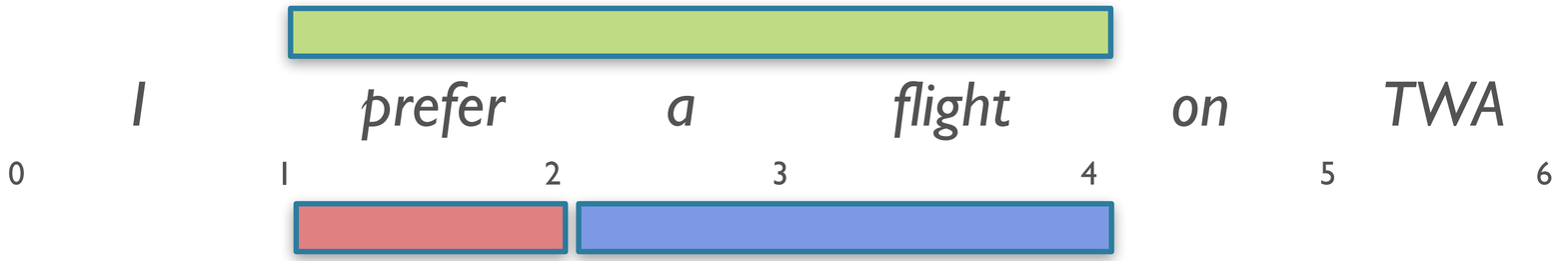

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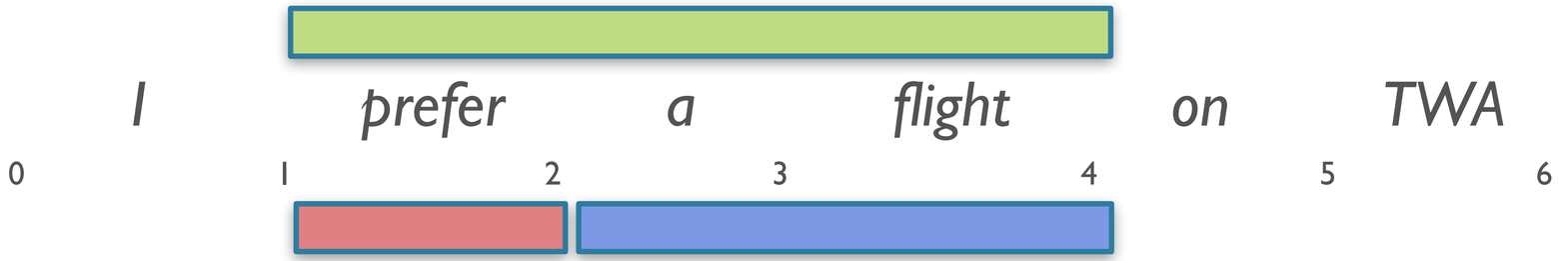

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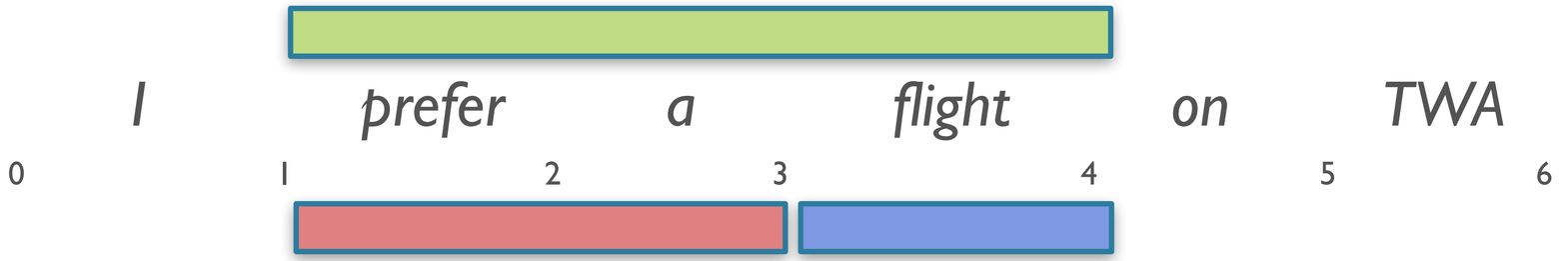

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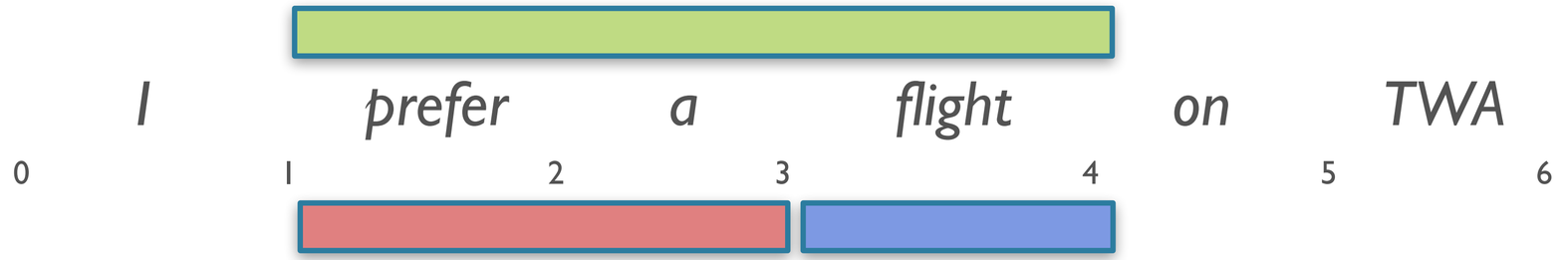

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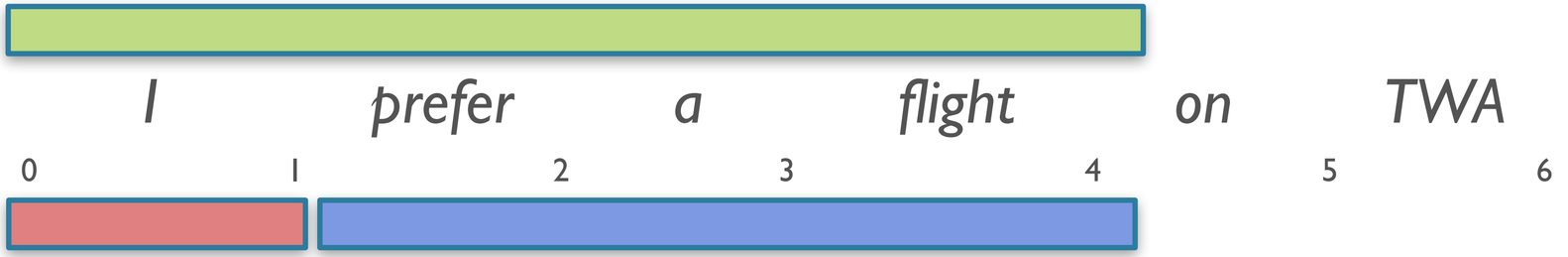

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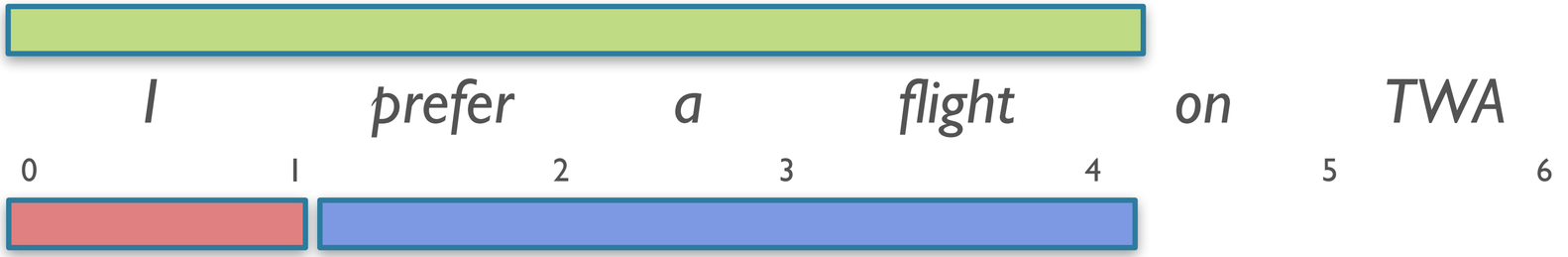

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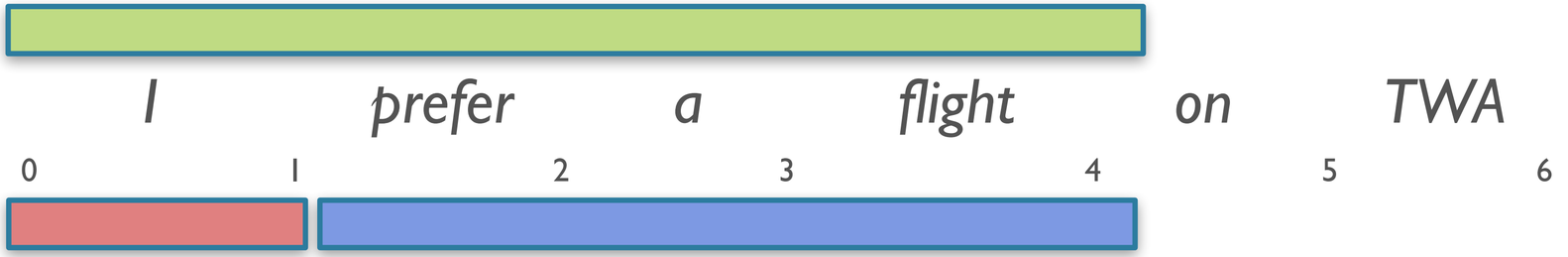
NP, Pronoun [0,1]	**S** [0,2]	[0,3]	**S** [0,4]		
	Verb, VP, S [1,2]	[1,3]	**VP, X2, S** [1,4]		
		Det [2,3]	**NP** [2,4]		
			Noun, Nom [3,4]		

Lexicon

Det → that | this | a
Noun → book | flight | meal | money
Pronoun → I | she | me
Proper-Noun → Houston | TWA
Aux → does
Preposition → from | to | on | near | through
Verb → book | include | prefer



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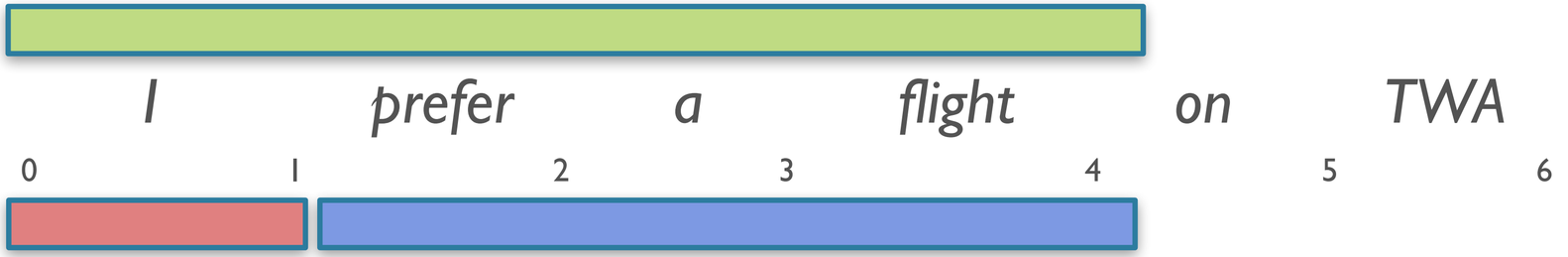

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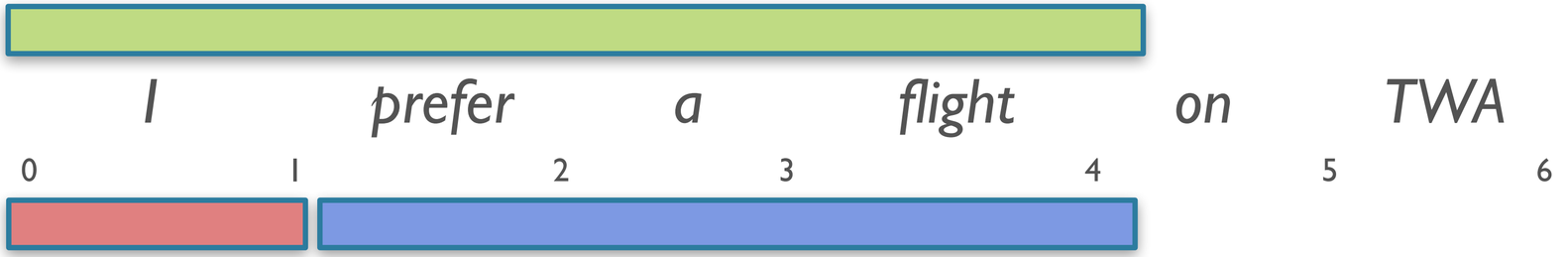

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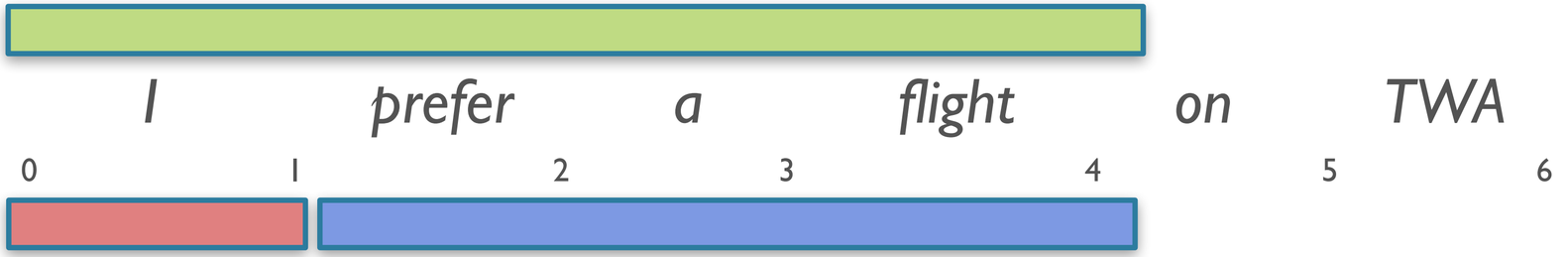

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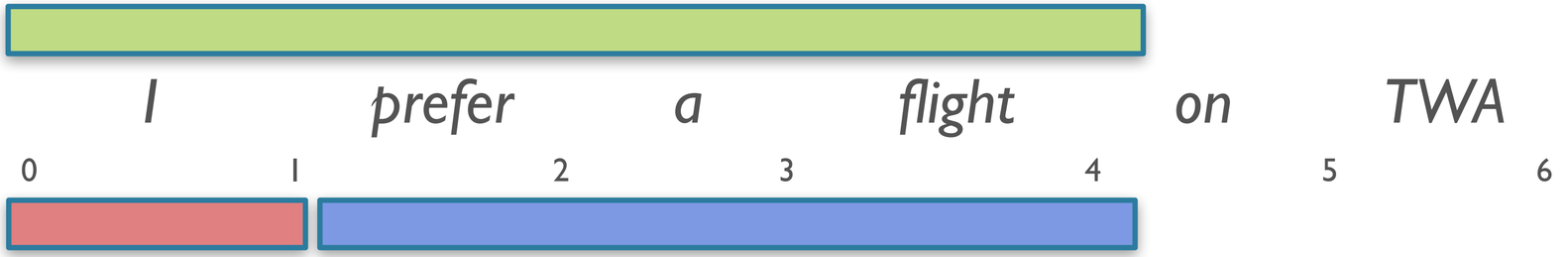

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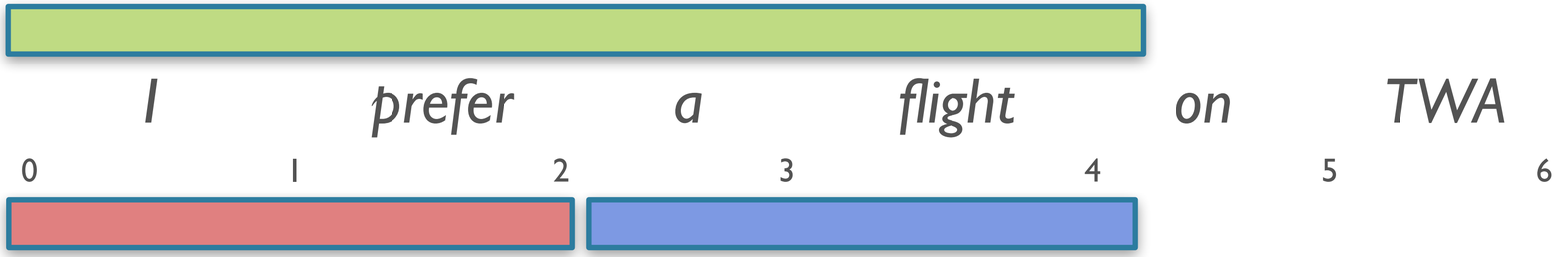

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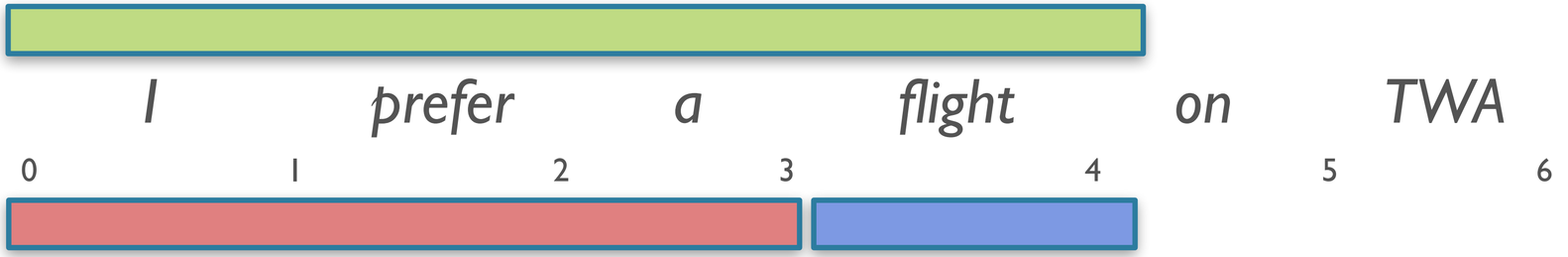

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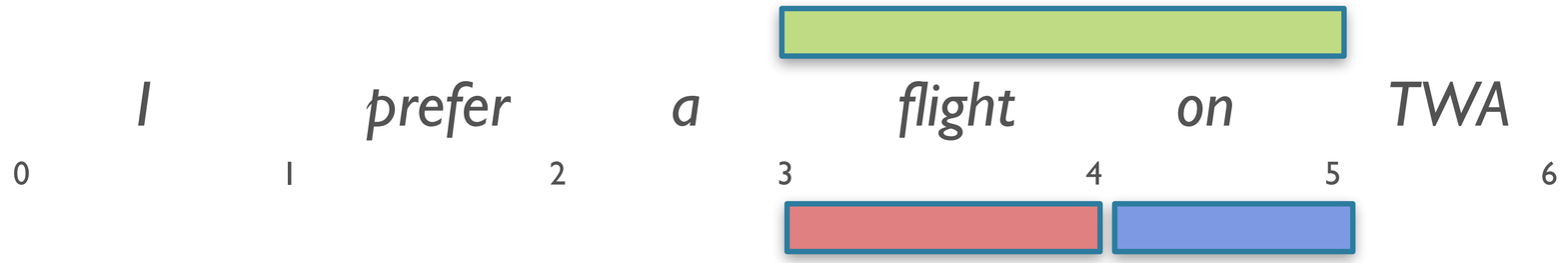

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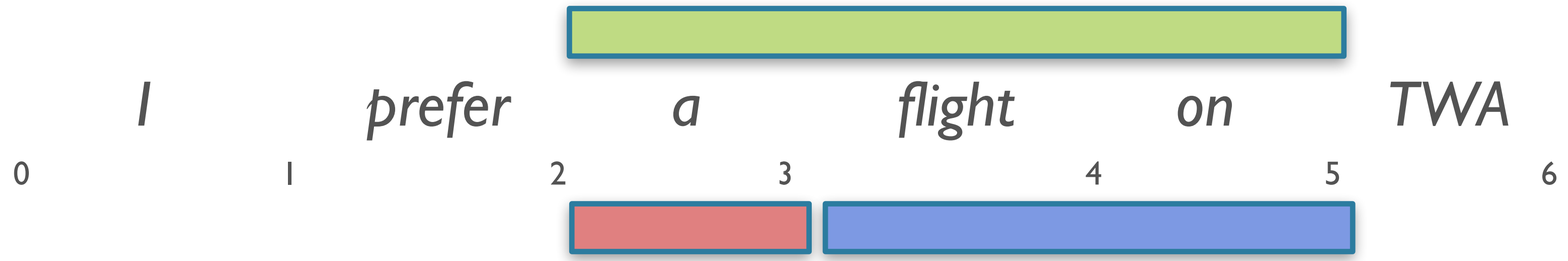

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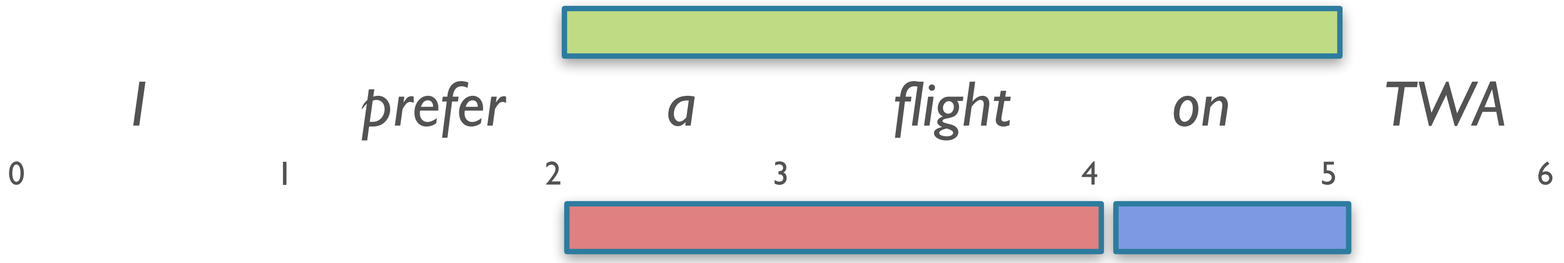

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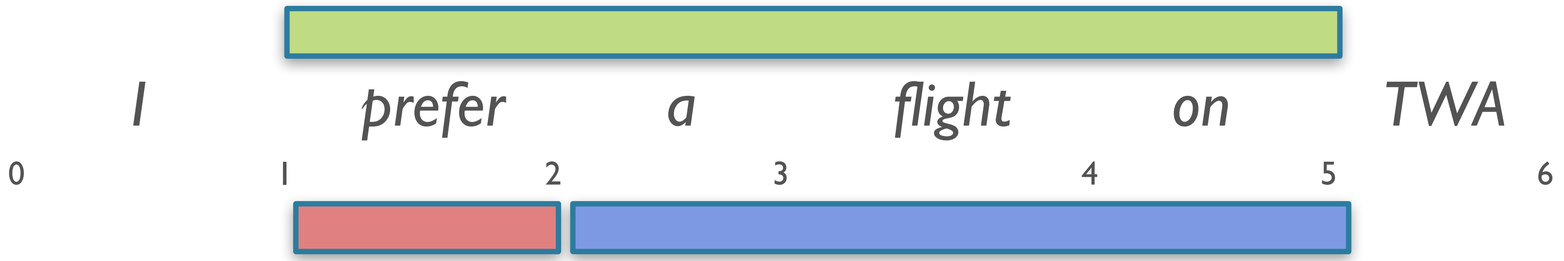

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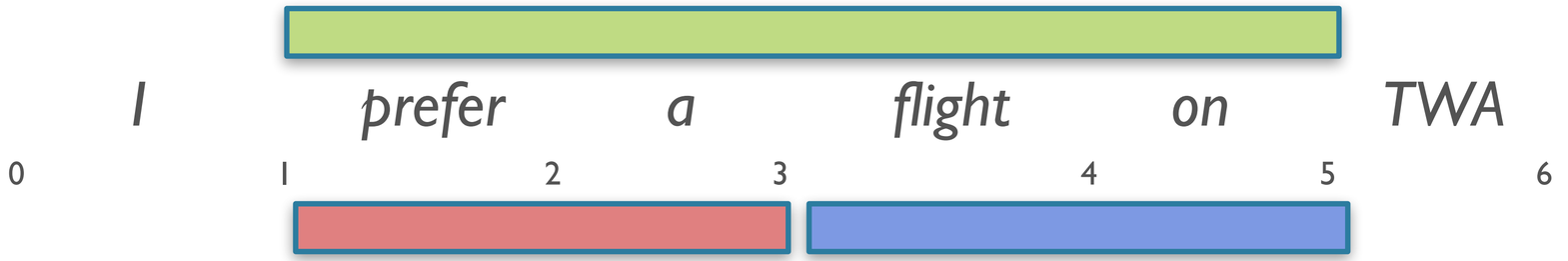

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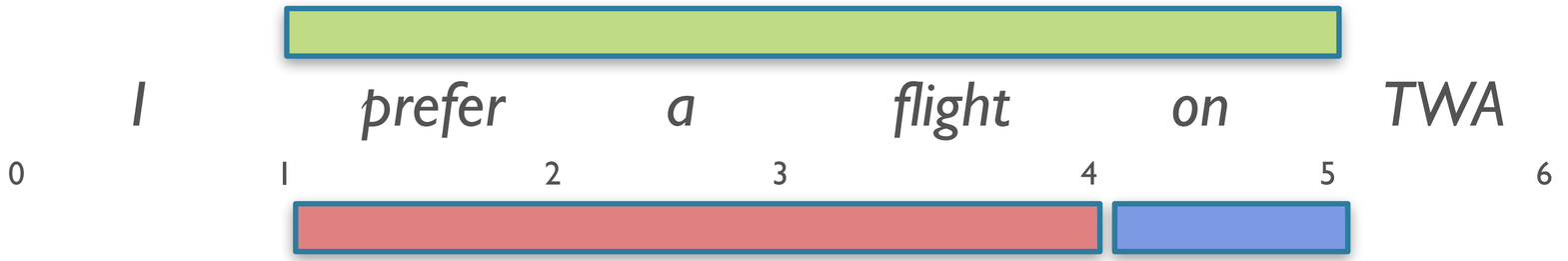

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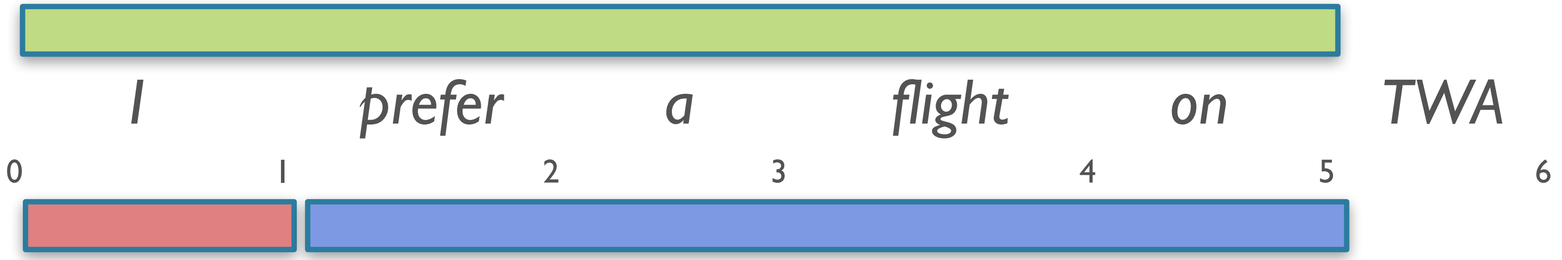

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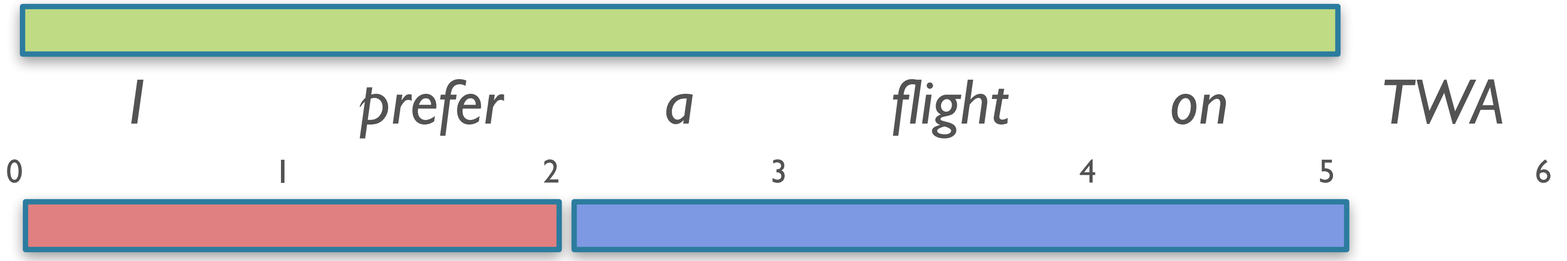

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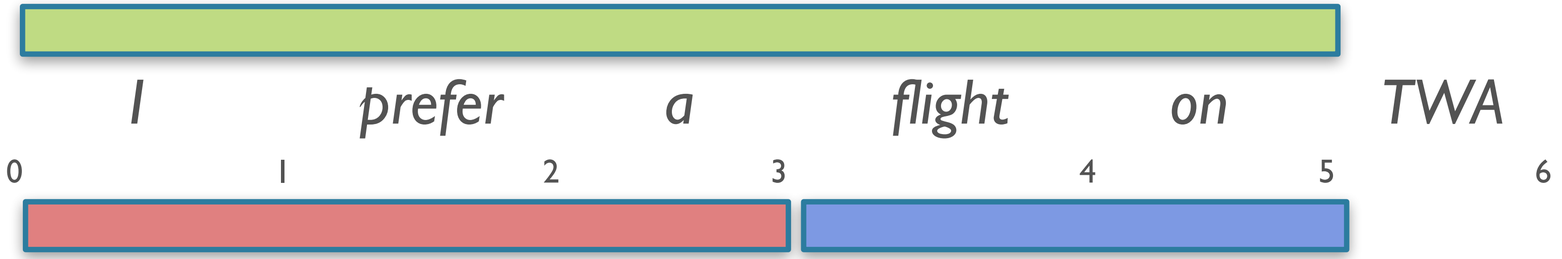
			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	

Lexicon

Det → that | this | a
Noun → book | flight | meal | money
Pronoun → I | she | me
Proper-Noun → Houston | TWA
Aux → does
Preposition → from | to | on | near | through
Verb → book | include | prefer



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S → Verb NP
S → X2 PP
S → Verb PP
S → VP PP

NP → I | she | me
NP → TWA | Houston
NP → Det Nominal

Nominal → book | flight | meal | money
Nominal → Nominal Noun
Nominal → Nominal PP

VP → book | include | prefer
VP → Verb NP
VP → X2 PP
X2 → Verb NP
VP → Verb PP
VP → VP PP

PP → Preposition NP

NP, Pronoun [0,1]	**S** [0,2]	[0,3]	**S** [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	**VP, X2, S** [1,4]	[1,5]	
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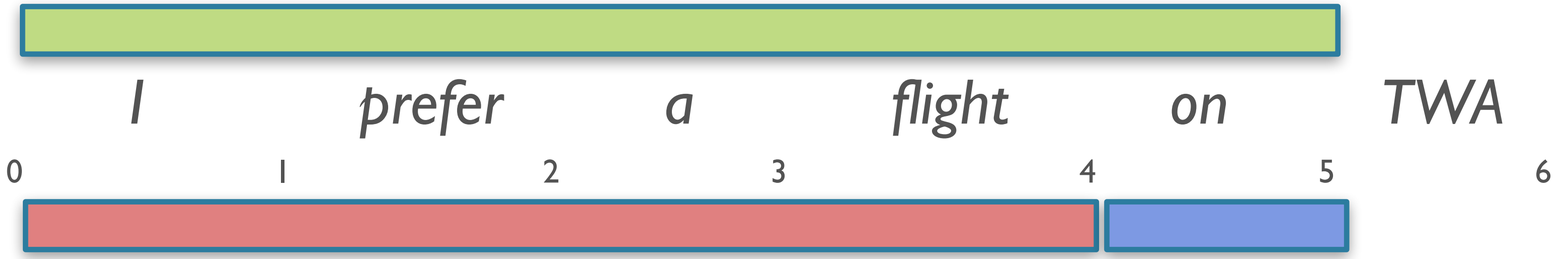

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			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	
					NNP, NP [5,6]

I *prefer* *a* *flight* *on* *TWA*

0 1 2 3 4 5 6

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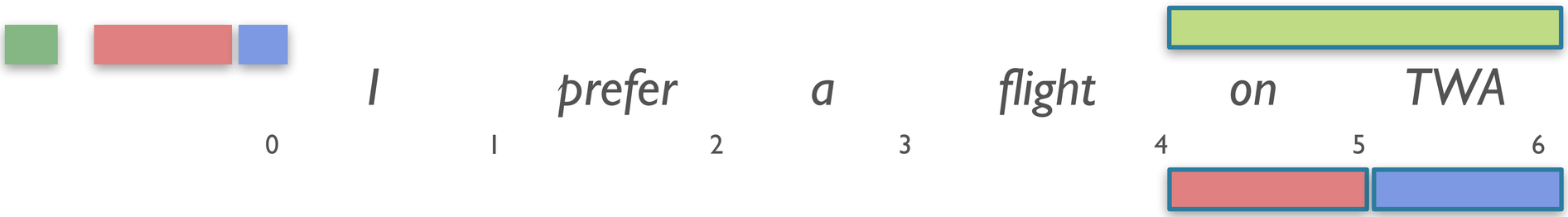

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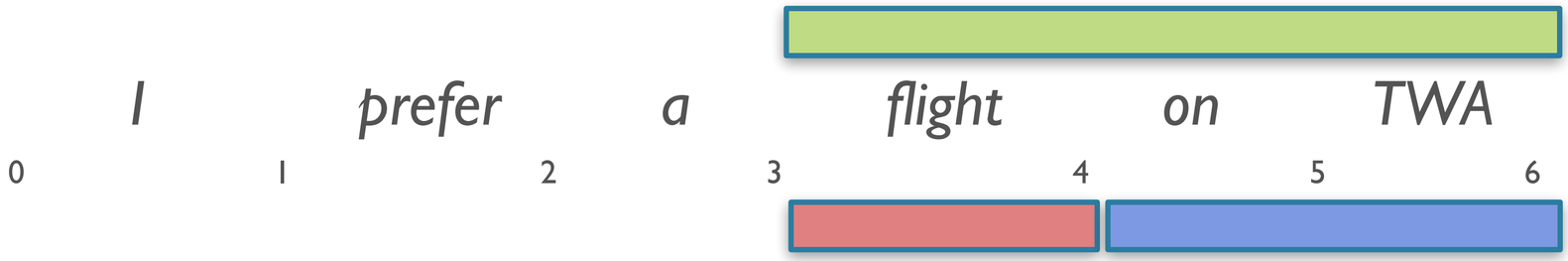

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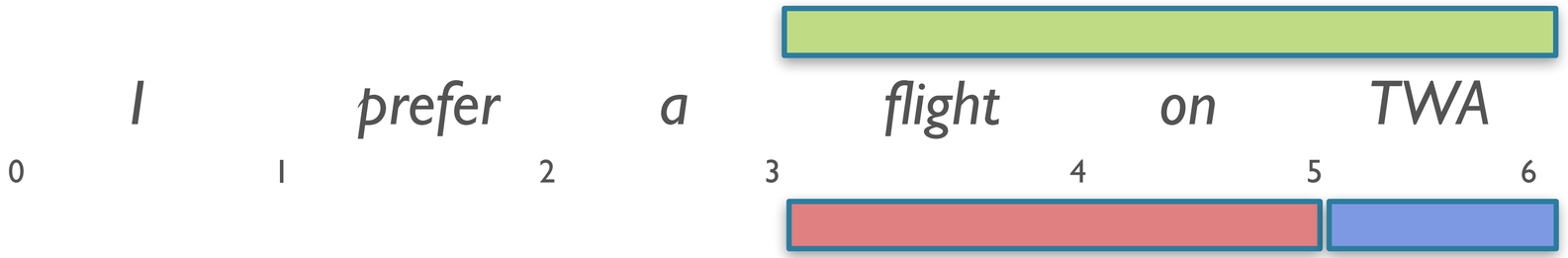

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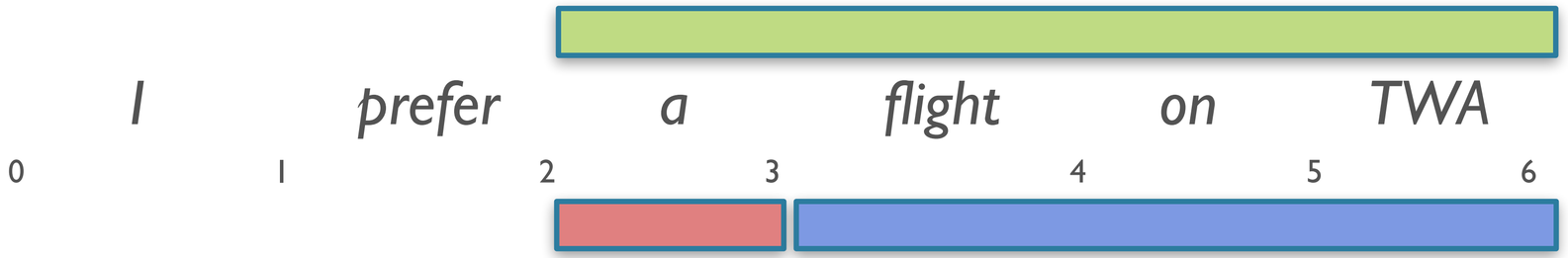

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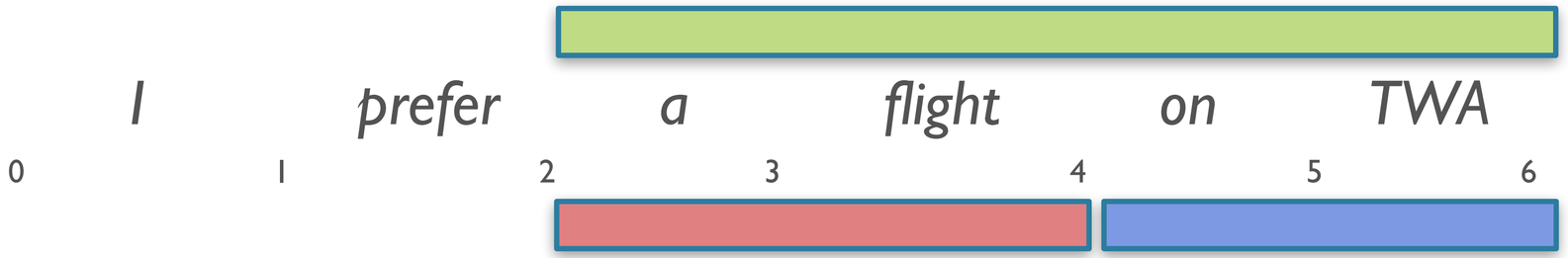

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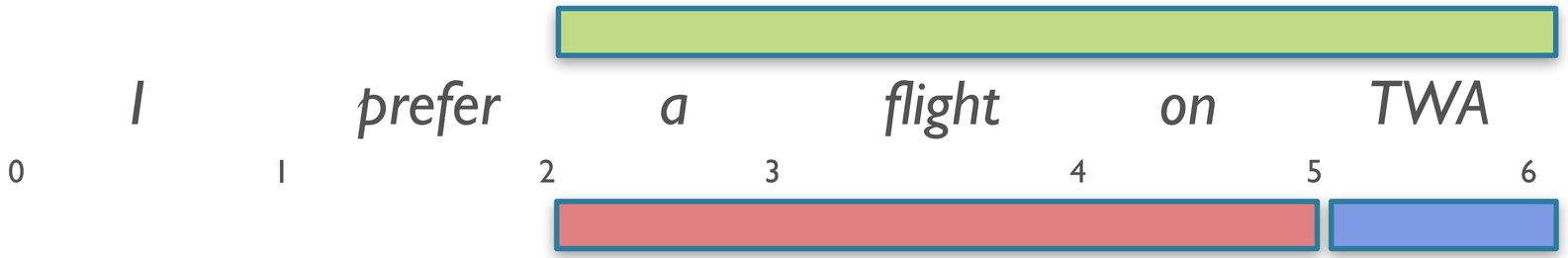

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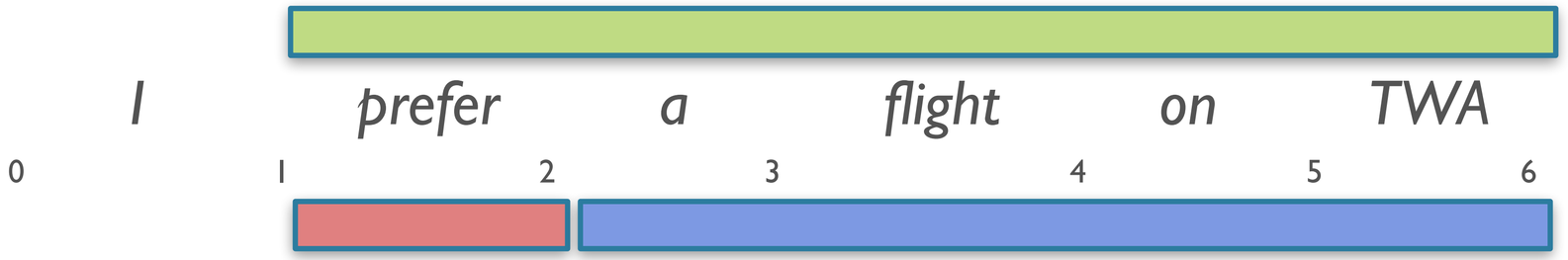

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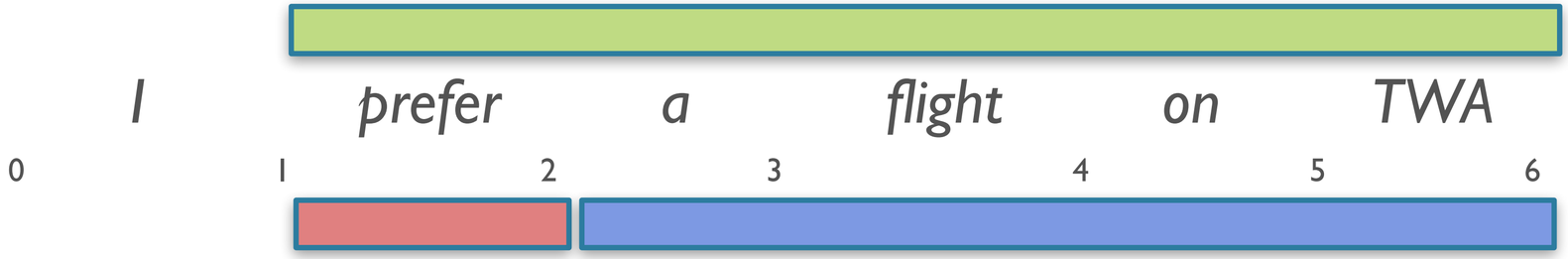

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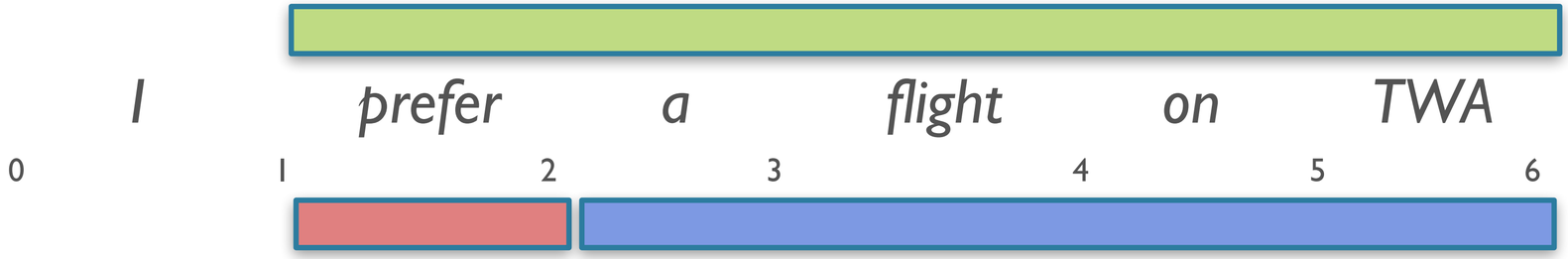

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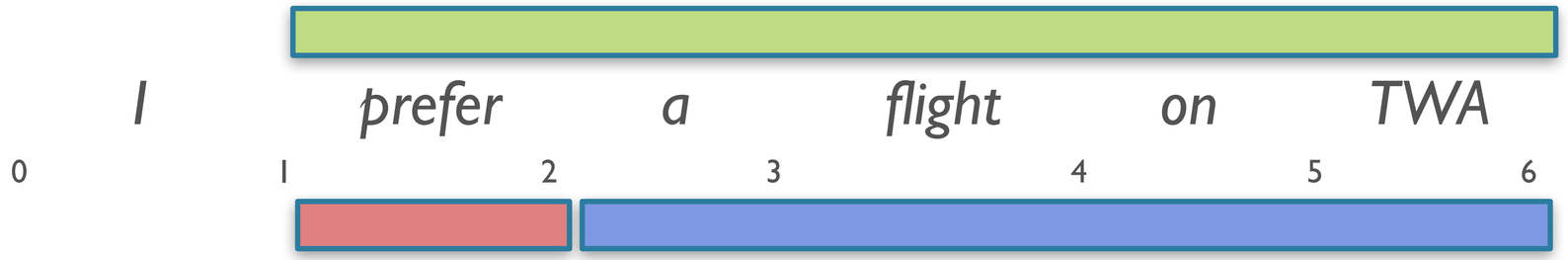

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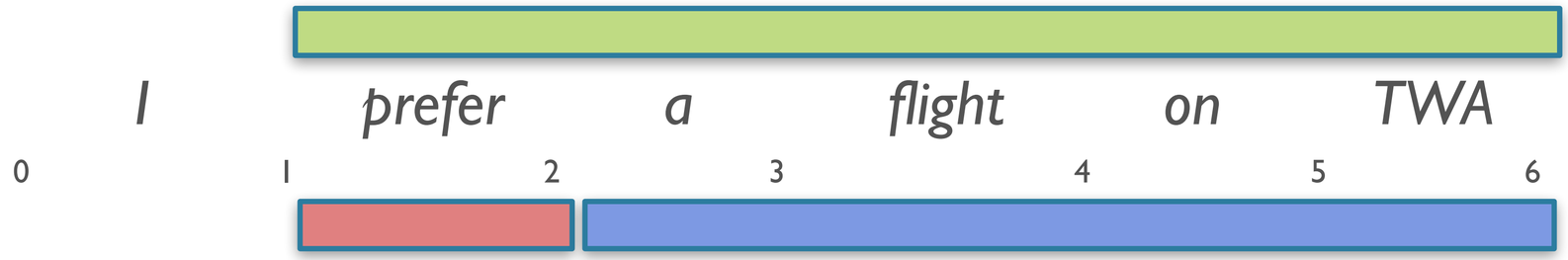

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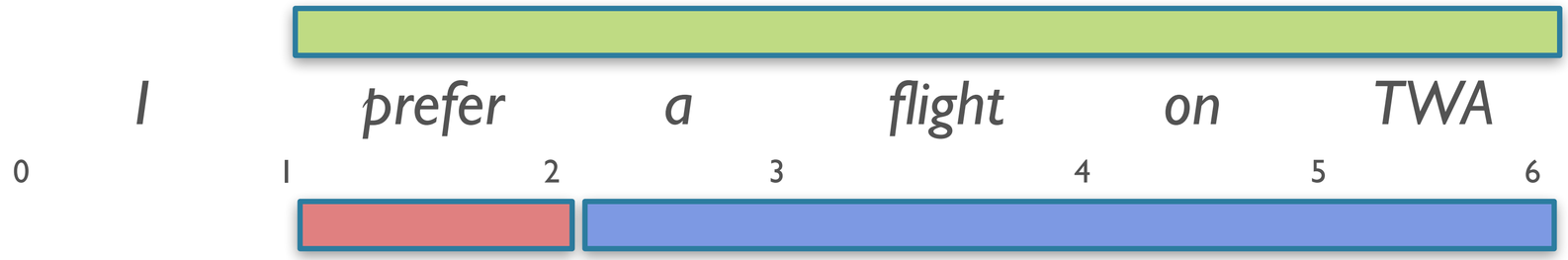

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NP → Det Nominal

Nominal → book | flight | meal | money
Nominal → Nominal Noun
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VP → Verb NP
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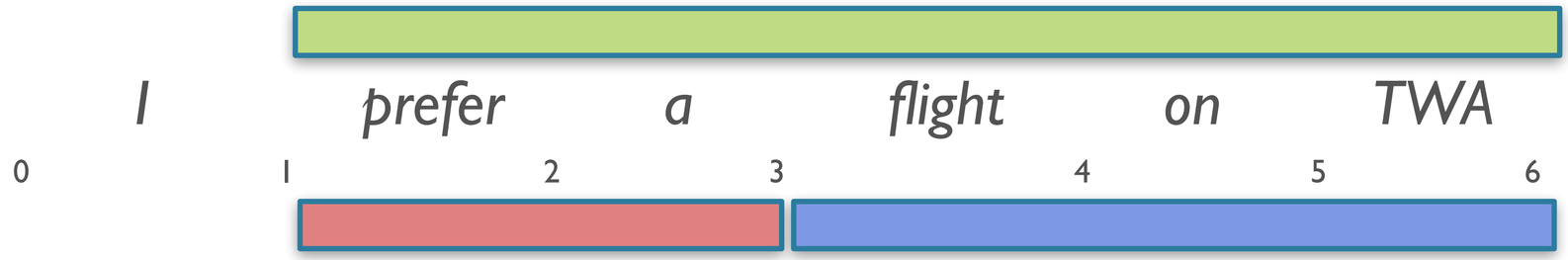

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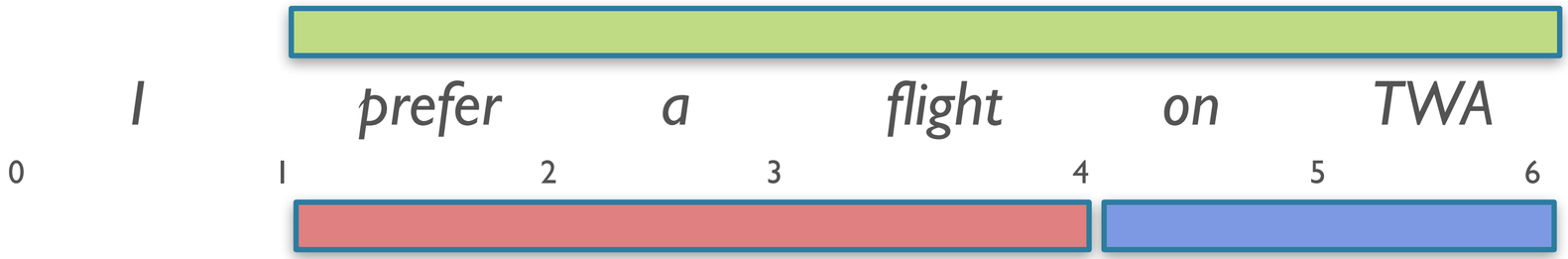

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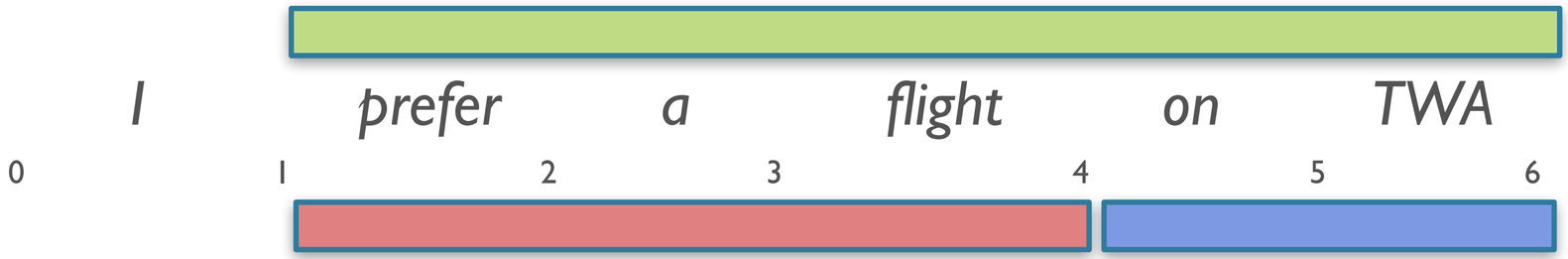

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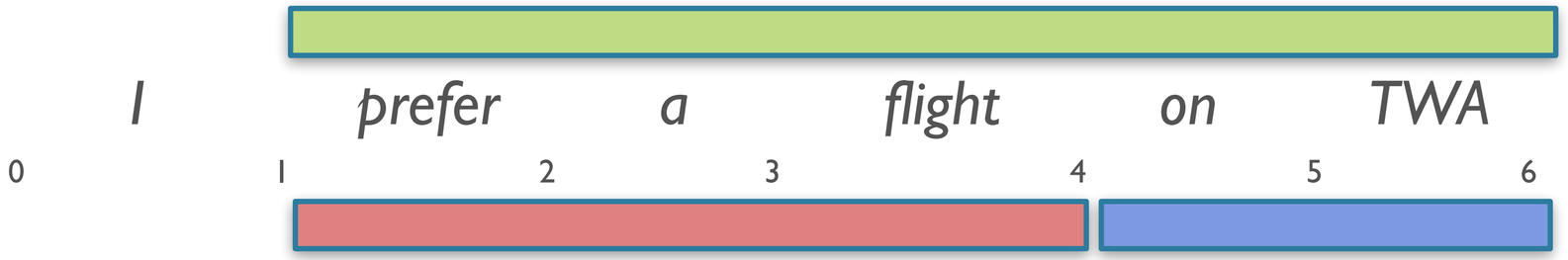

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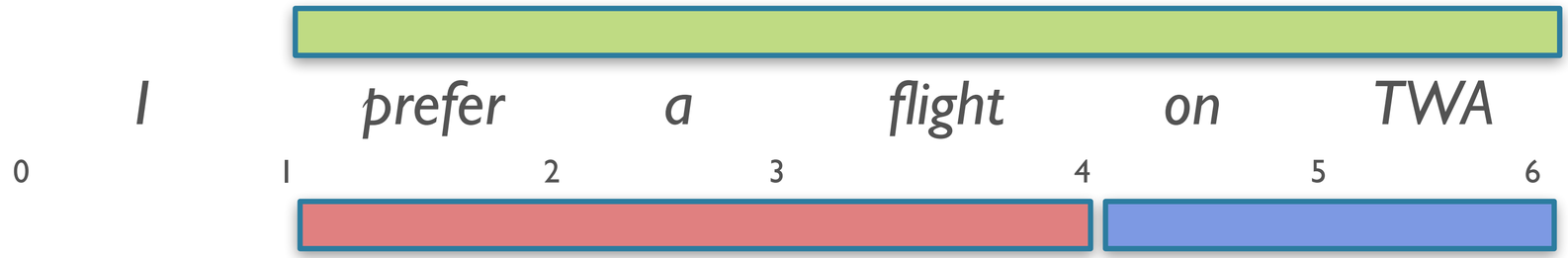

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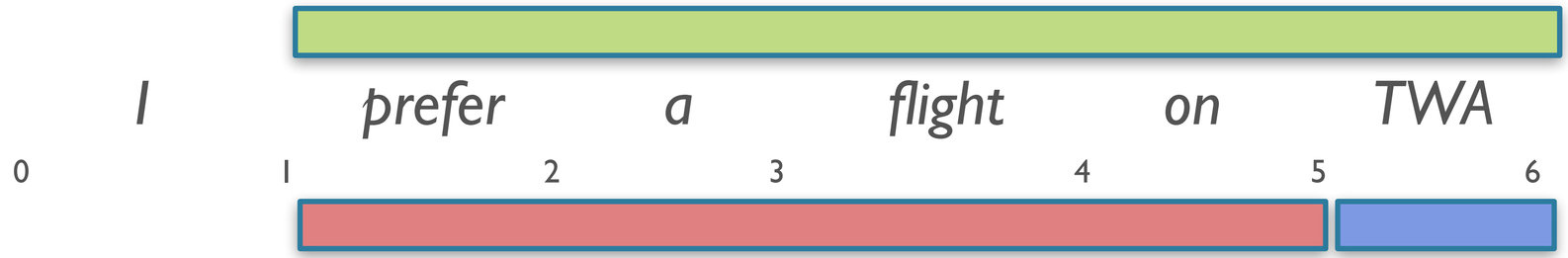

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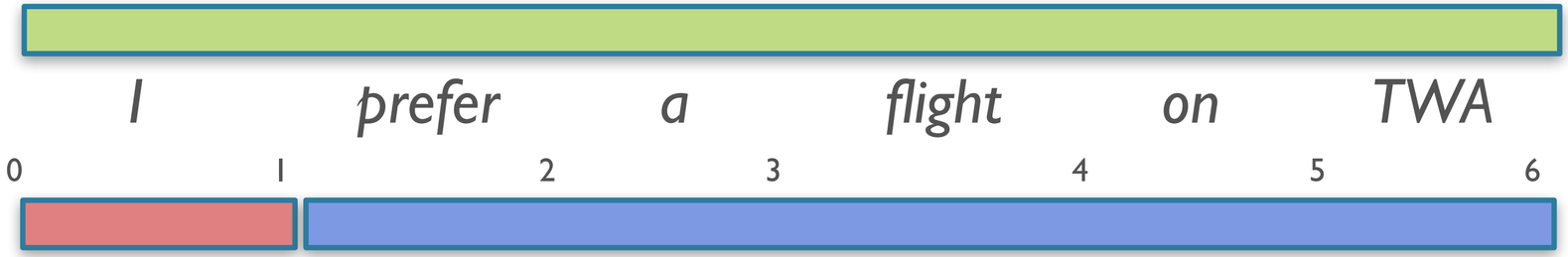

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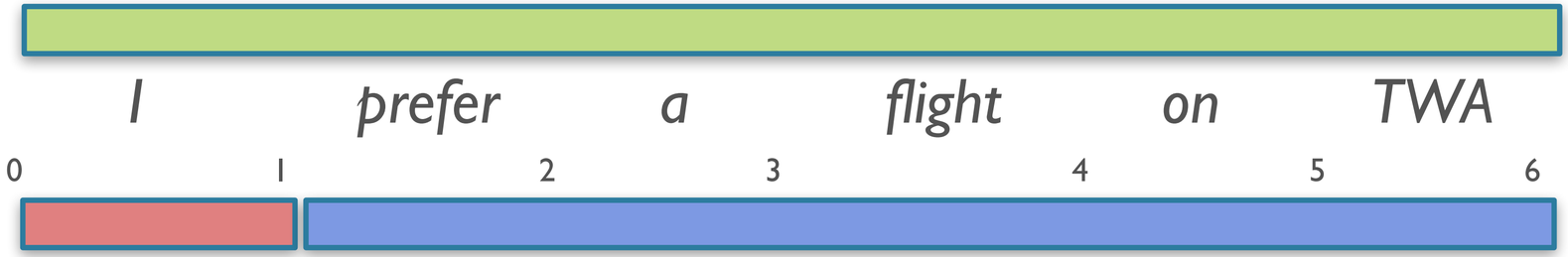

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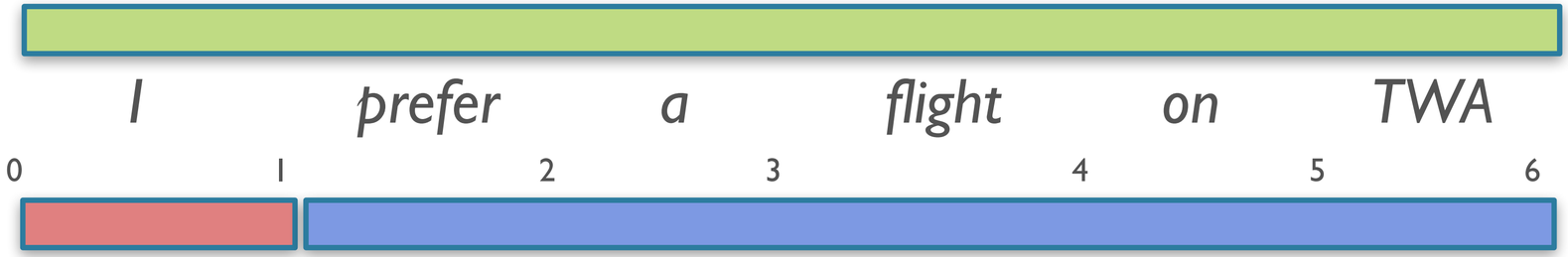

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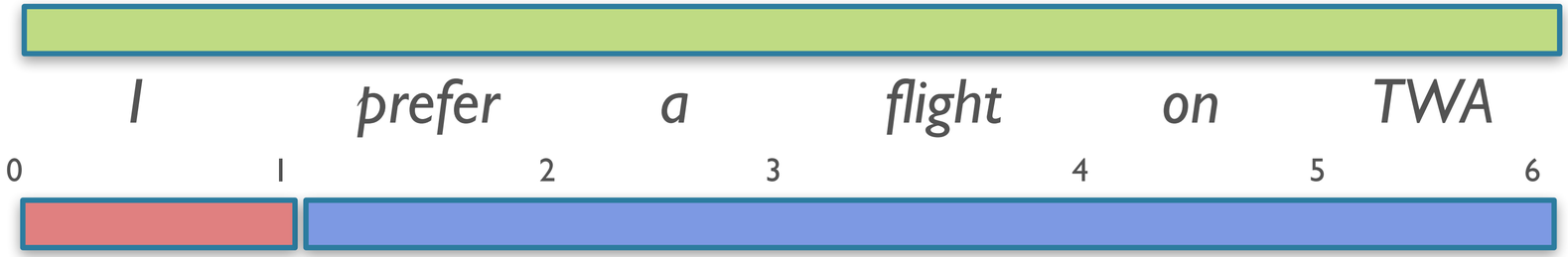

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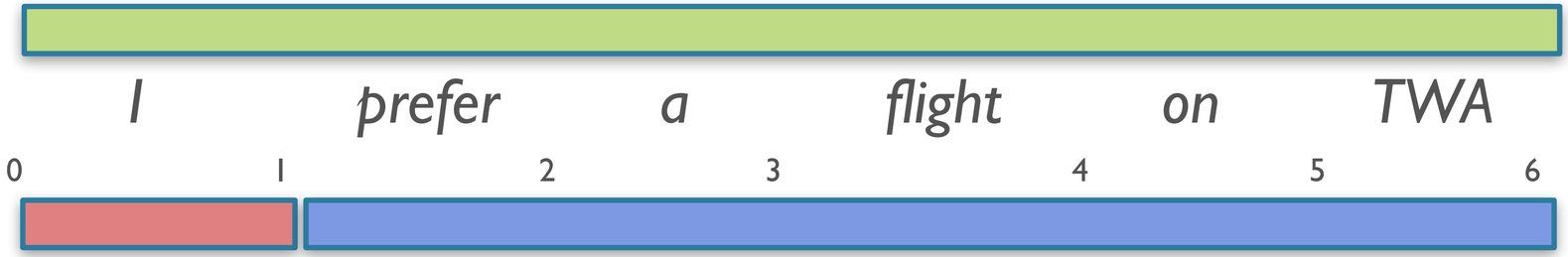

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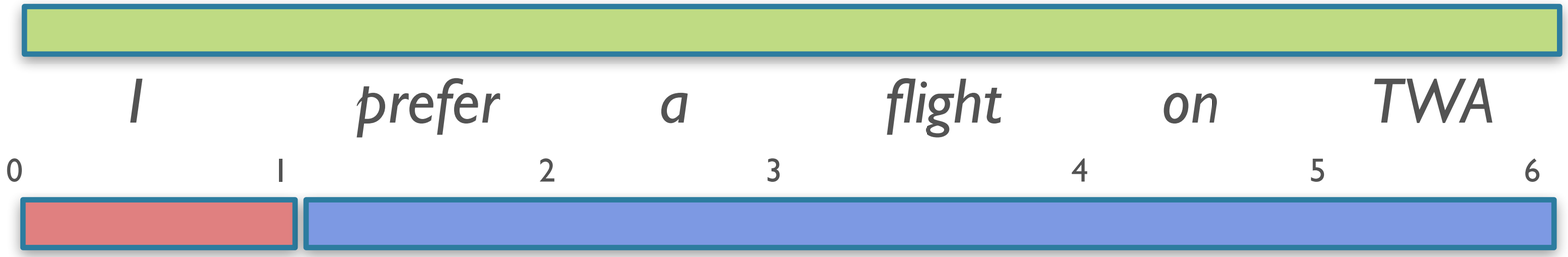

Current CKY Algorithm

Limitations:

Only stores non-terminals in cell

Not rules or cells corresponding to RHS

Stores SETS of non-terminals

Multiple rules with same LHS collide

Currently only ***acceptance/recognition***

CKY *Parser*

- Pair each nonterminal with back-pointer to cells from which it was derived
- Last step:
  - construct trees from back-pointers in [0, *n*]

NP, Pronoun [0,1]	**S** [0,2]	[0,3]	**S** [0,4]	[0,5]	
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		Det [2,3]	**NP** [2,4]	[2,5]	**NP** [2,6]
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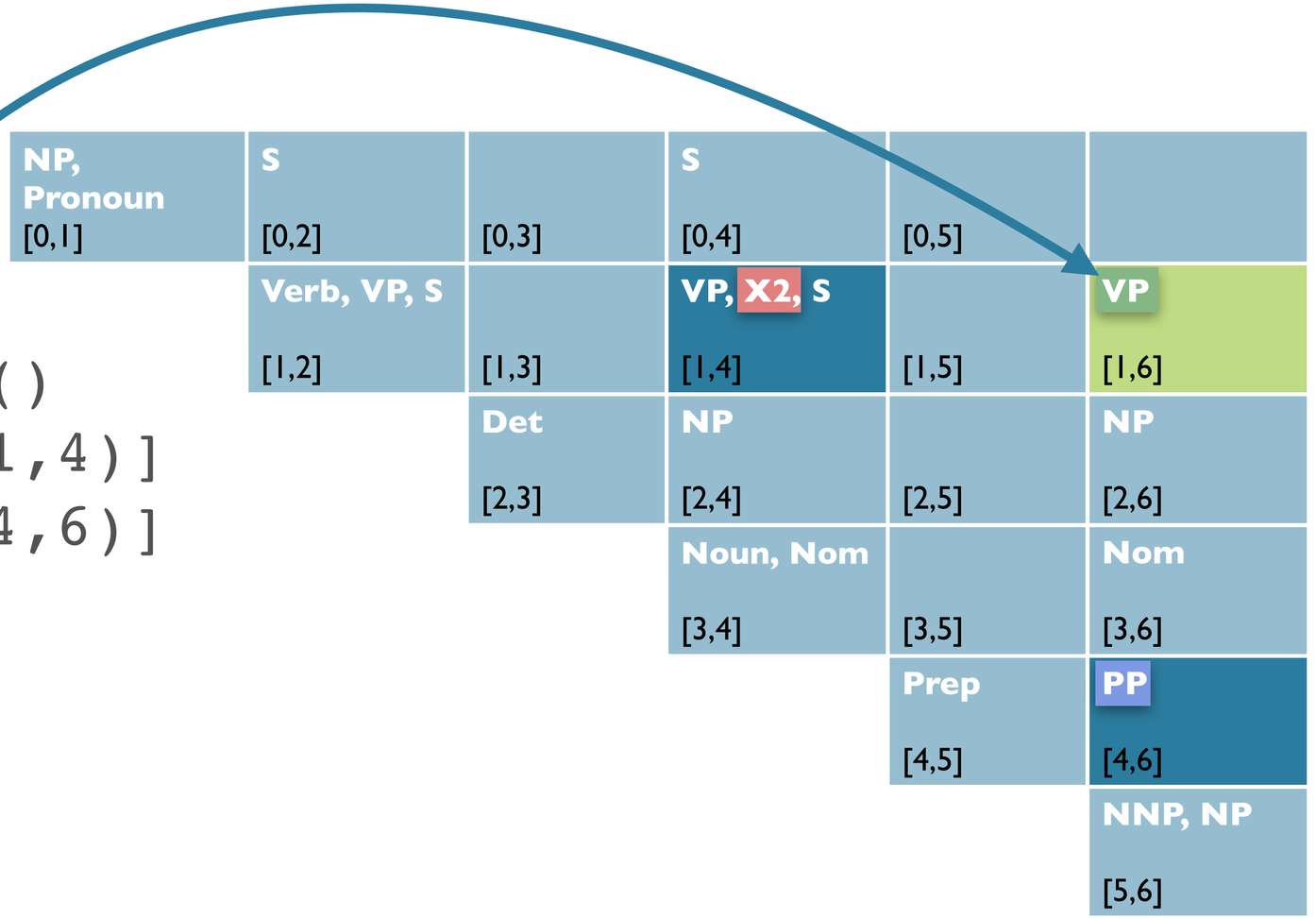
```
   bp_1 = BackPointer()
bp_1.l_child = [VP, (1,4)]
bp_1.r_child = [PP, (4,6)]
```

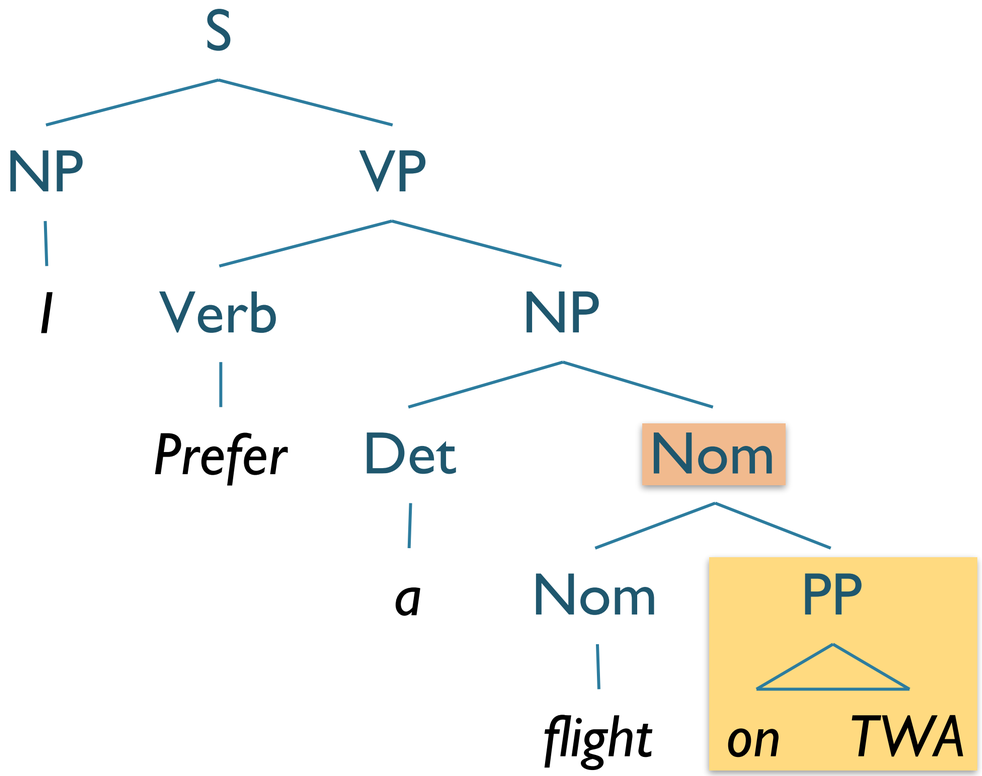
NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

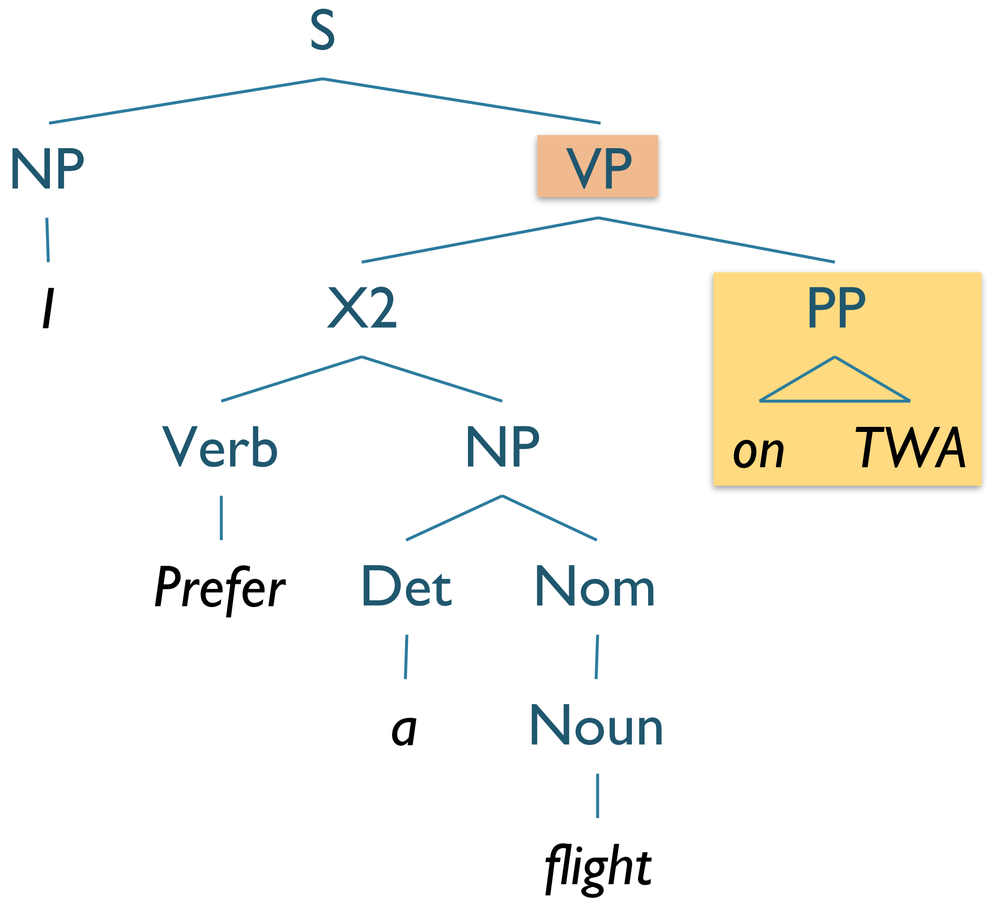
VP → X2 PP

```
    bp_2 = BackPointer()
bp_2.l_child = [X2, (1,4)]
bp_2.r_child = [PP, (4,6)]
```



Resulting Parses





CKY Discussion

Running time:

- **O(n^3)** where ***n*** is the length of the input string
- Inner loop grows as square of # of non-terminals

Expressiveness:

- As implemented, requires CNF
  - Weak equivalence to original grammar
  - Doesn't capture full original structure
  - Back-conversion?
    - Can do binarization, terminal conversion
    - Unit productions requires change in CKY