# CKY Parsing

LING 571 — Deep Processing Techniques for NLP
October 12, 2020
Shane Steinert-Threlkeld

#### Announcements

- HW1: grades posted
  - In general, we will aim for Monday release of grades.
- Some notes:
  - Make sure dir structure of your submission is correct [generally, no nested directories]
    - Run check\_hwX.sh before submitting!
  - Pay careful attention to format of output files; make sure there's exact match on toy examples
  - patas/dryas, etc: will regrade HW1 if that was an issue
    - Use full paths for python binaries to be safe

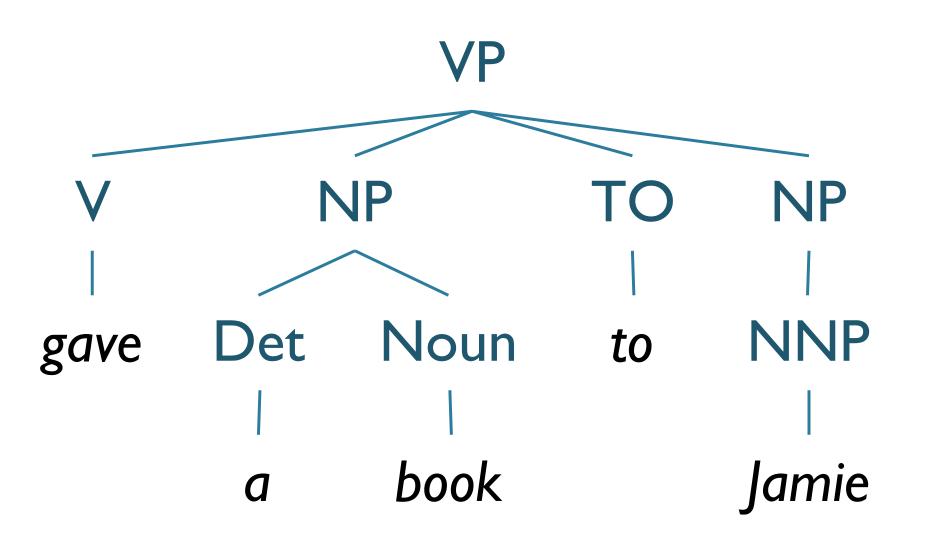
#### Announcements: HW #2

- Due Wednesday, 11:00pm Pacific
- Grammar output format:
  - Terminals quoted (single or double)
  - One production per line
  - Optionally, RHS alternates with vertical bar/pipe character: I
    - Start symbol is first nonterminal encountered in file

### Variation in CNF: Binarization

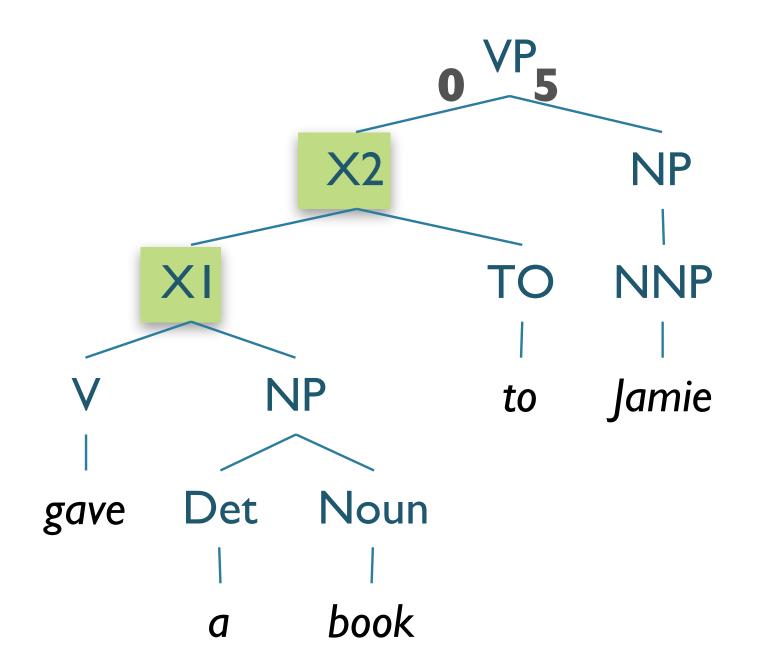
#### Original Rule

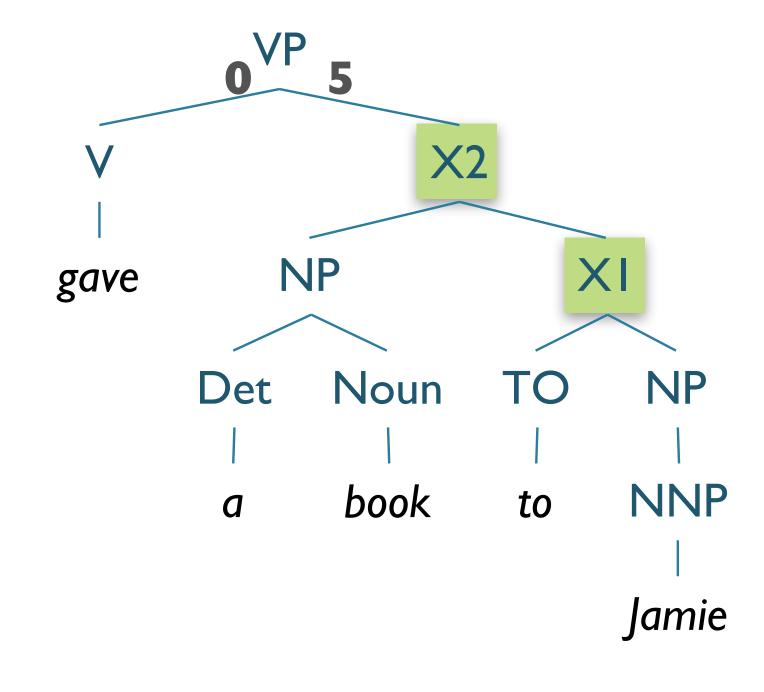
 $VP \rightarrow V NP TO NP$ 



### Variation in CNF: Binarization

Original Rule			
VP → V NP TO NP			
Left to Right Reduction		Right to Left Reduction	
$VP \rightarrow X1 TO NP$	$X1 \rightarrow V NP$	$VP \rightarrow V NP X1$	$X1 \rightarrow TO NP$
VP → X2 NP	$X2 \rightarrow X1 TO$	$VP \rightarrow V X2$	$X2 \rightarrow NP X1$





Seattle/Sealth/Si'ahl



- Seattle/Sealth/Si'ahl
- For those of you taking 550:
  - The Lushootseed spelling [IPA] of Chief Seattle/Sealth/Si'ahl:
    - si?at ['si?ar4]



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    - si?at ['si?ar4]
- Duwamish Dx<sup>w</sup>dəw?abš [dx<sup>w</sup>dɐw?ab∫]
- IPA resources:
  - https://en.wikipedia.org/wiki/International\_Phonetic\_Alphabet
  - http://web.mit.edu/6.mitx/www/24.900%20IPA/IPAapp.html



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  - Many interesting phenomena in non-Indo-European languages
- <u>Lushootseed</u> exhibits debatable distinction between verbs and nouns [link to Glottolog page for more references]
  - ?uxw ti sbiaw
    goes that-which is-a-coyote
    "The/a coyote goes"
  - sbiaw ti ?uxw is-a-coyote that-which goes "The one who goes is a coyote"
  - (Translation distinction provided for clarity semantically equivalent)

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is-a-coyote that-which goes
"The one who goes is a coyote"
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via Beck, 2013

- (Translation distinction provided for clarity semantically equivalent)
- Lillooet Salish quantification has repercussions for e.g. English (Matthewson 2001)

- UW American Indian Studies Courses
  - (Sometimes including language courses, e.g. Southern Lushootseed)
- Lushootseed resources: <a href="https://tulaliplushootseed.com/">https://tulaliplushootseed.com/</a>

### Roadmap

- CKY Parsing Algorithm
  - Dynamic programming structure
  - Algorithm and example
  - Analysis & Discussion

- Probabilistic CFGs
  - Motivation: Ambiguity
  - Augmenting CFGs with probabilities

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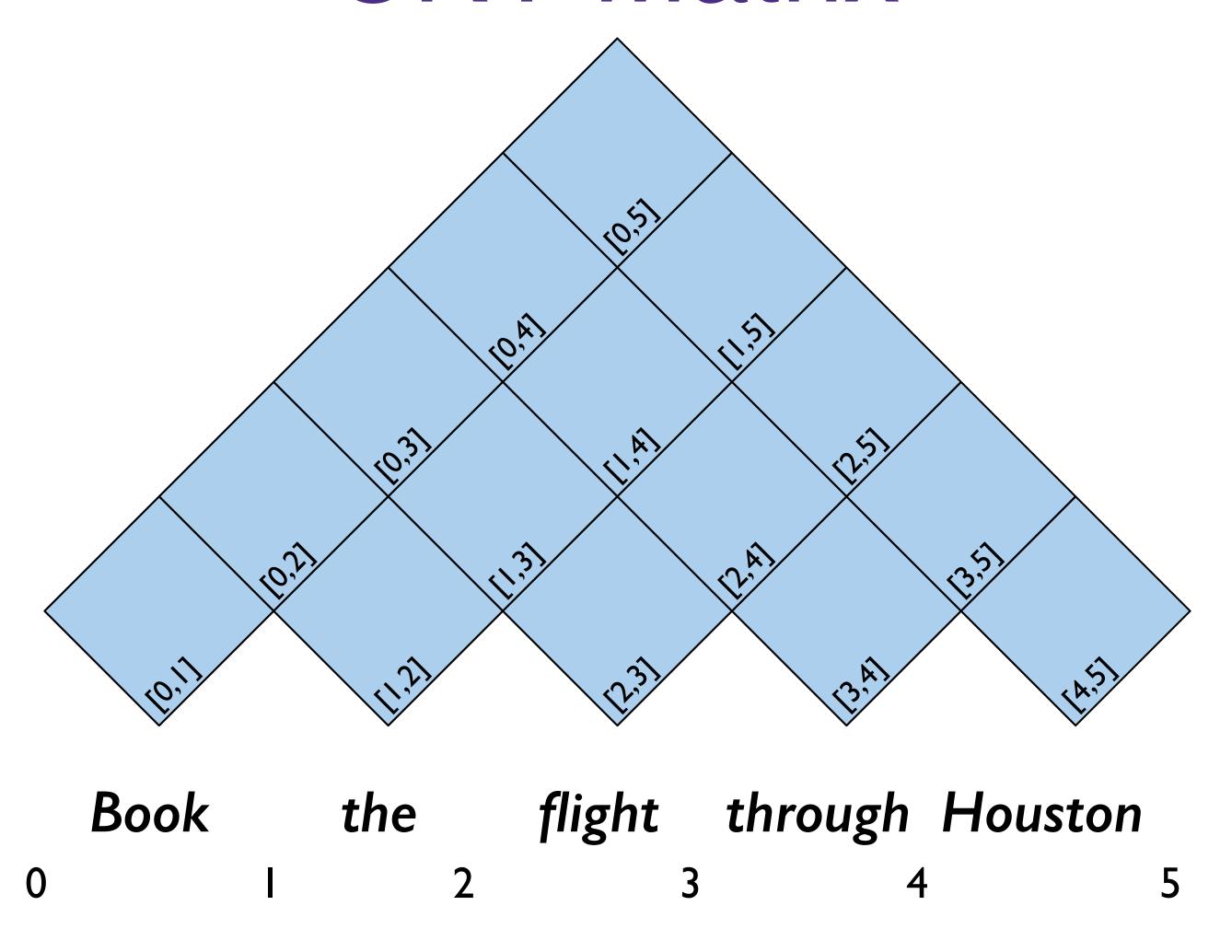
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  - Book 1 that 2 flight 3

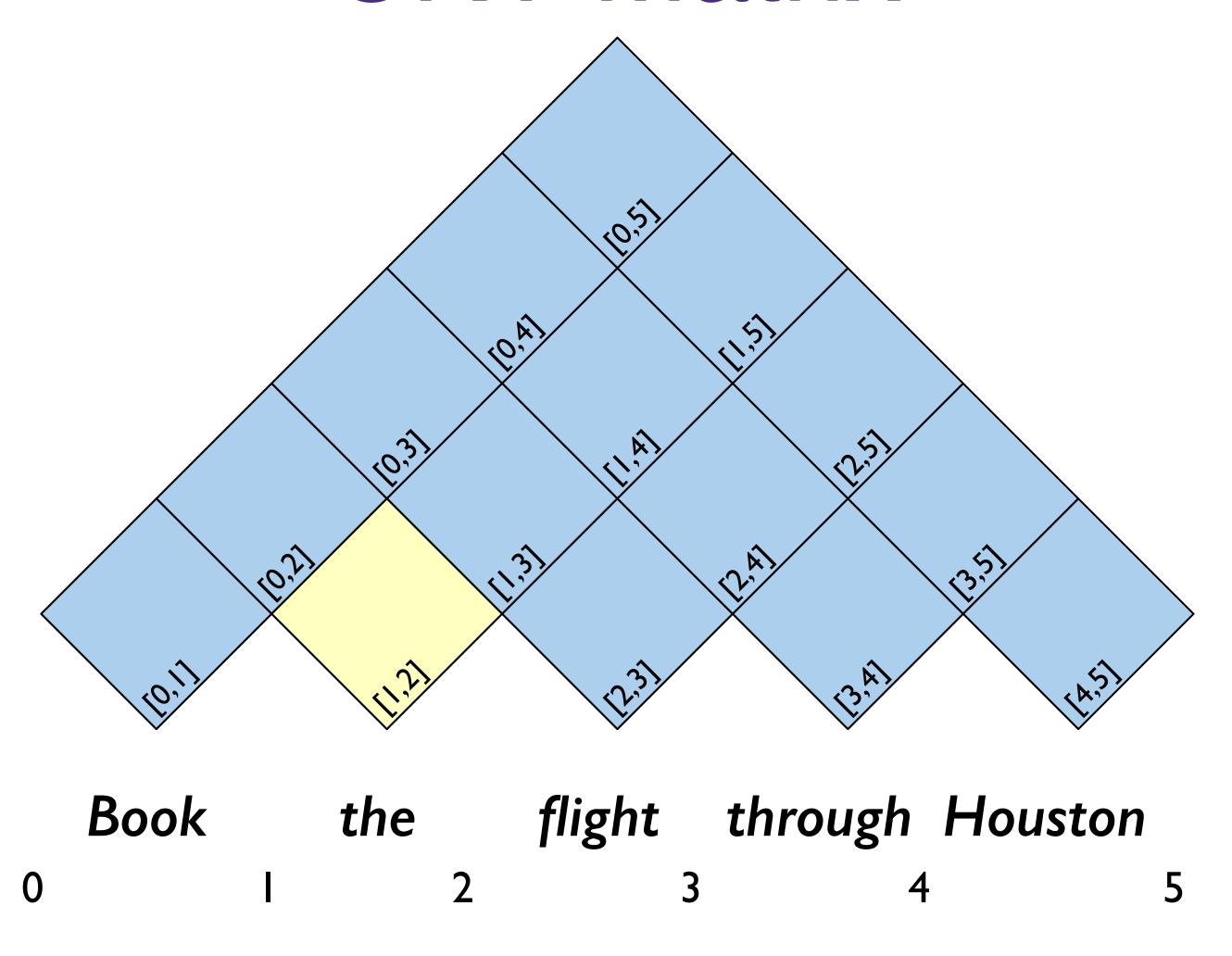
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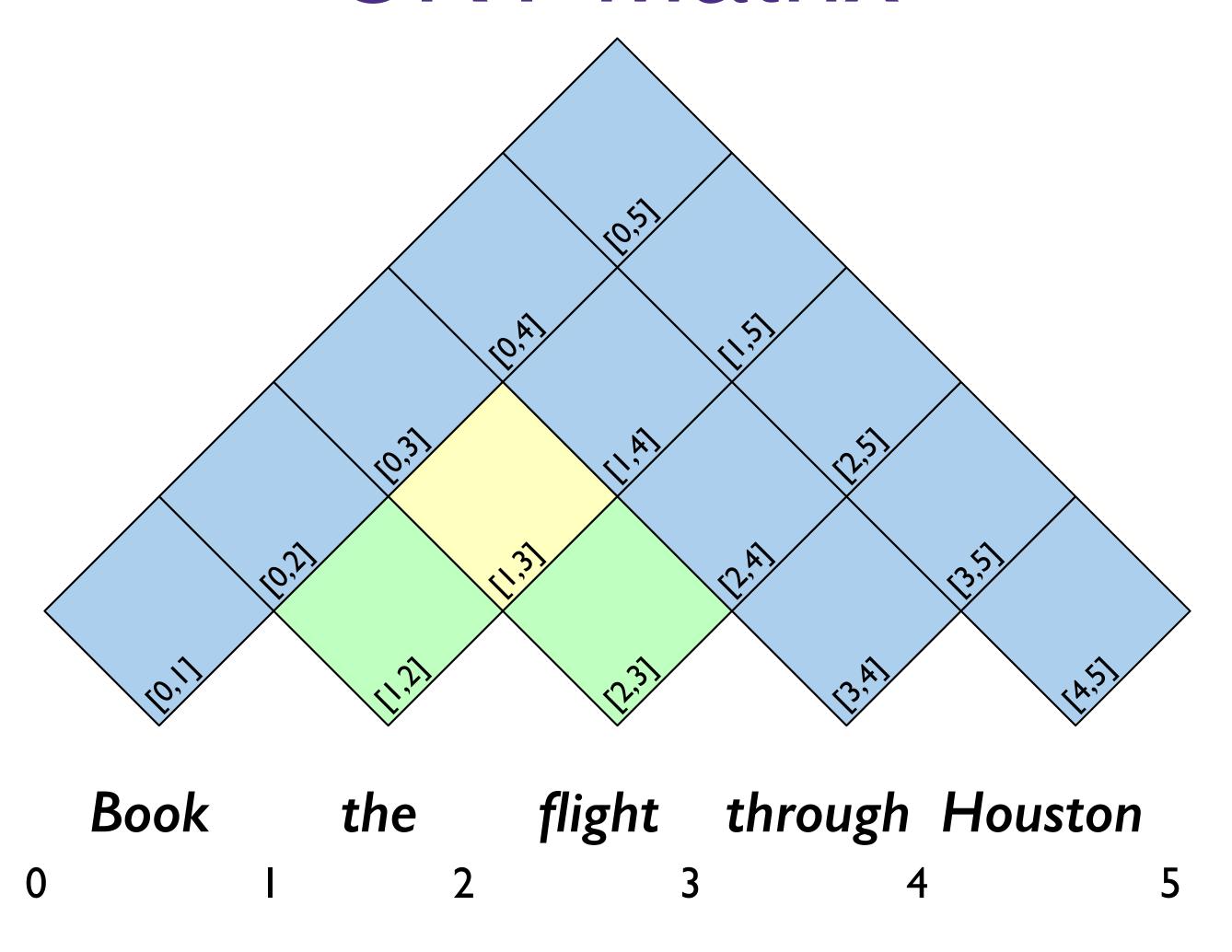
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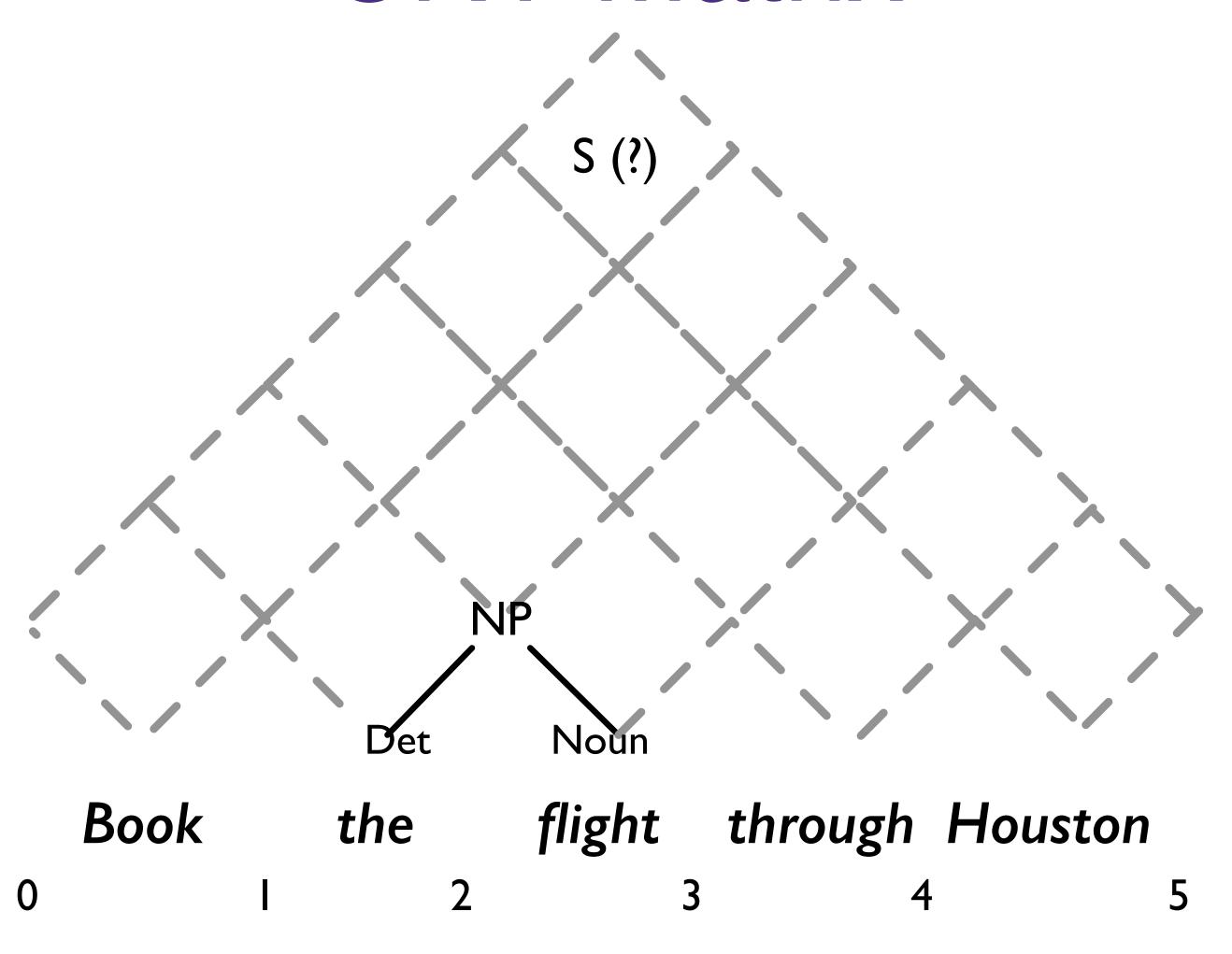
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- Each cell [ i, j ] contains SETS { } of non-terminals with derivations spanning i, j
  - [j-1, j] contains pre-terminals
  - if [0, n] contains the start symbol, the sentence is *accepted*

Book	the	flight	through	Houston
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	[1,2]	[1,3]	[1,4]	[1,5]
		[2,3]	[2,4]	[2,5]
			[3,4]	[3,5]
				[4,5]







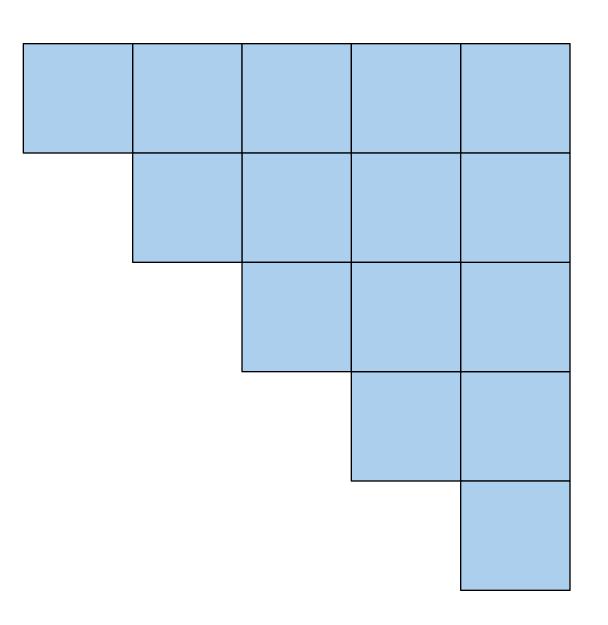


# Dynamic Programming in CKY

- Key idea:
  - for i < k < j</li>
  - ...and a parse spanning substring [ i, j ]
  - There is a **k** such that there are parses spanning [ **i**, **k** ] and [ **k**, **j** ]
  - We can construct parses for whole sentences by building from these partial parses
- So to have a rule  $A \rightarrow B C$  in [i, j]
  - Must have B in [i, k] and C in [k, j] for some i < k < j
  - CNF forces this for all j > i + 1

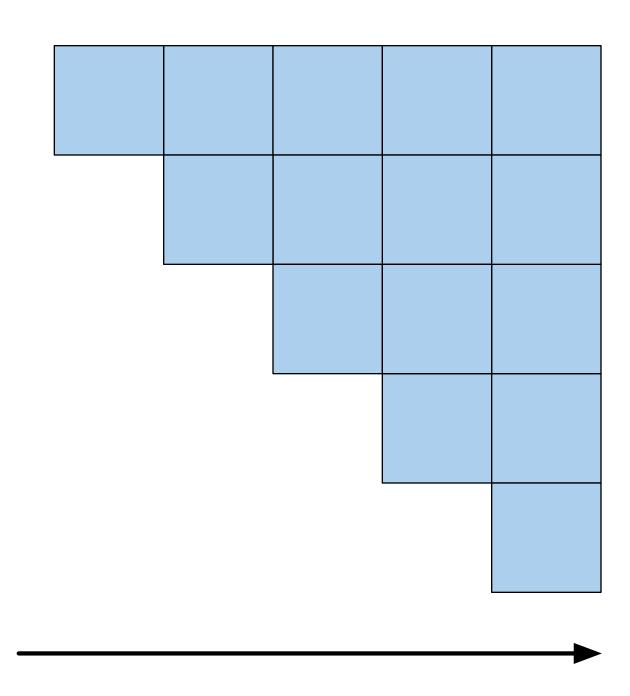


Table fills:



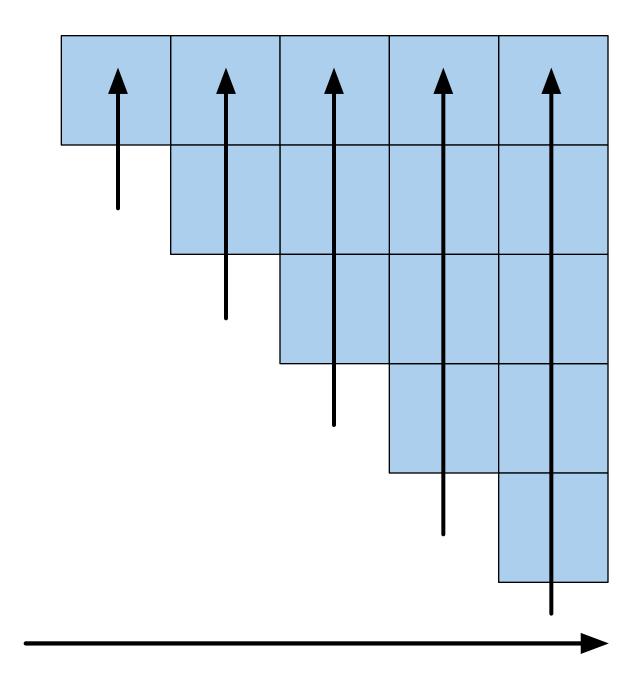


- Table fills:
  - Column-by-column, left-to-right



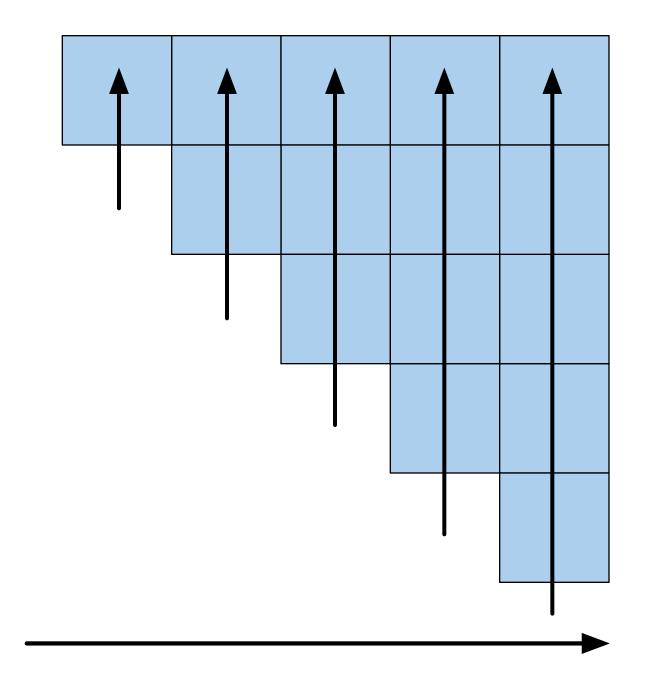


- Table fills:
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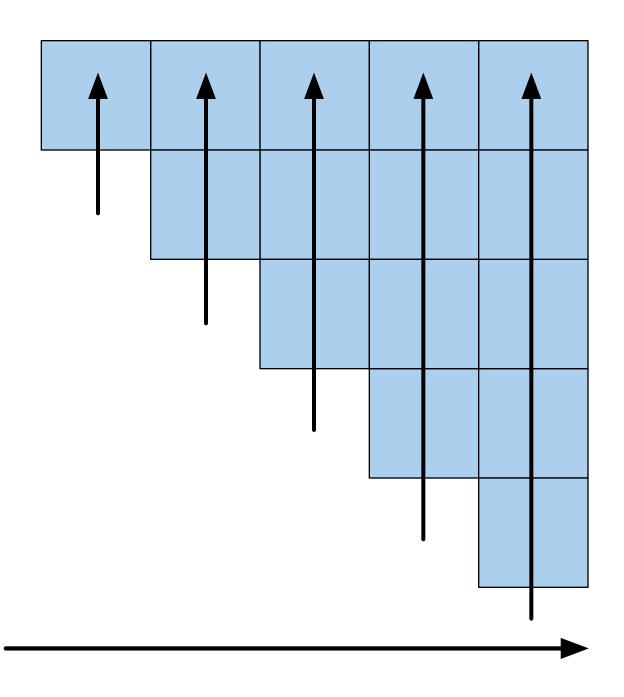




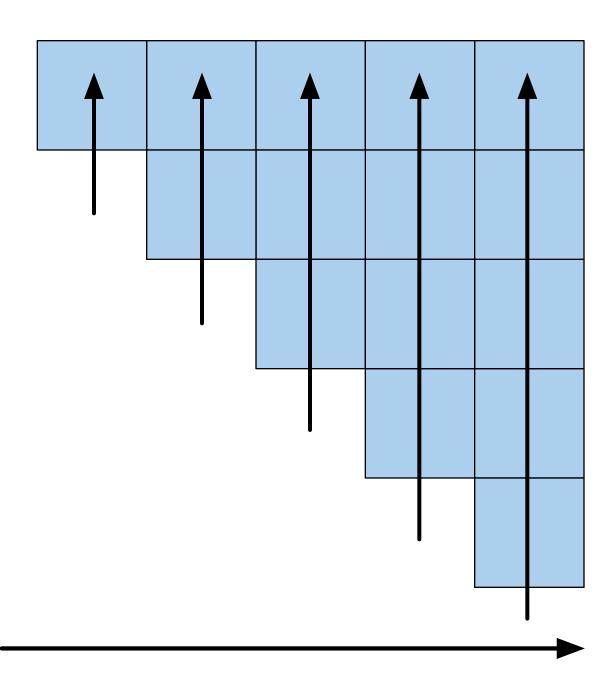
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- Why?



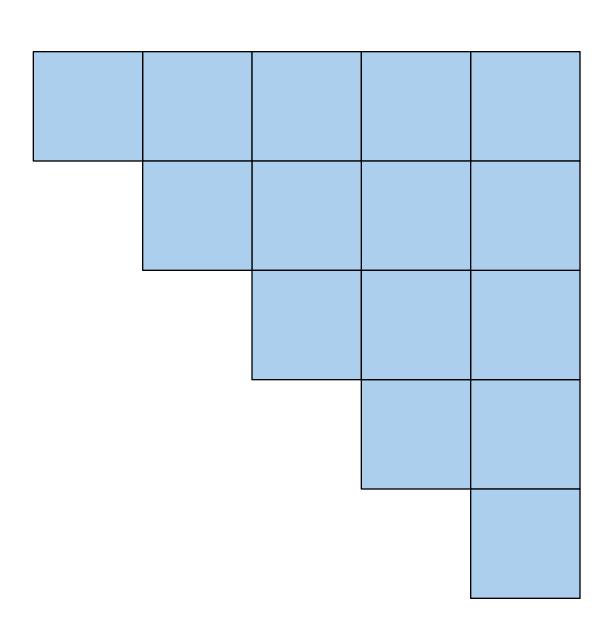
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  - To fill each cell, must have access to all sub-spans



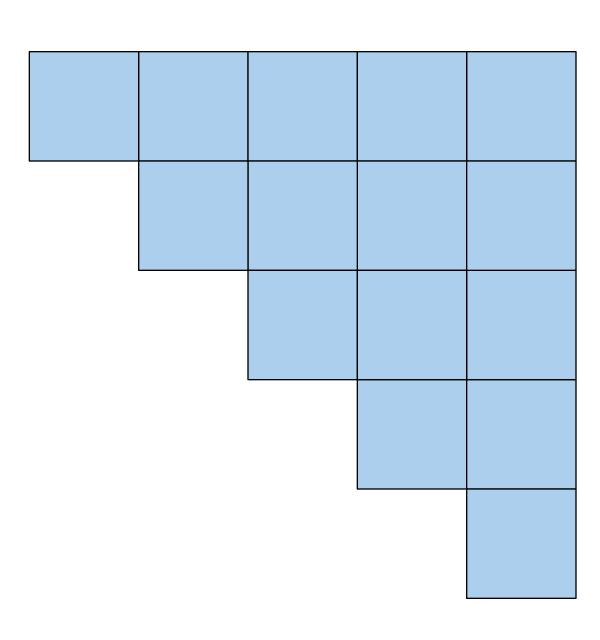
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  - Allows online sentence analysis

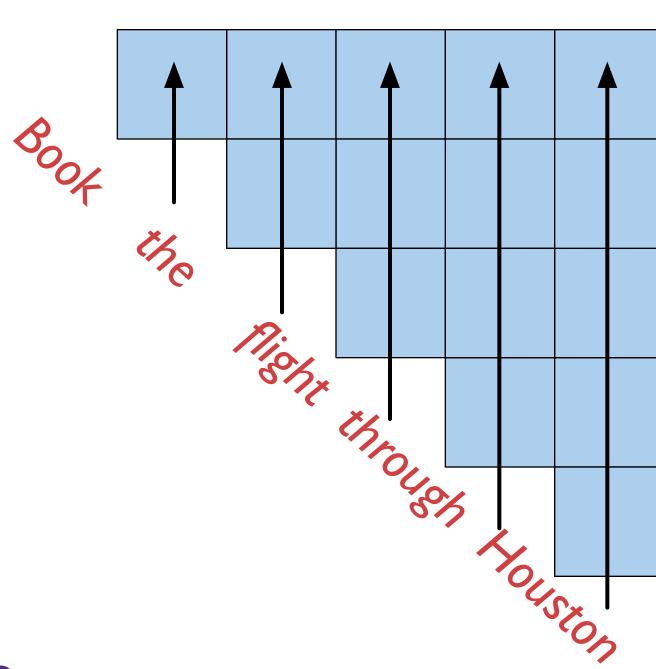


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# CKY Algorithm

```
function CKY-PARSE(words, grammar) returns table for j \leftarrow from \ 1 to LENGTH(words) do table[j - 1, j] \leftarrow \{A \mid A \rightarrow words[j] \in grammar\} for i \leftarrow from \ j - 2 downto 0 do for k \leftarrow i + 1 to j - 1 do table[i, j] \leftarrow table[i, j] \cup \{A \mid A \rightarrow BC \in grammar \ B \in table[i, k], \ C \in table[k, j]\}
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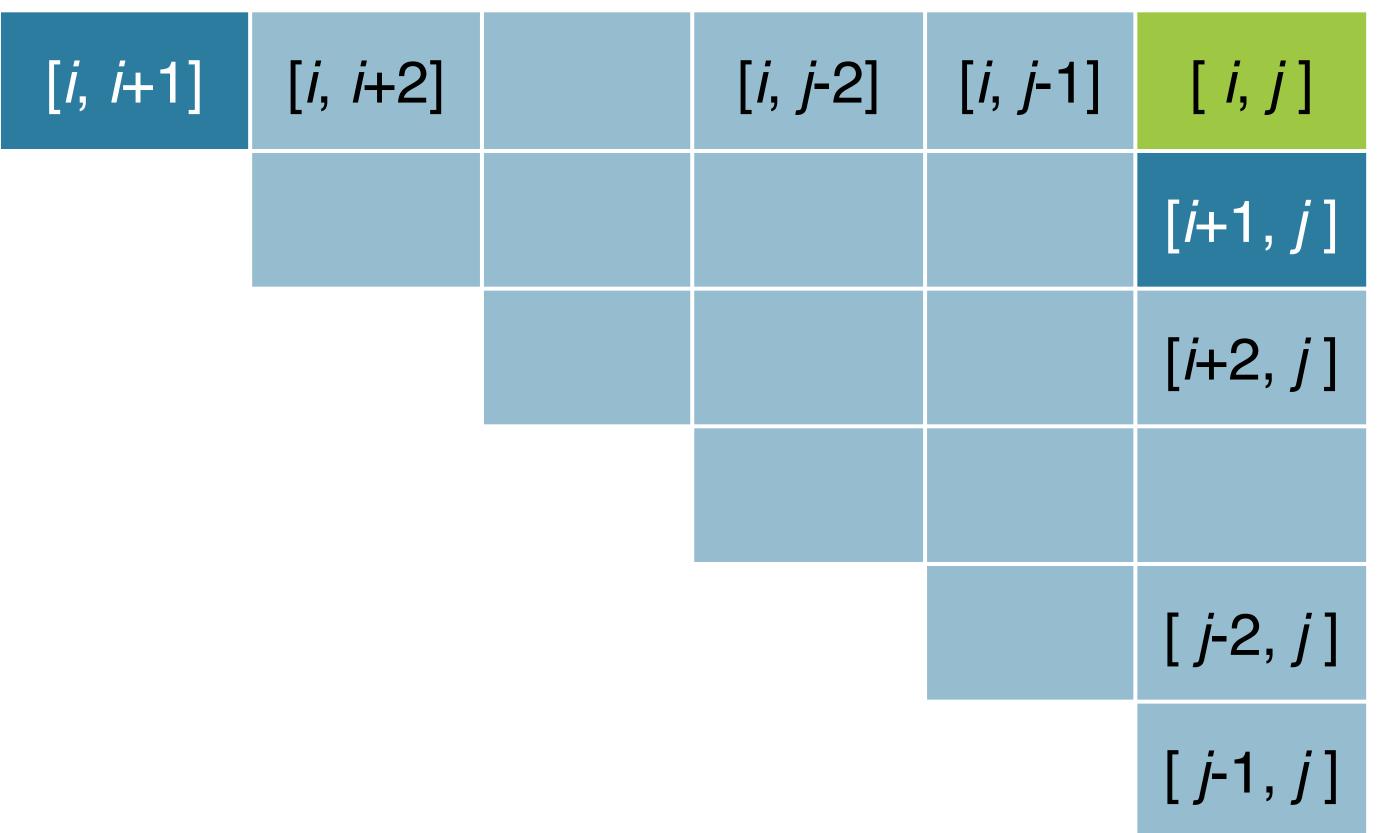
C \in table[k,j]\}
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Left-to-right, bottom-to-top
Sliding k as the pivot for child nodes

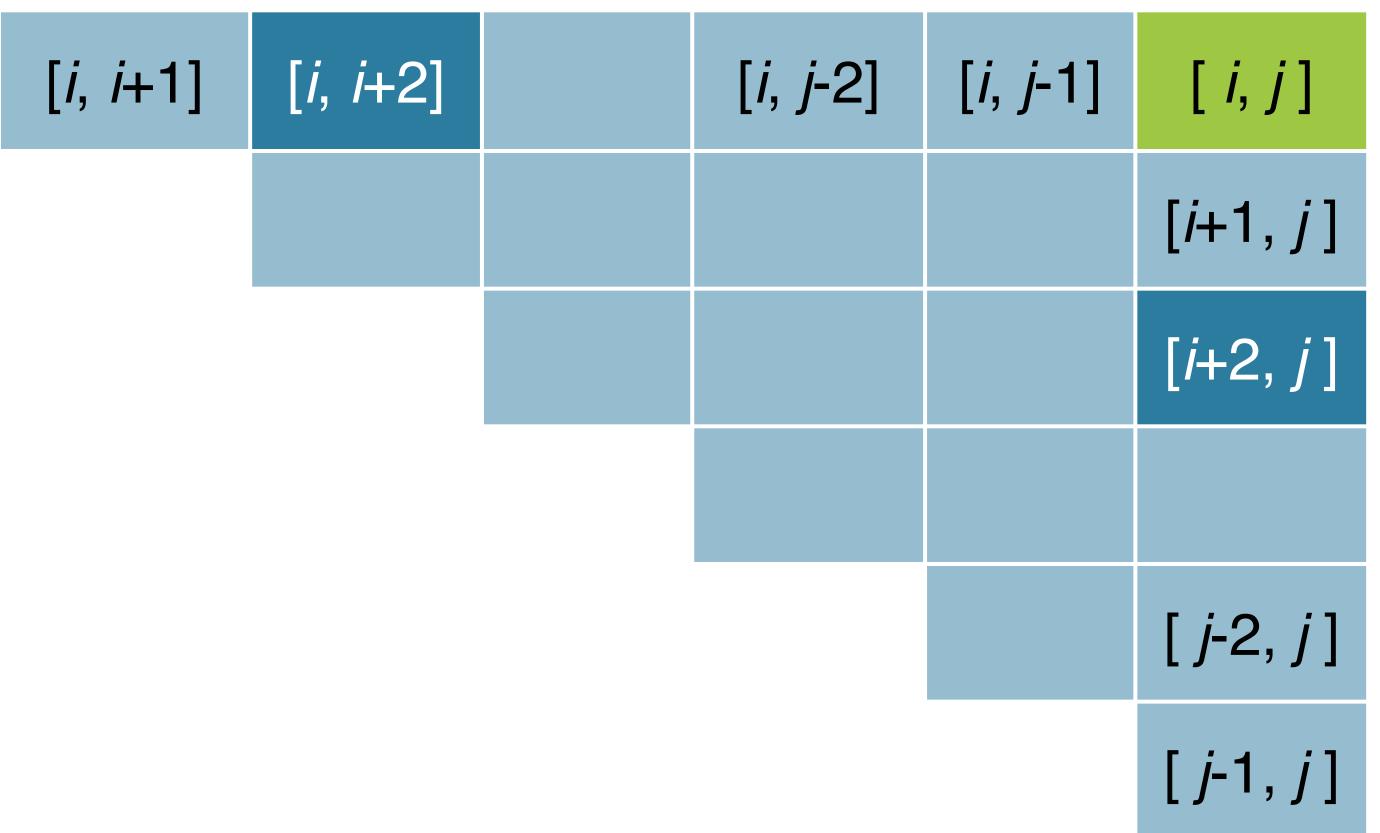
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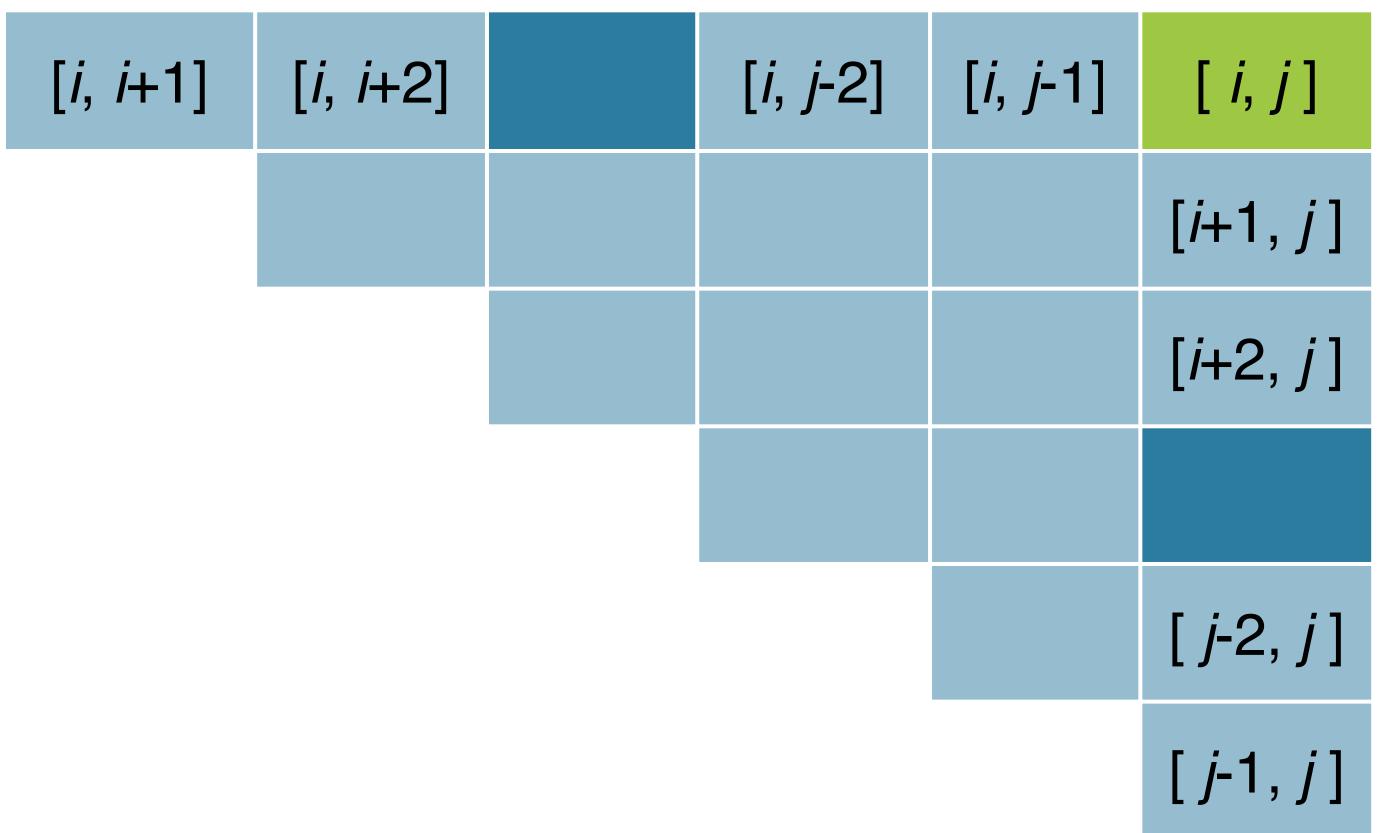
Left-to-right, bottom-to-top
Sliding k as the pivot for child nodes
Filling the cell with set of applicable rules



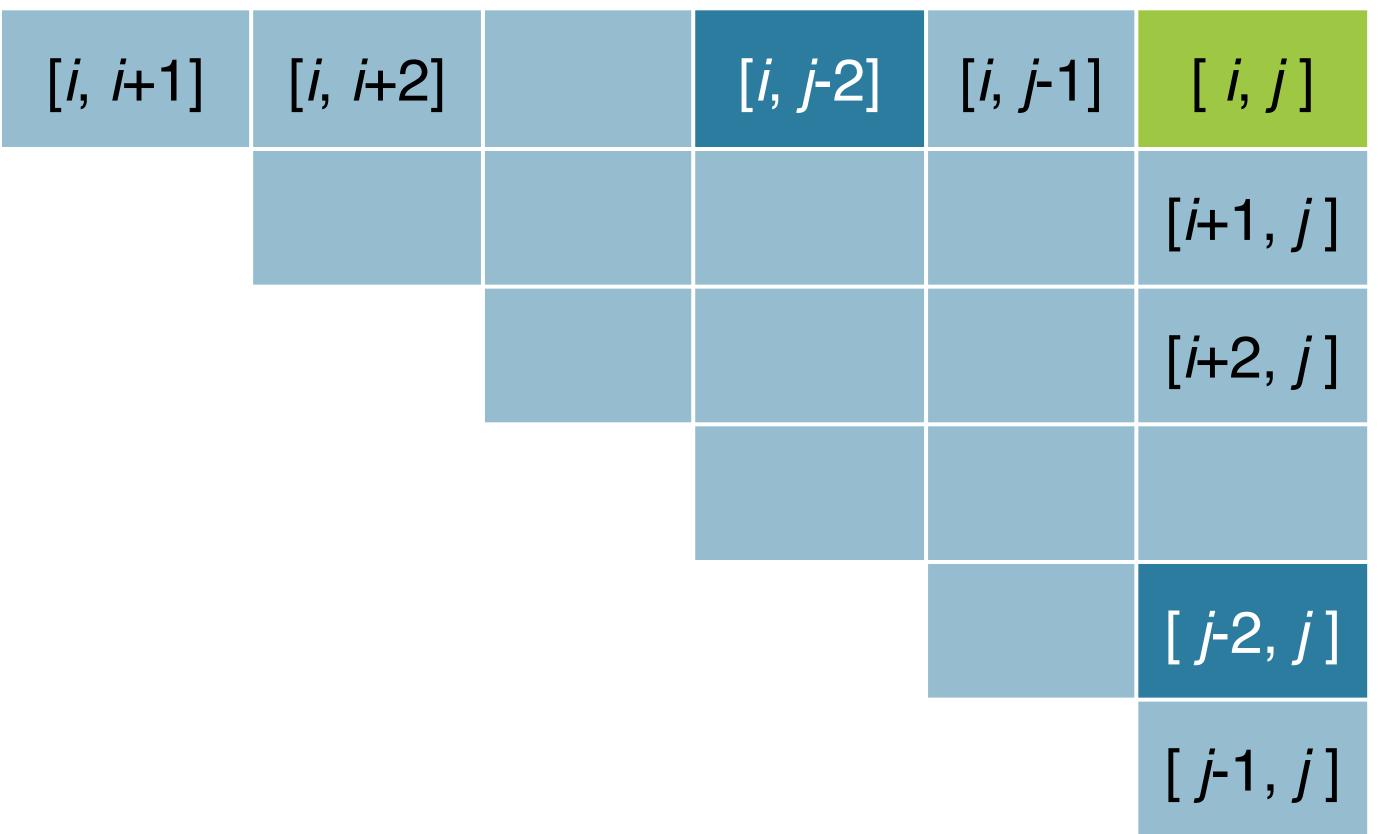
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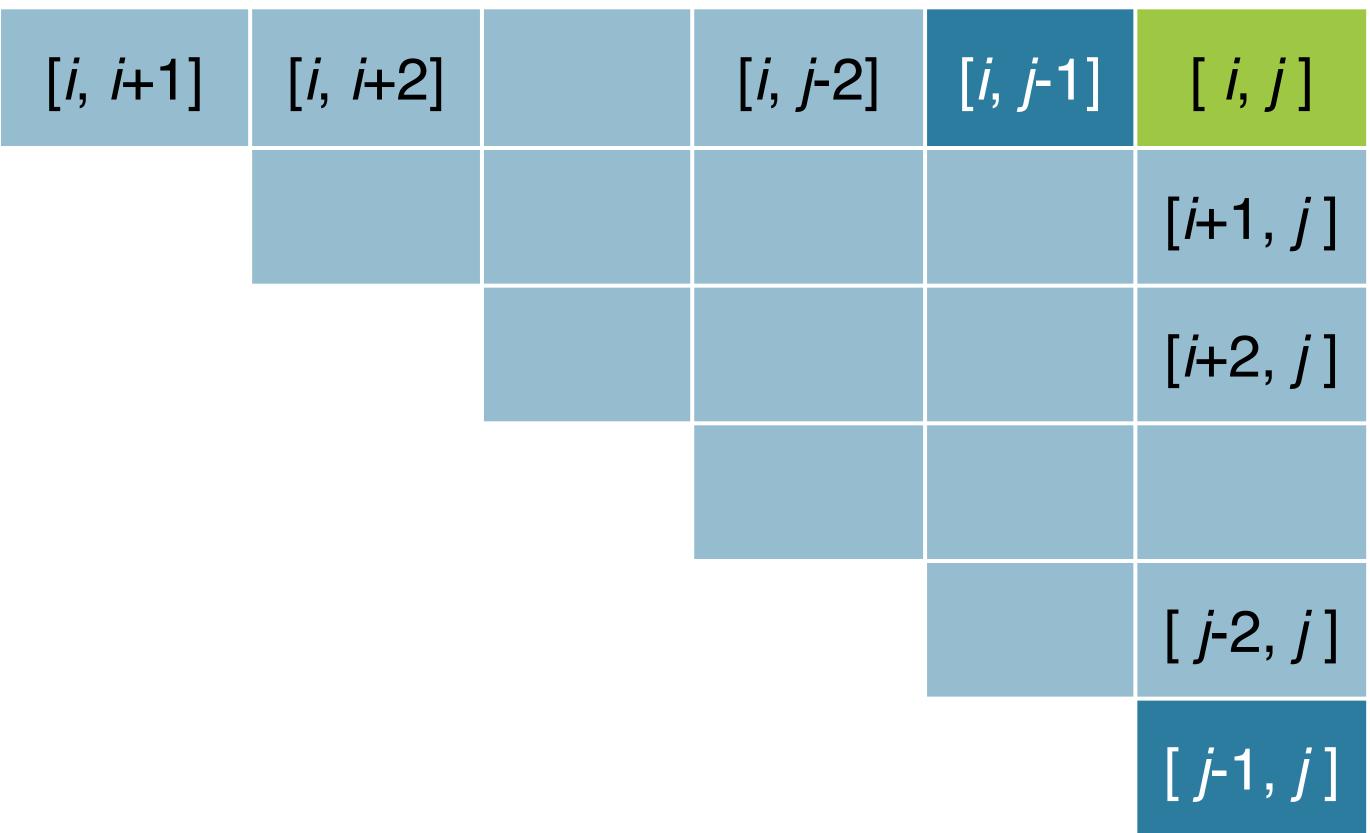
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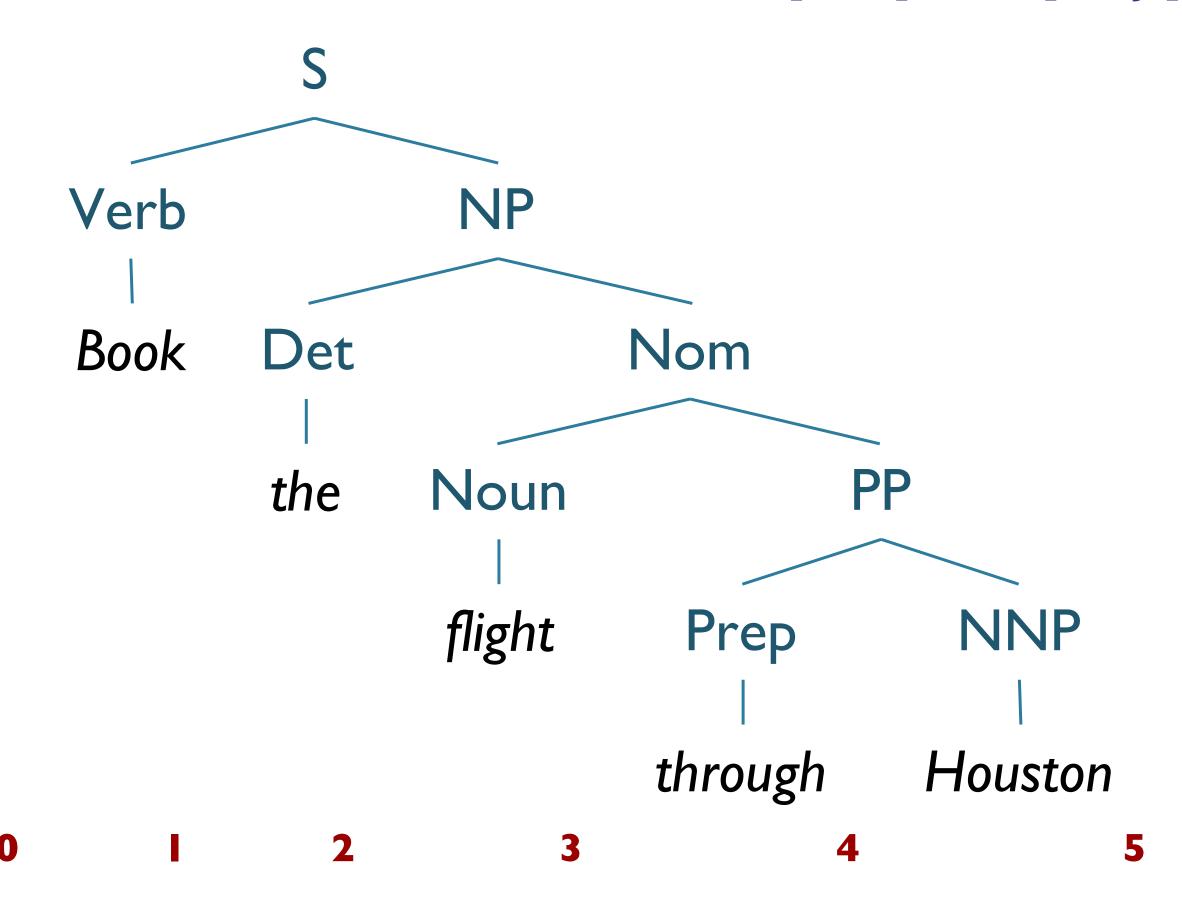
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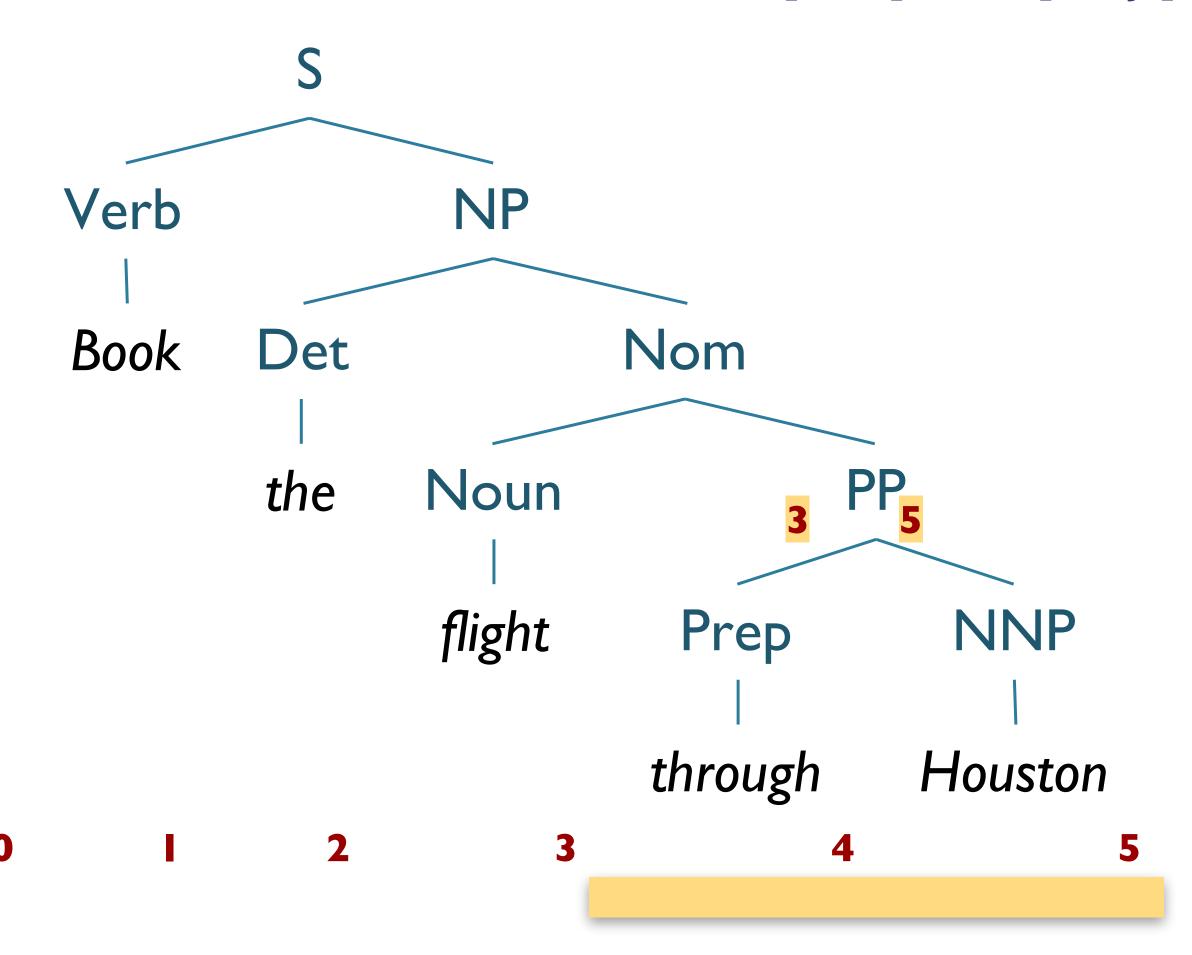


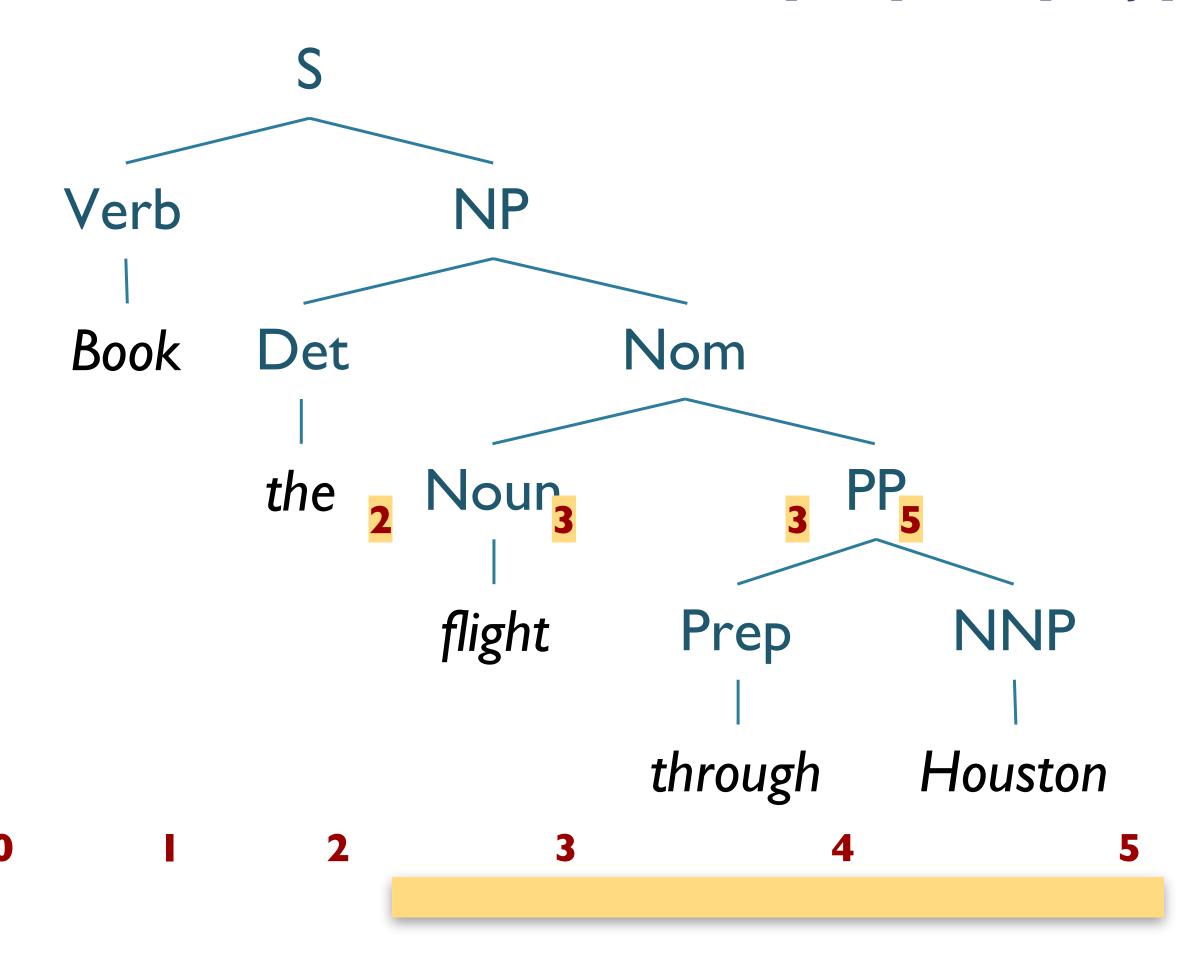
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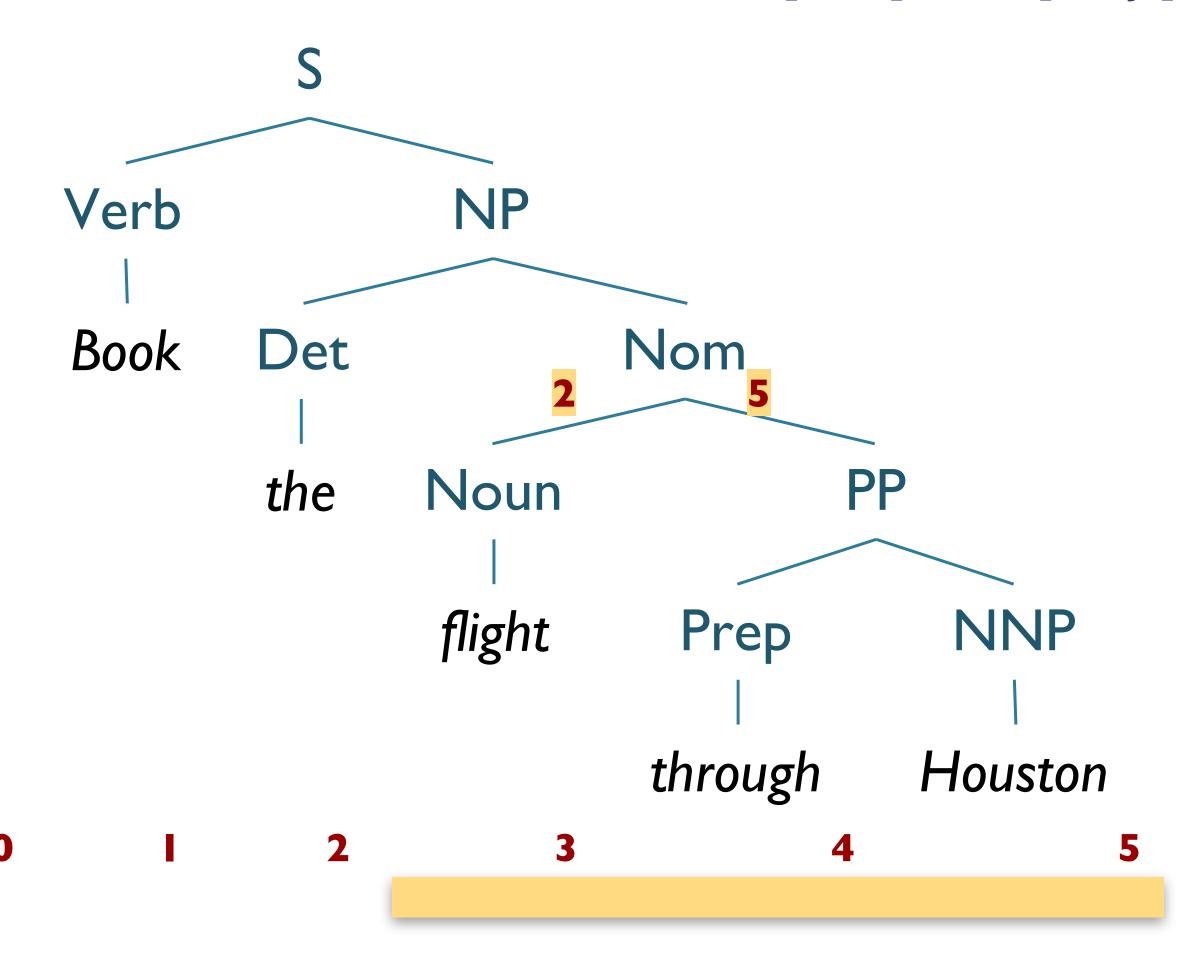


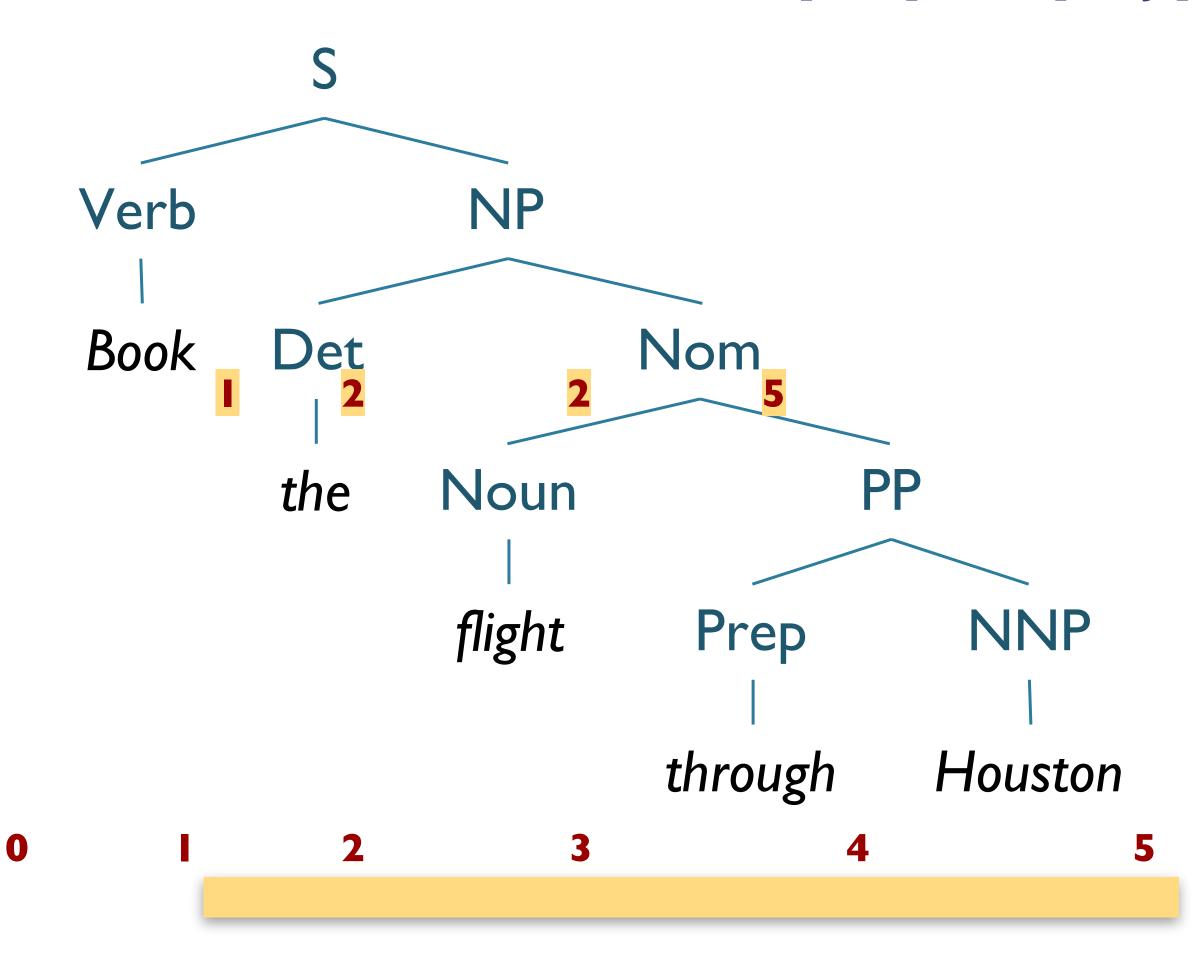
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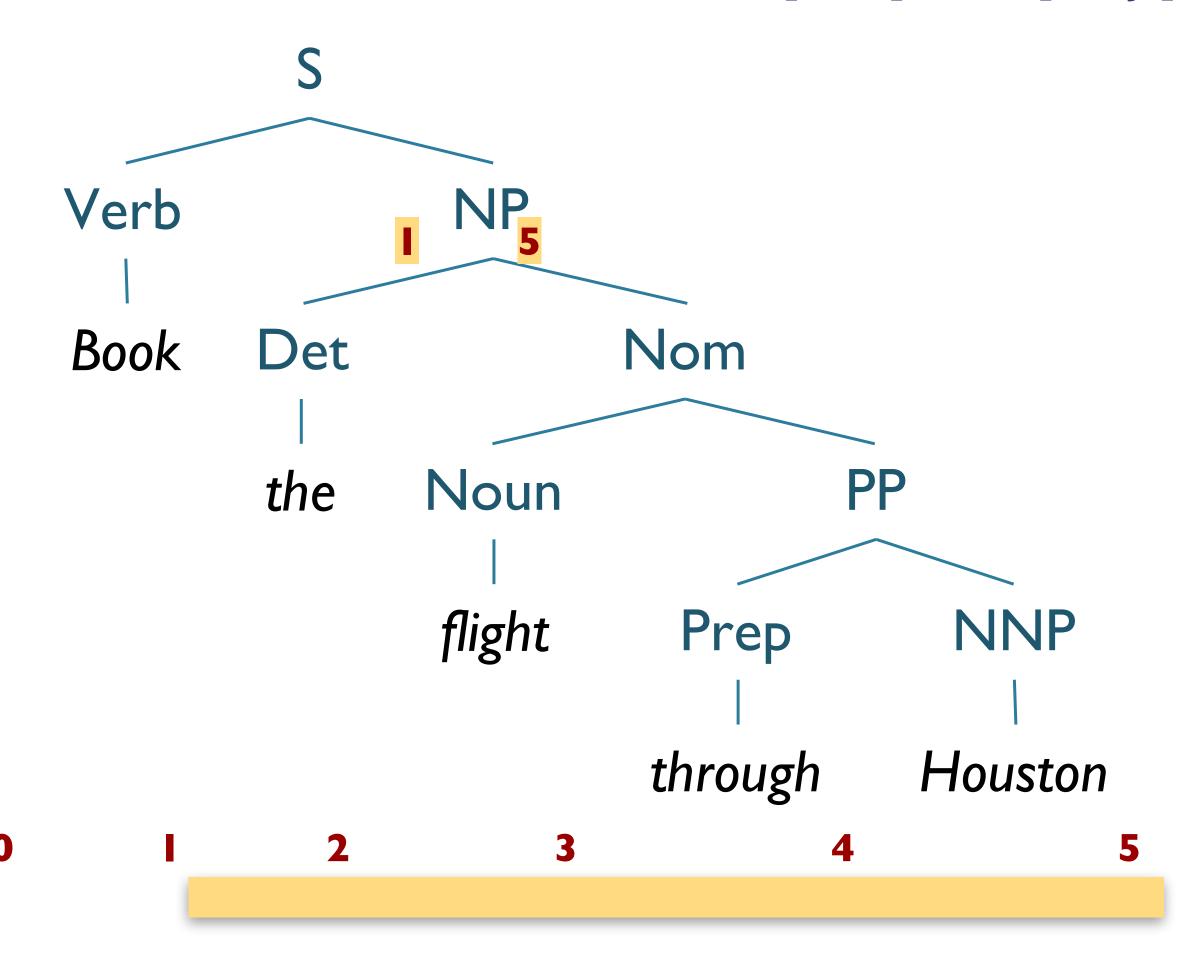


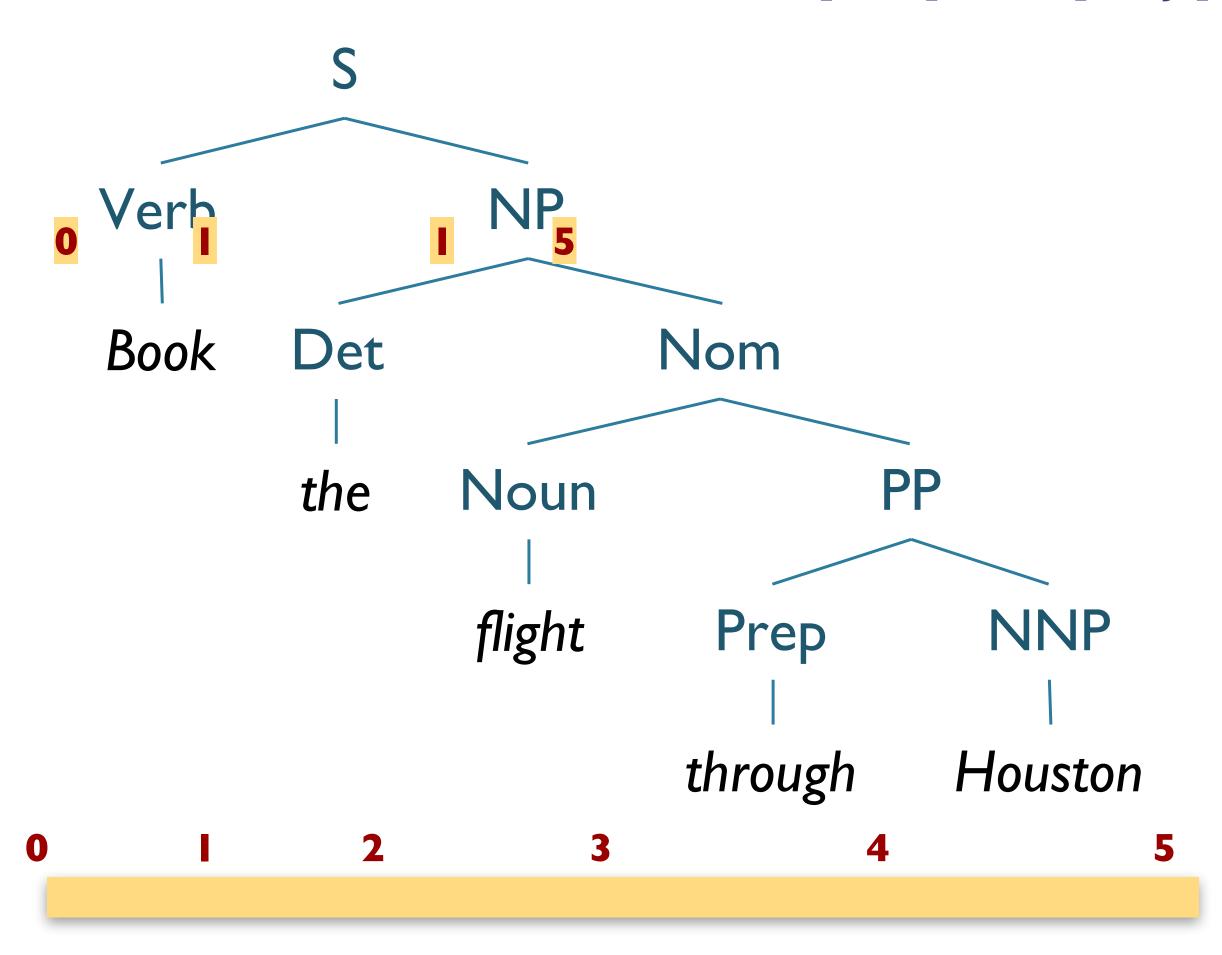


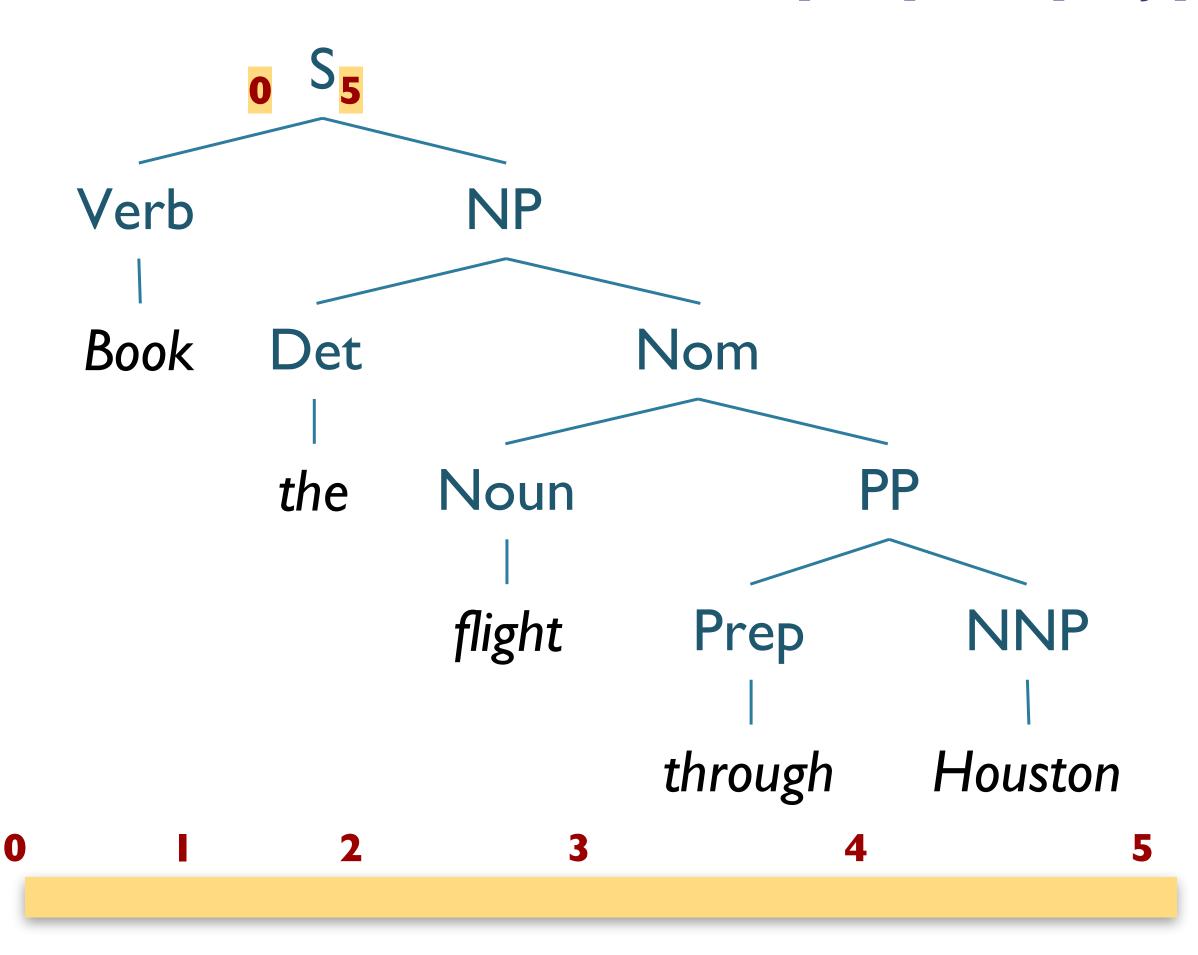


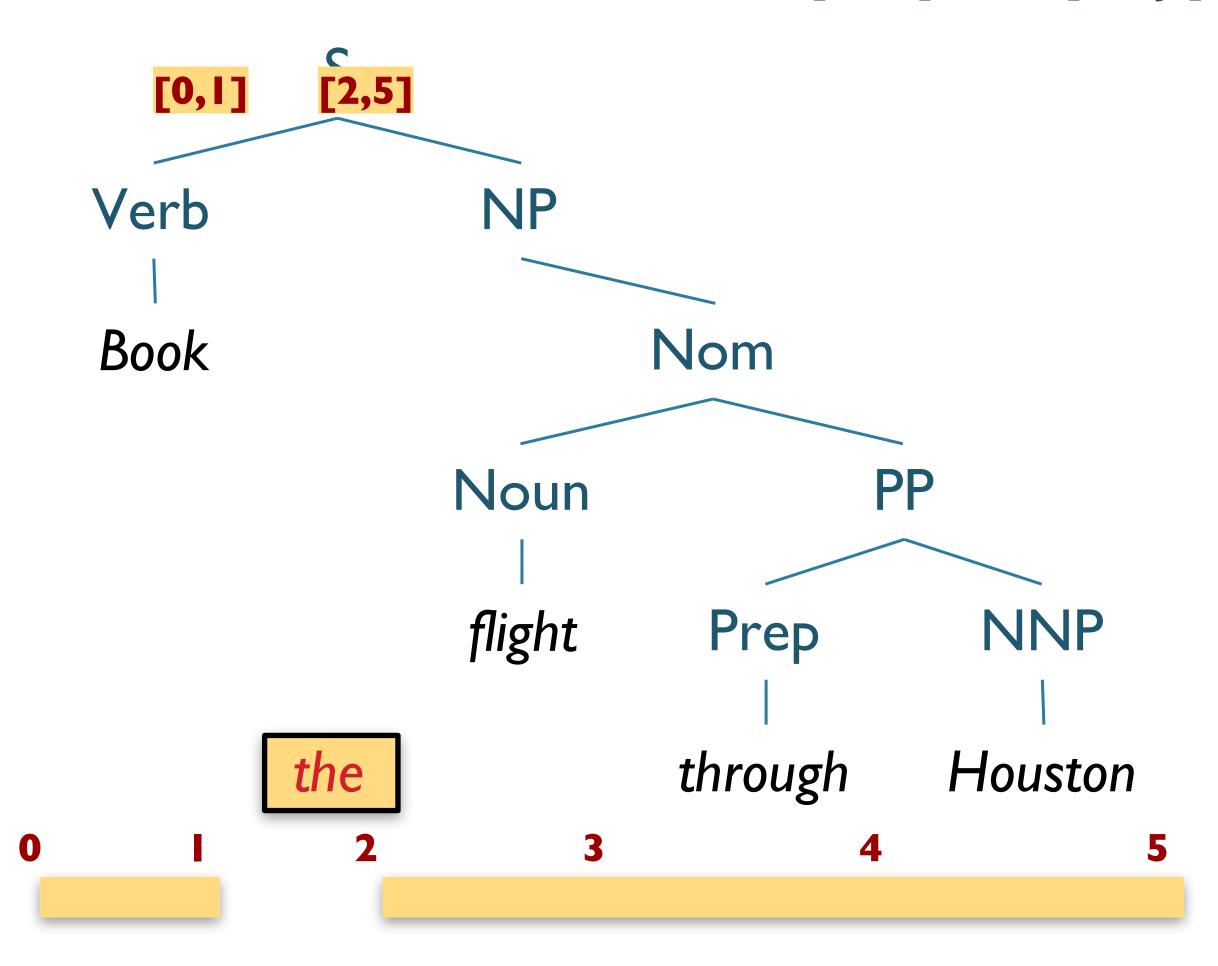


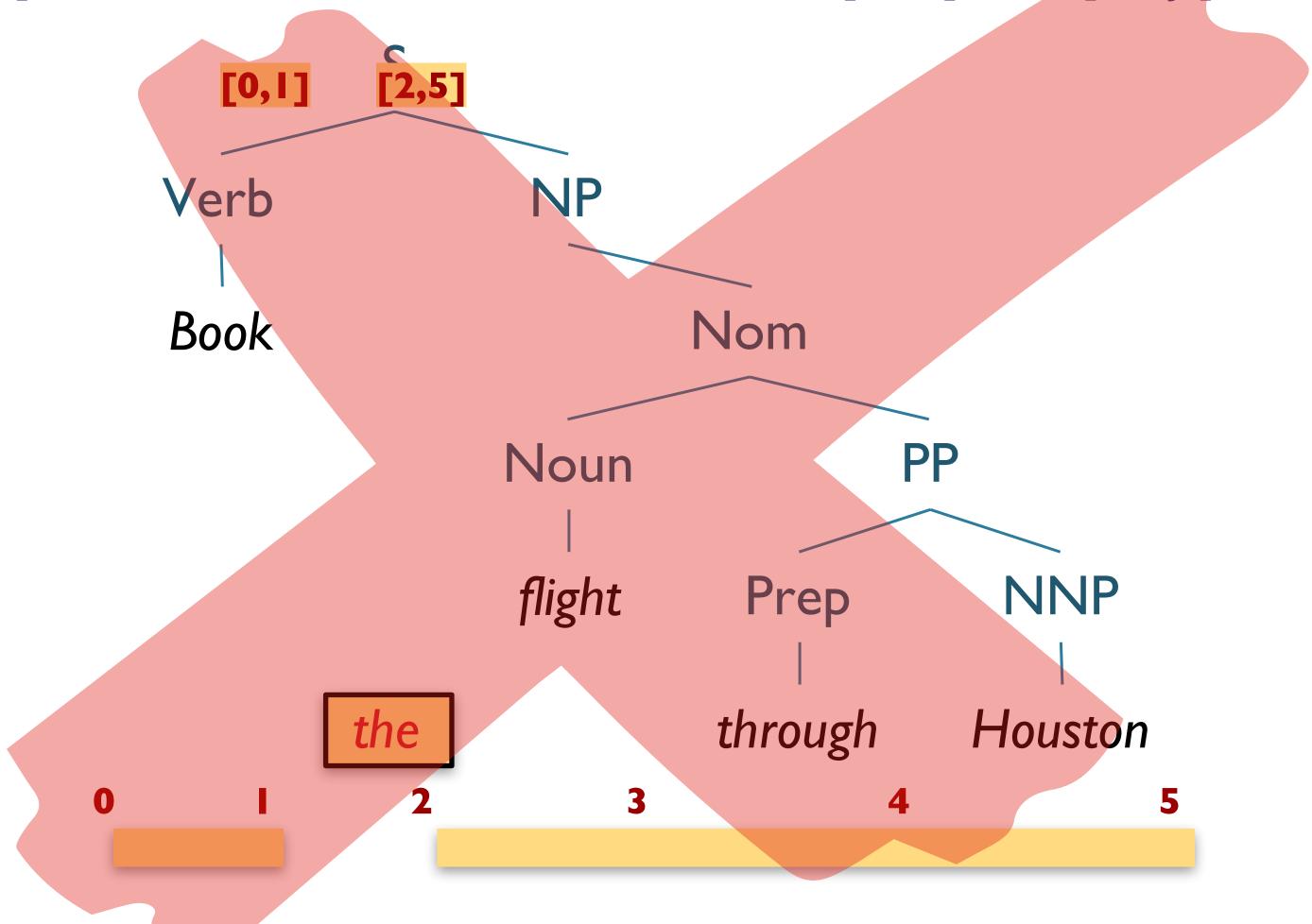




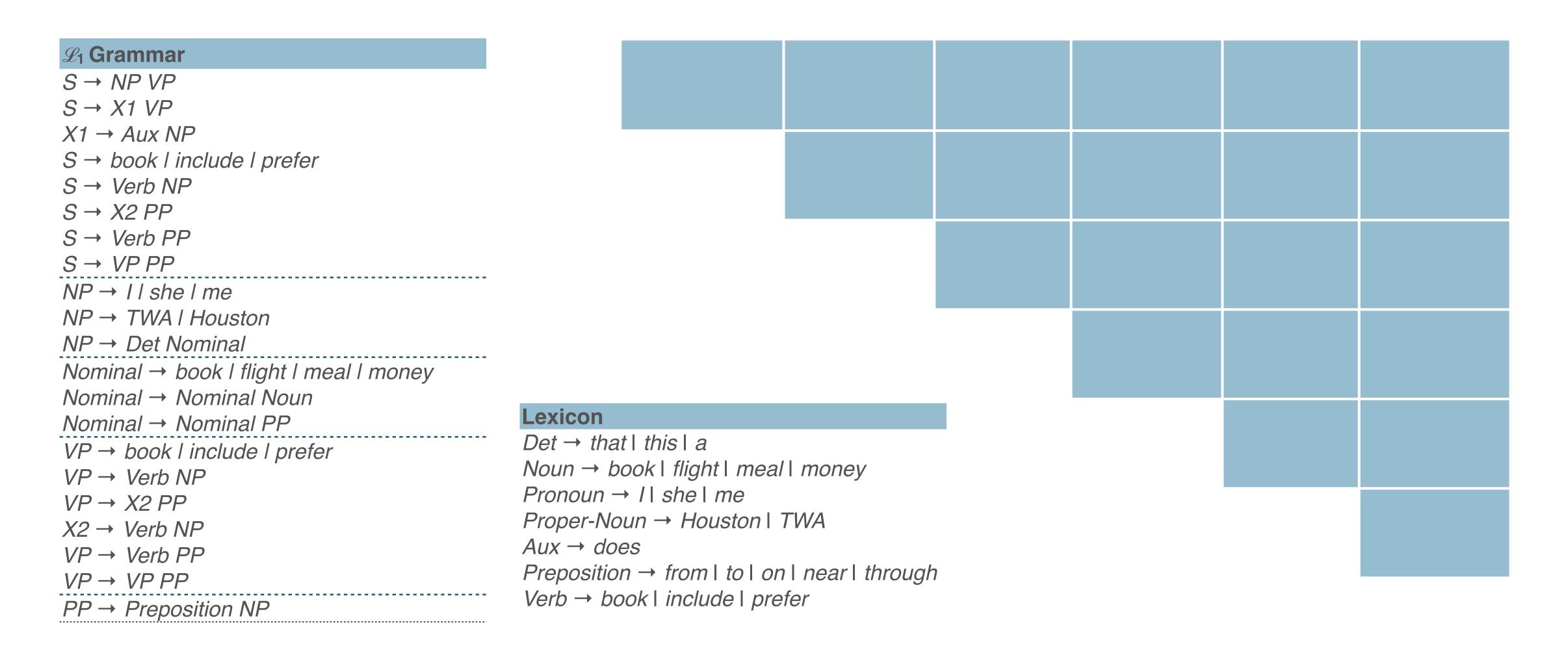


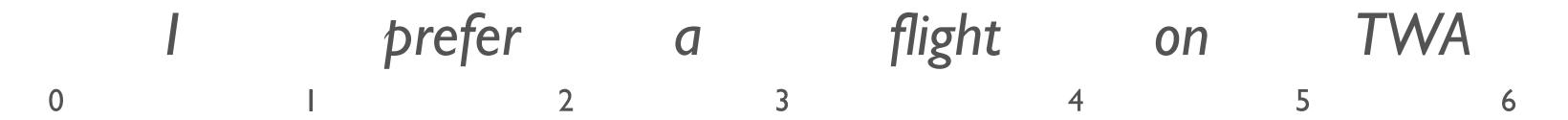


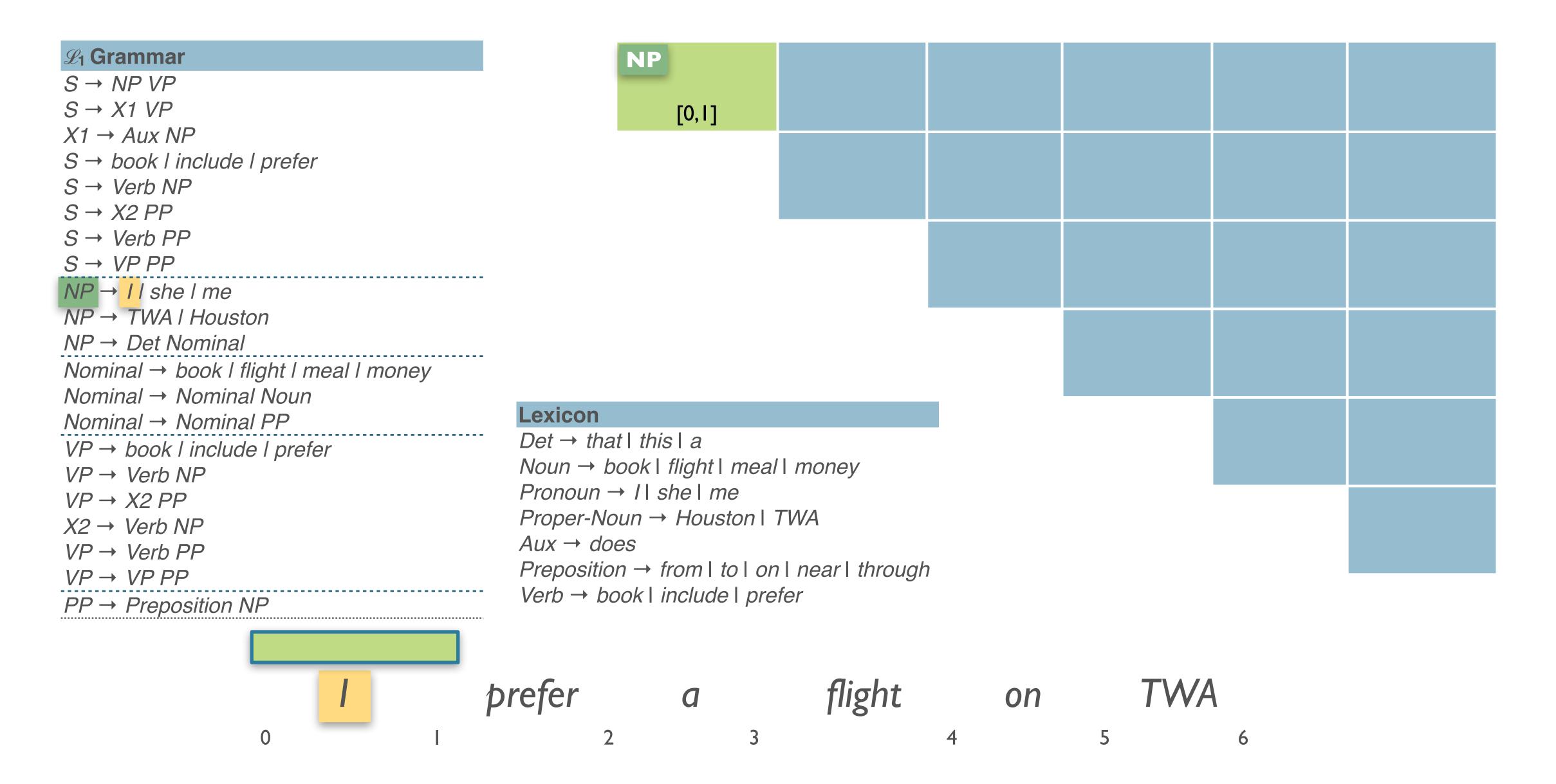


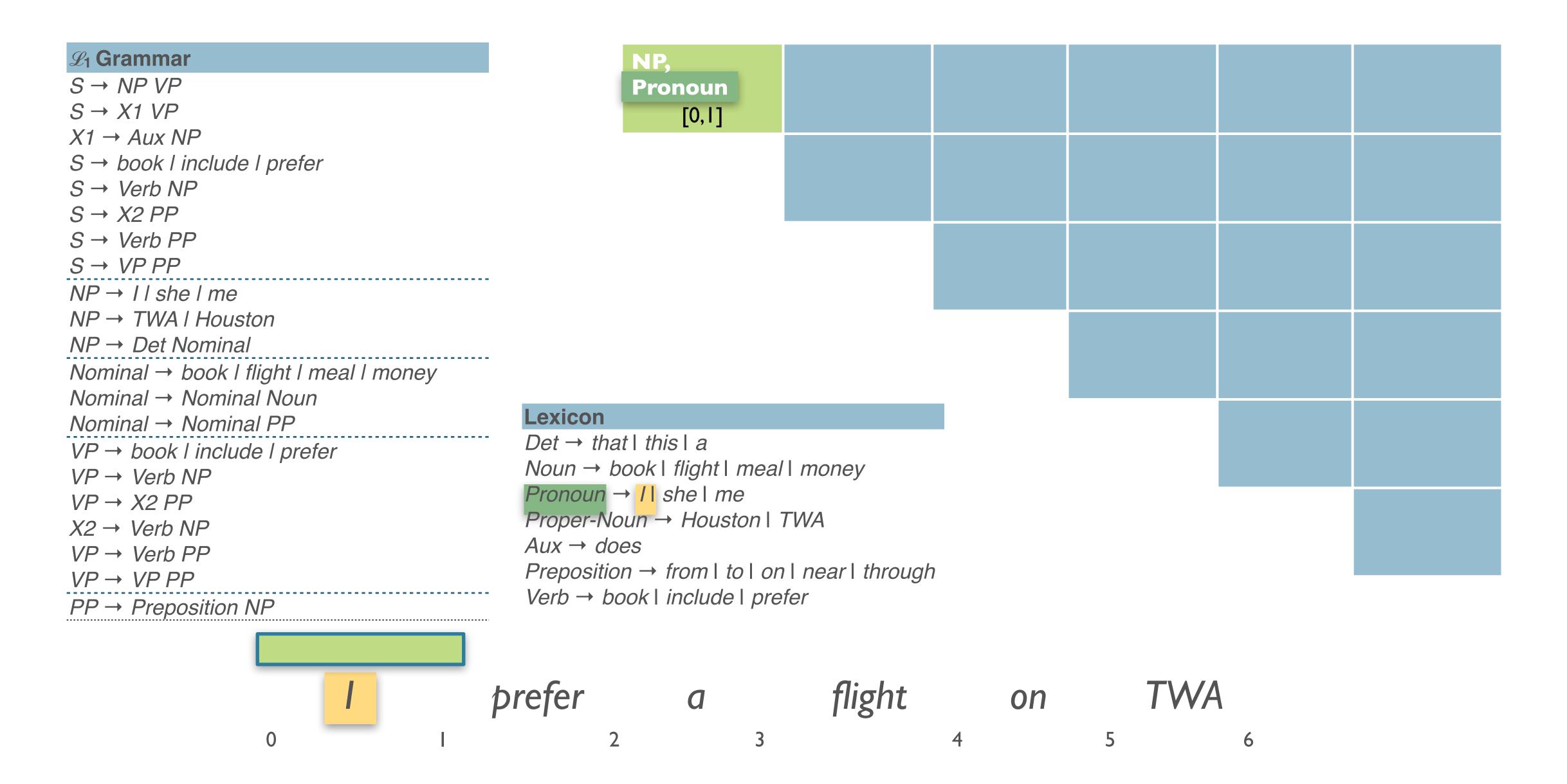


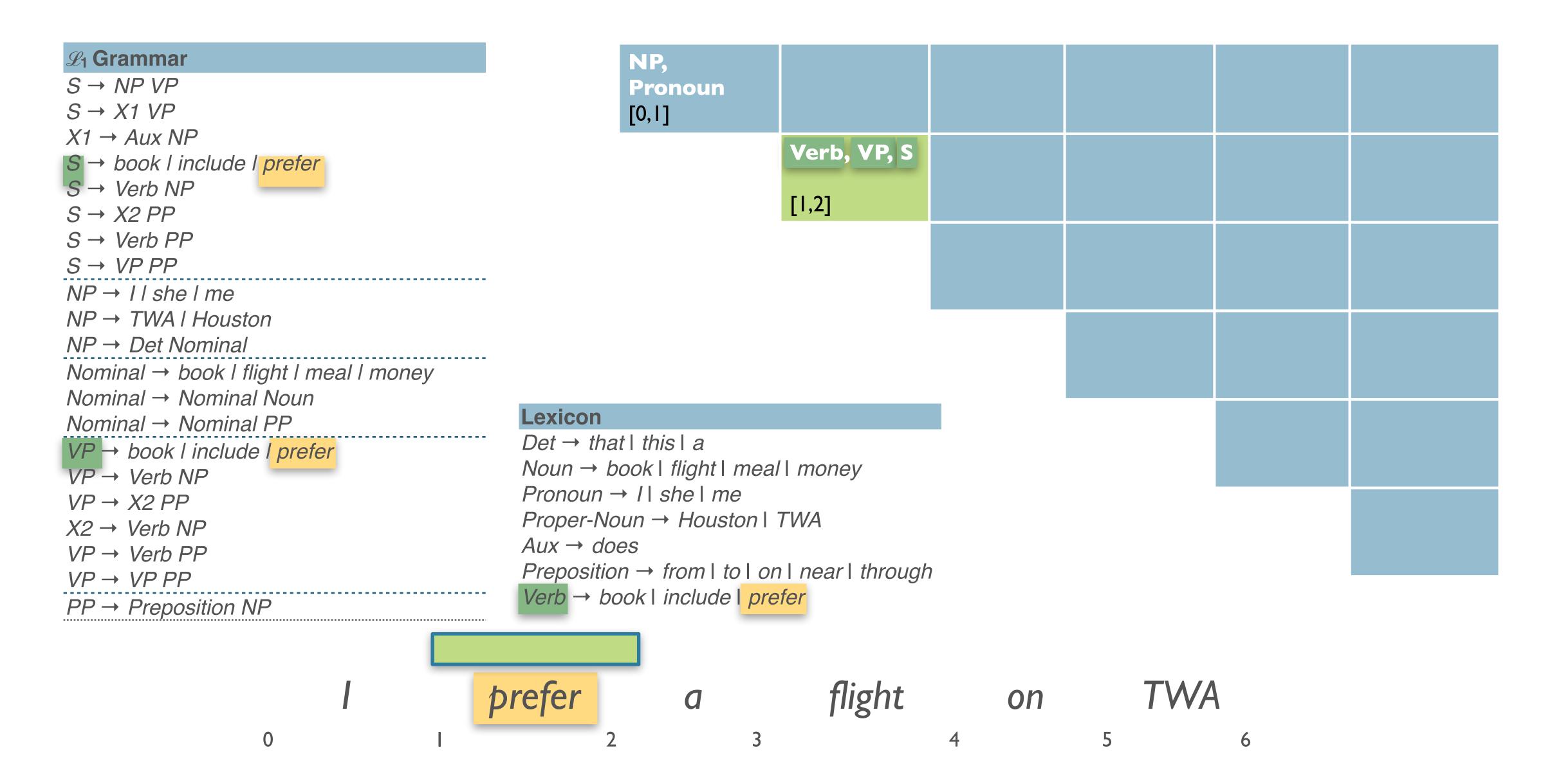
## CKY Parsing Example

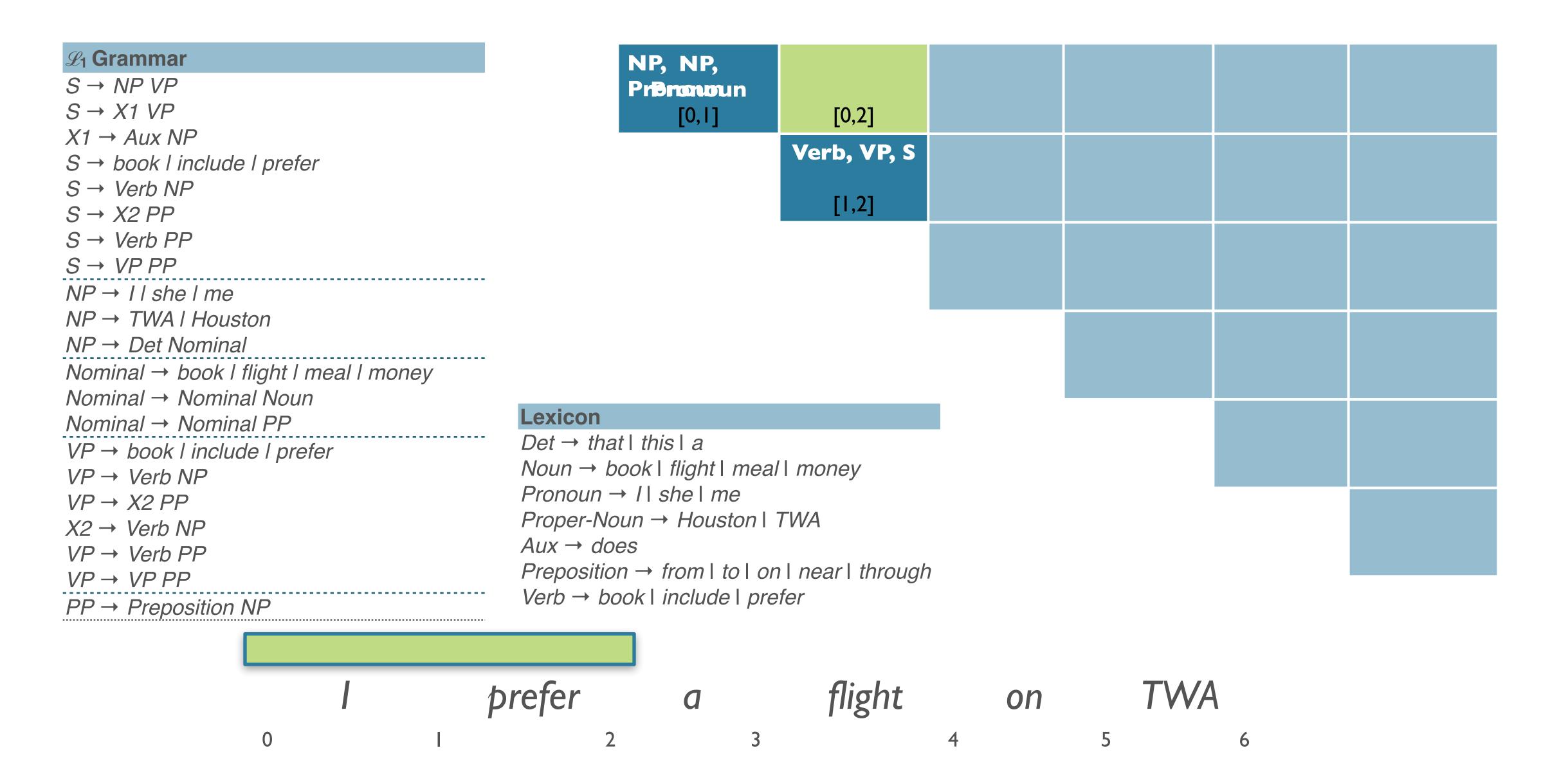


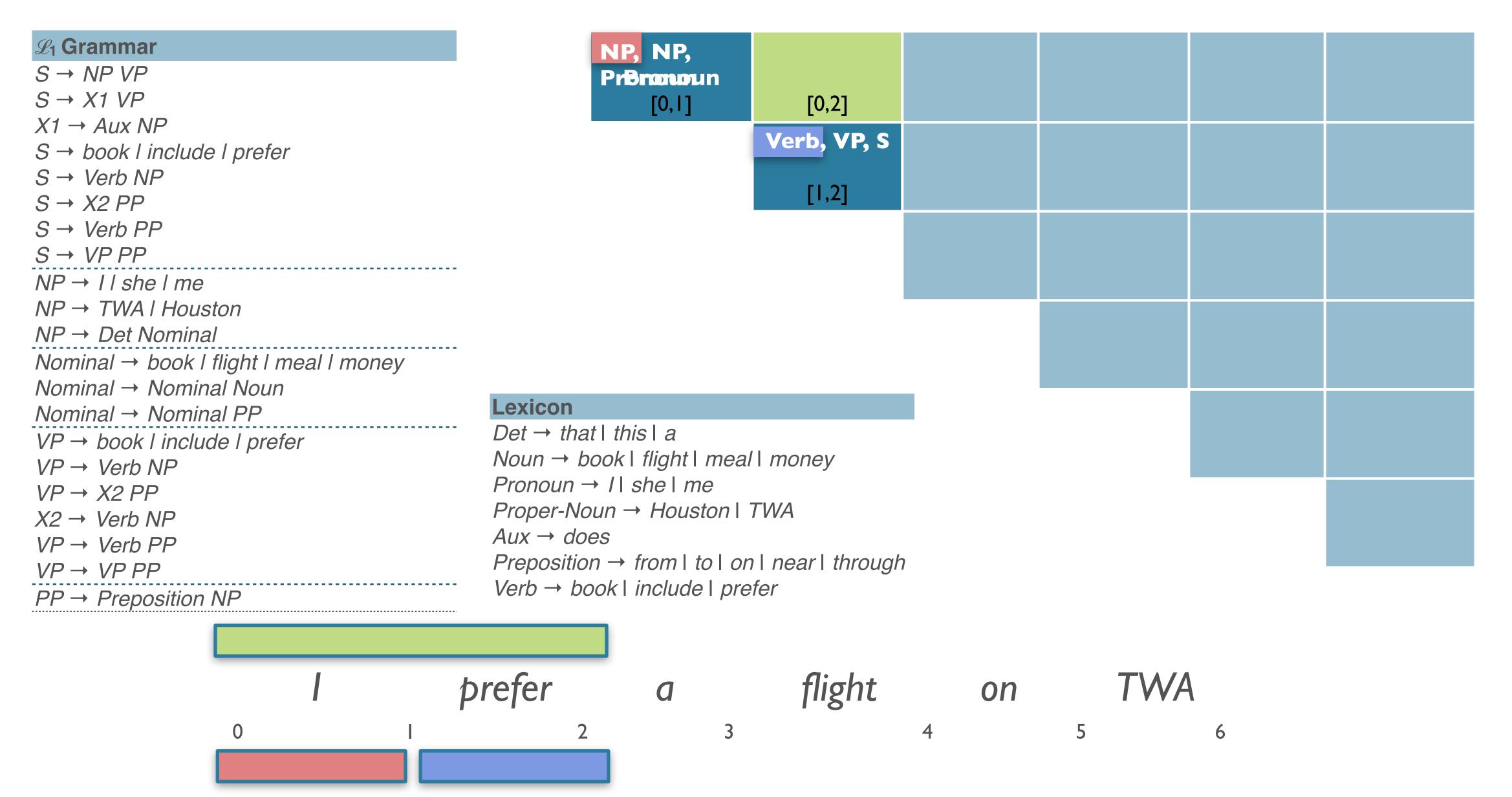


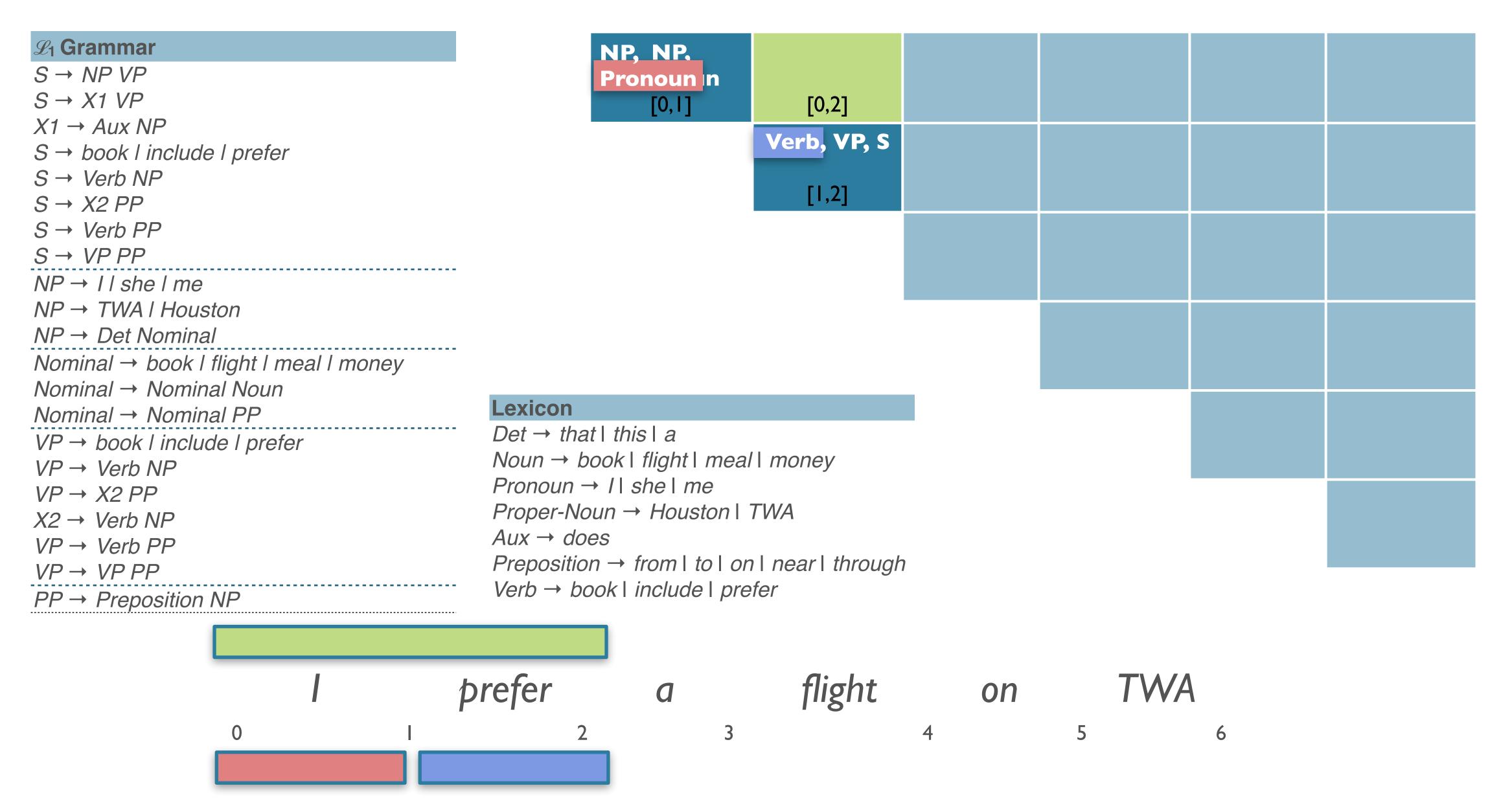


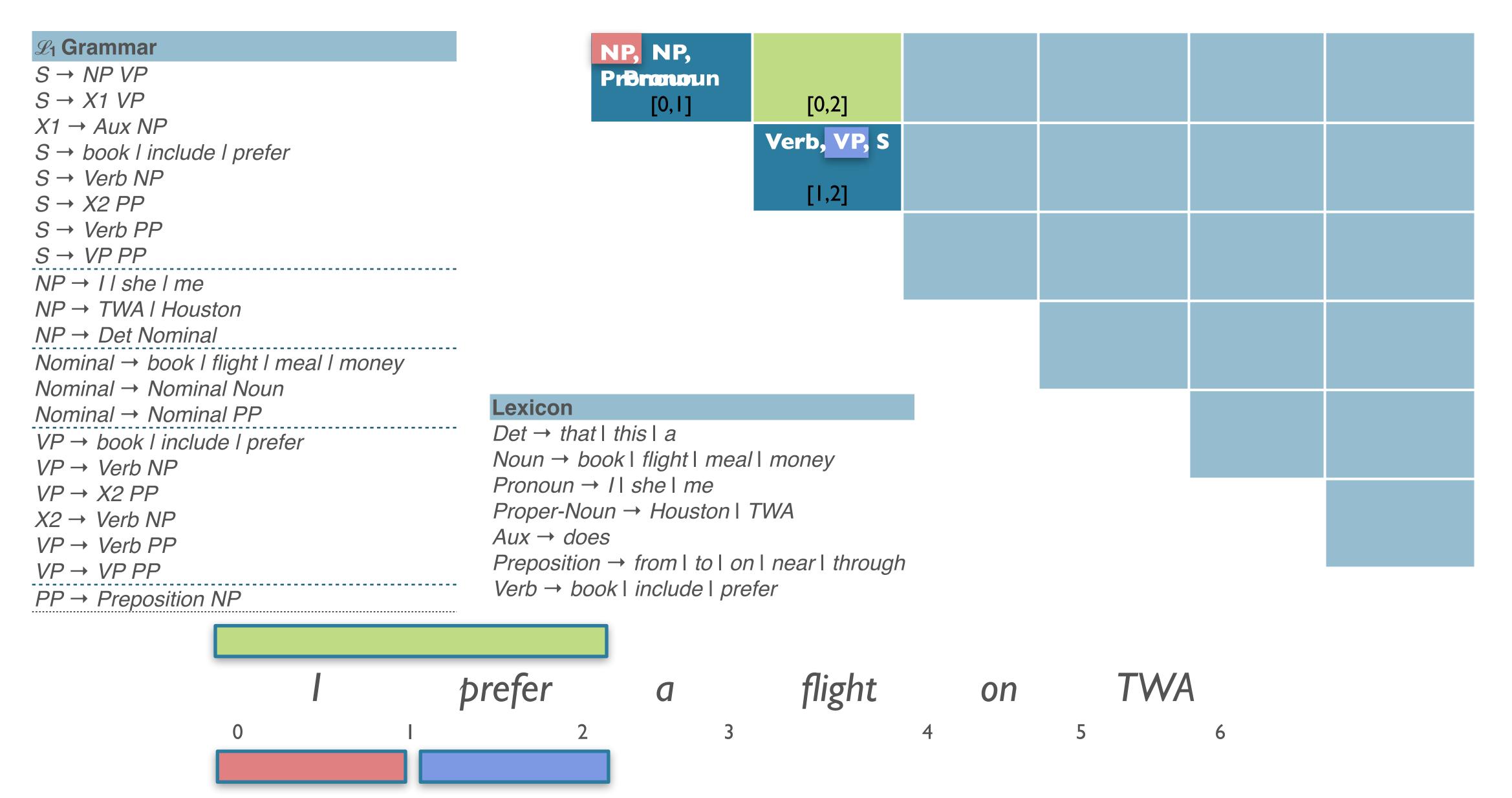


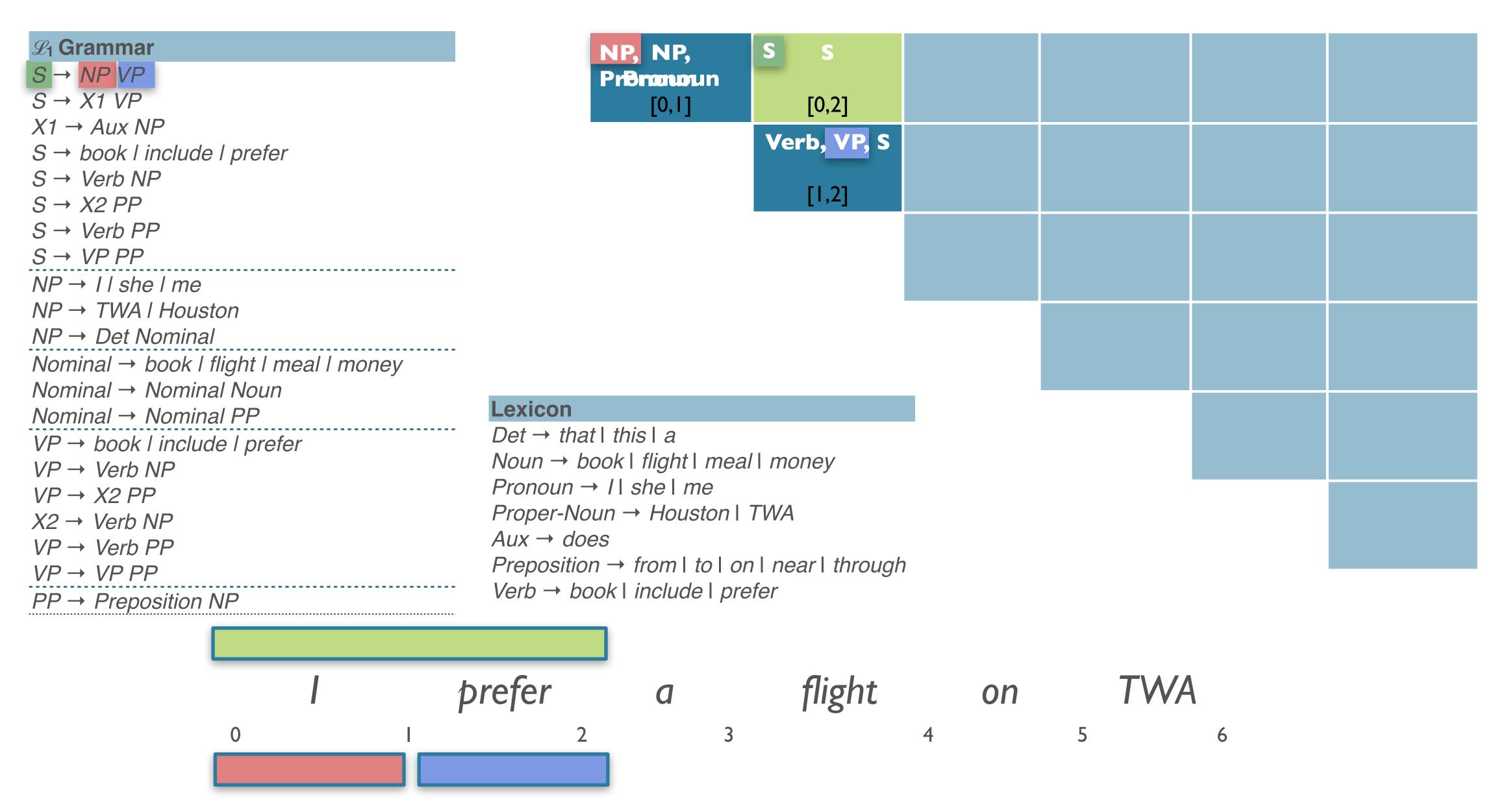


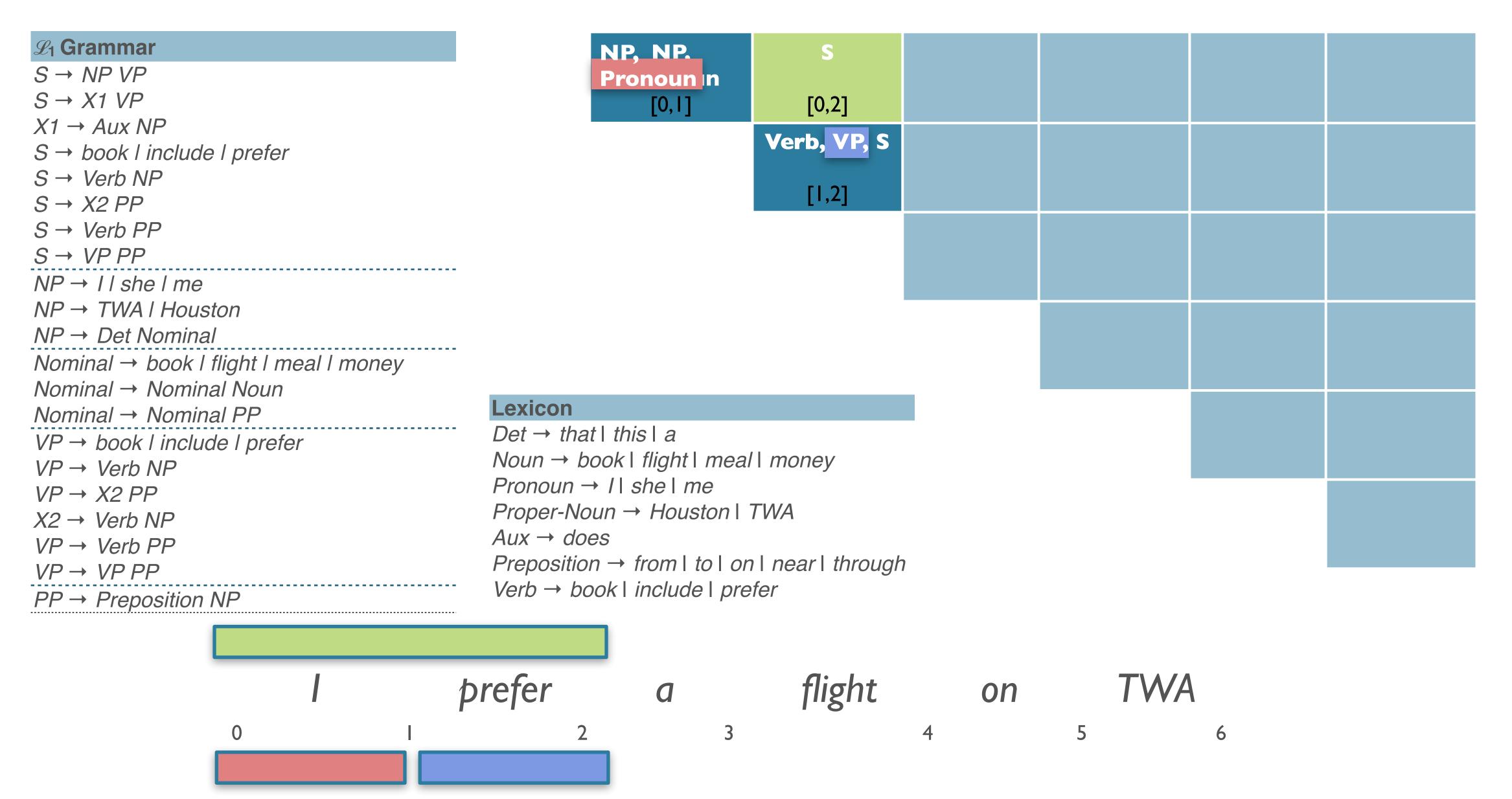


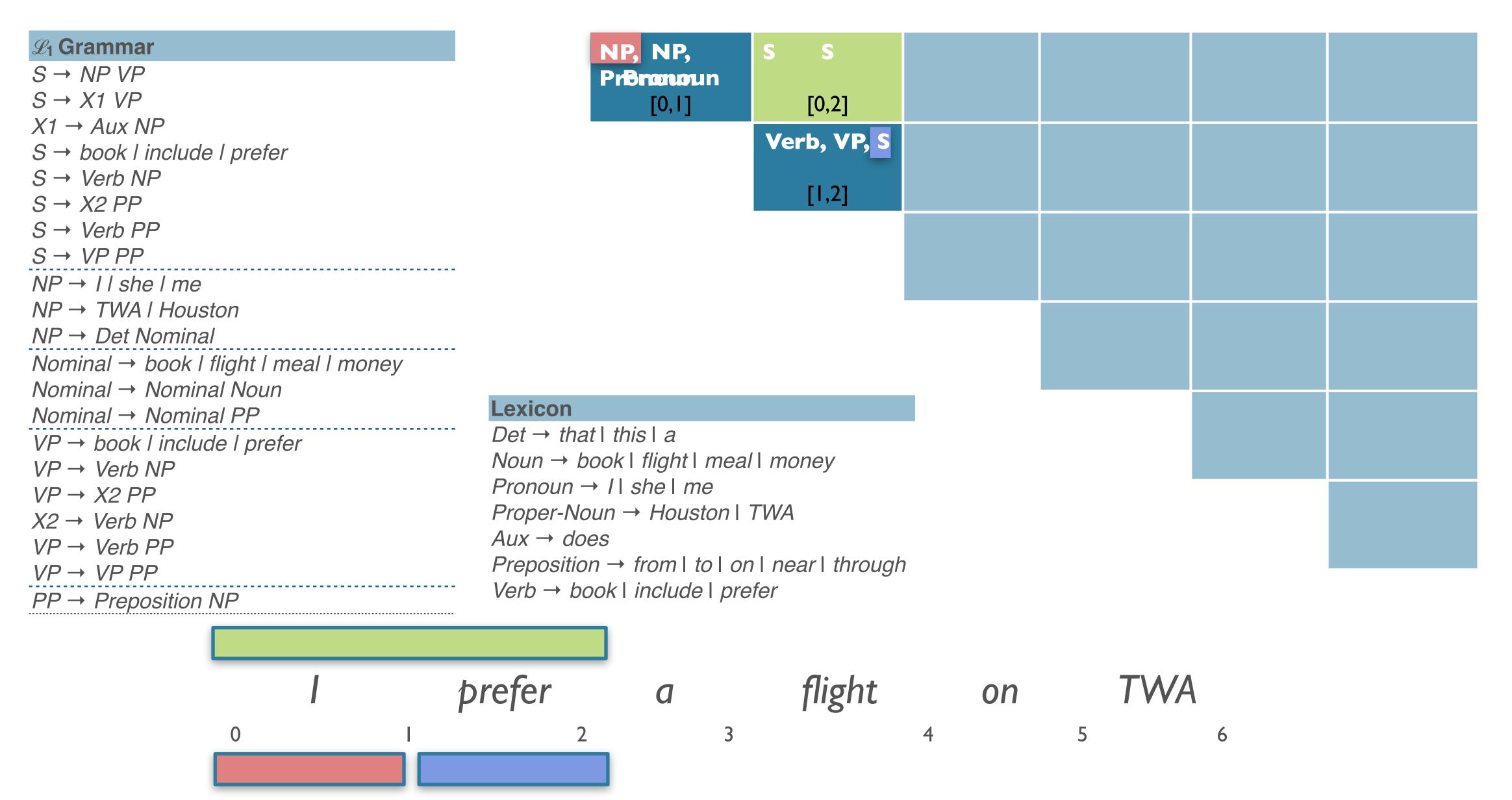


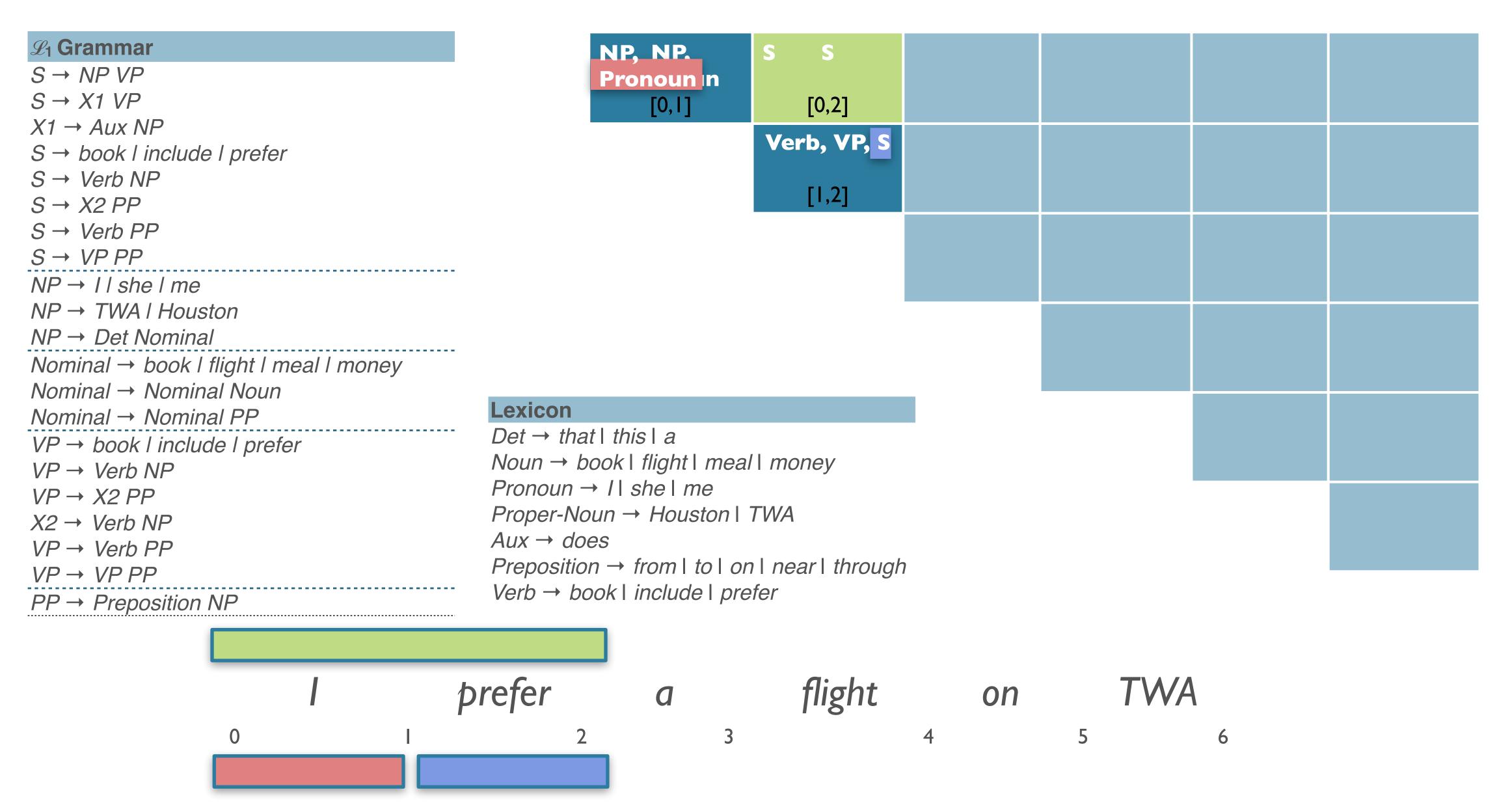


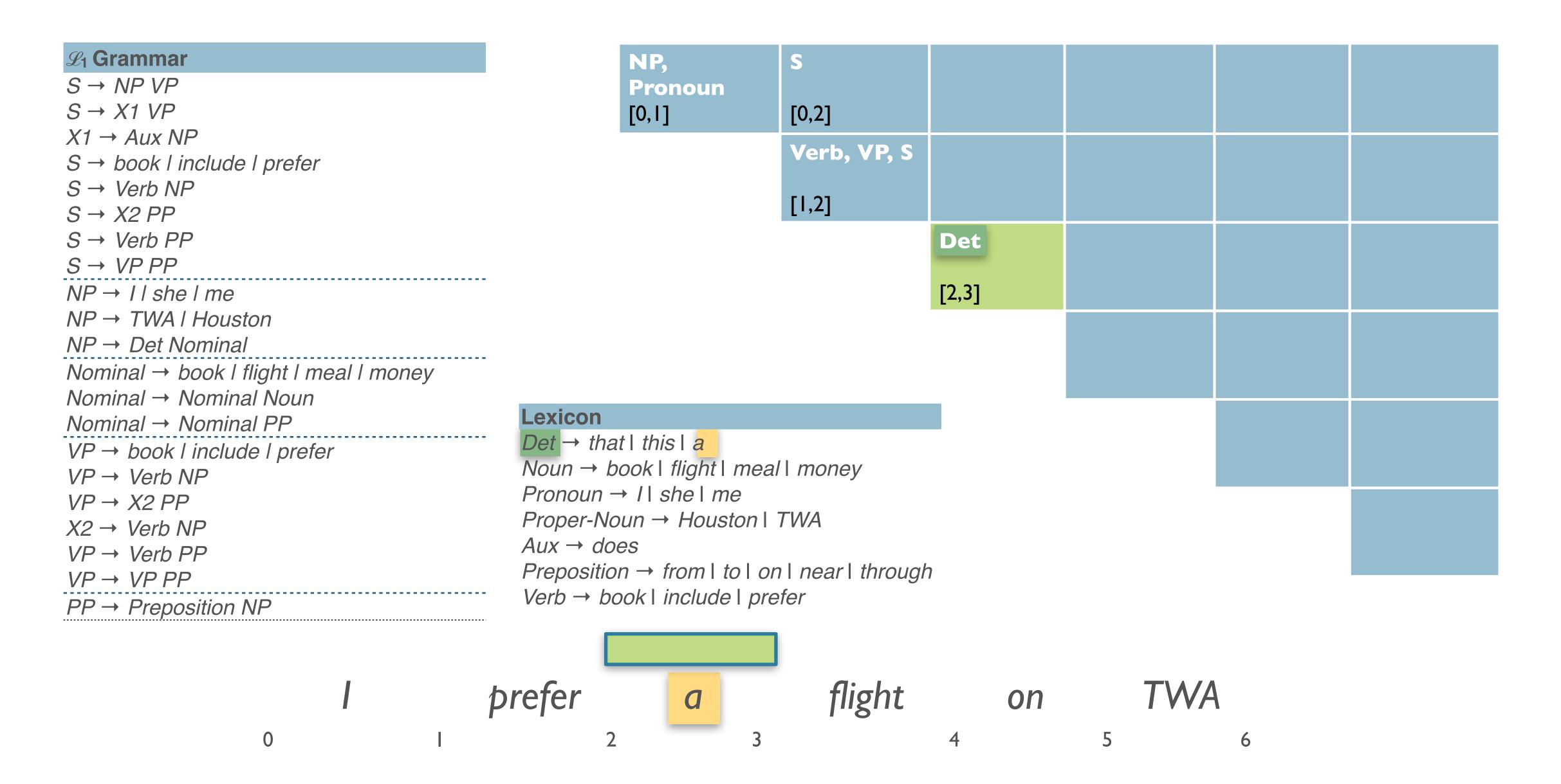


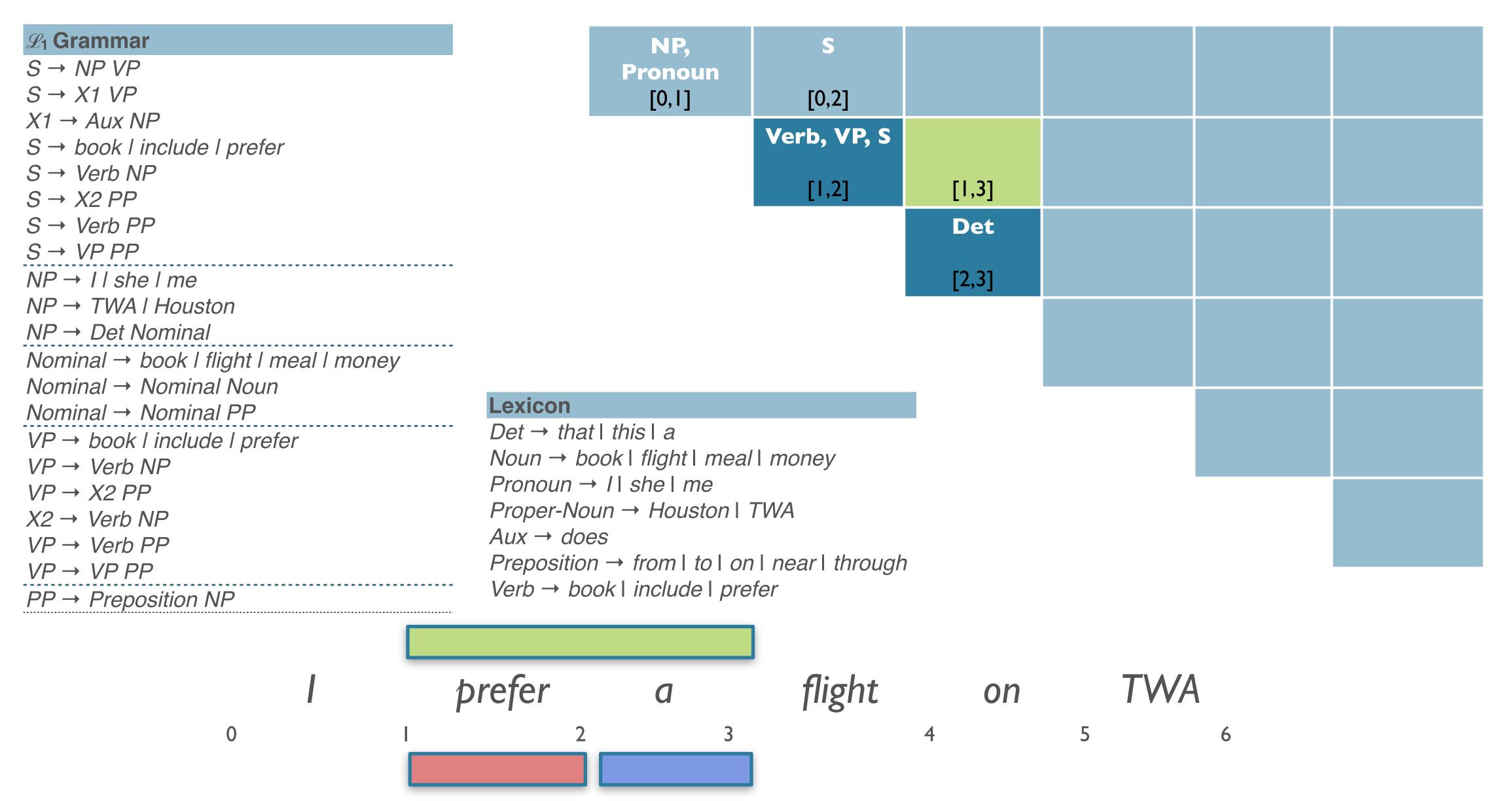


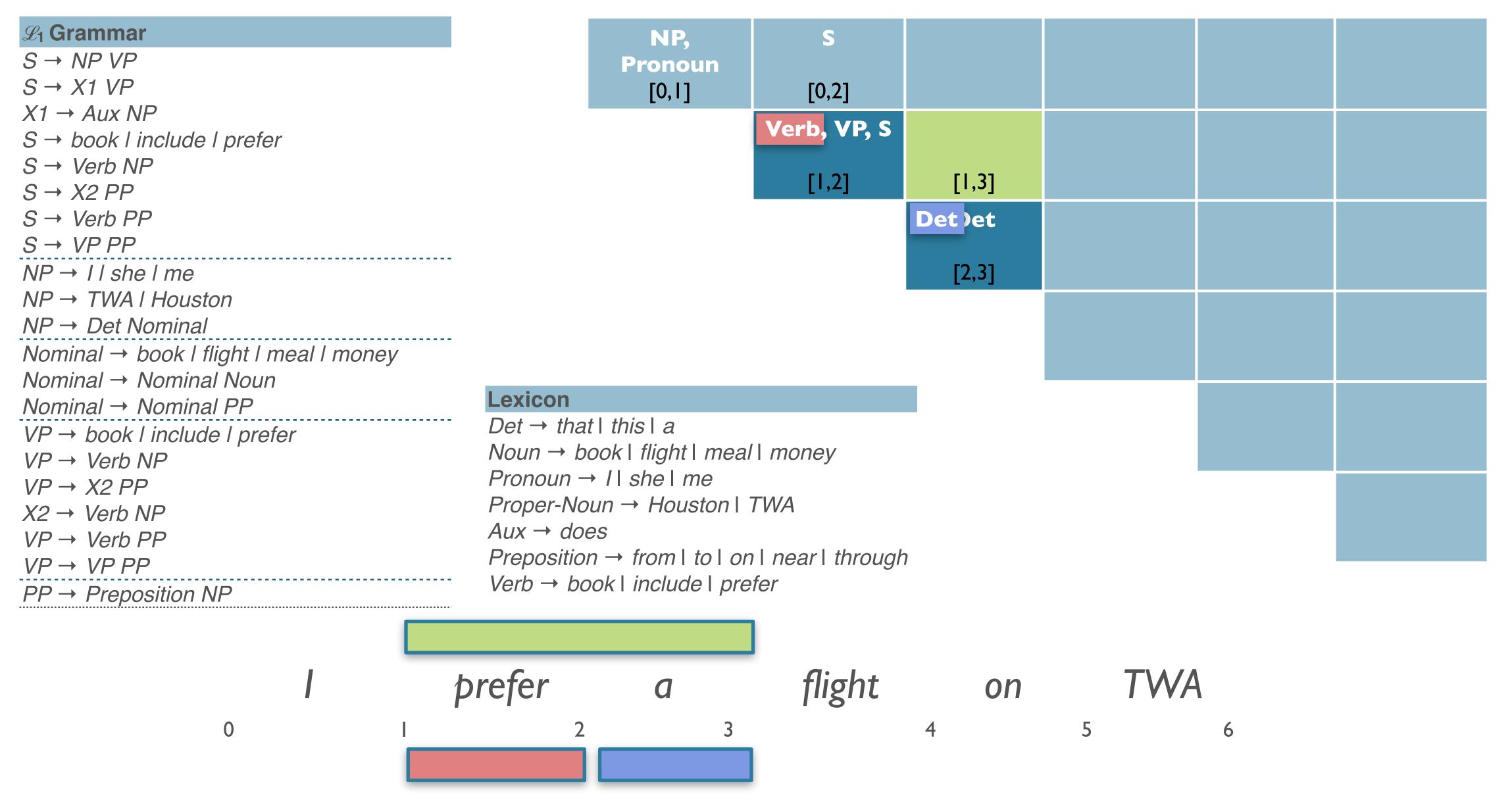


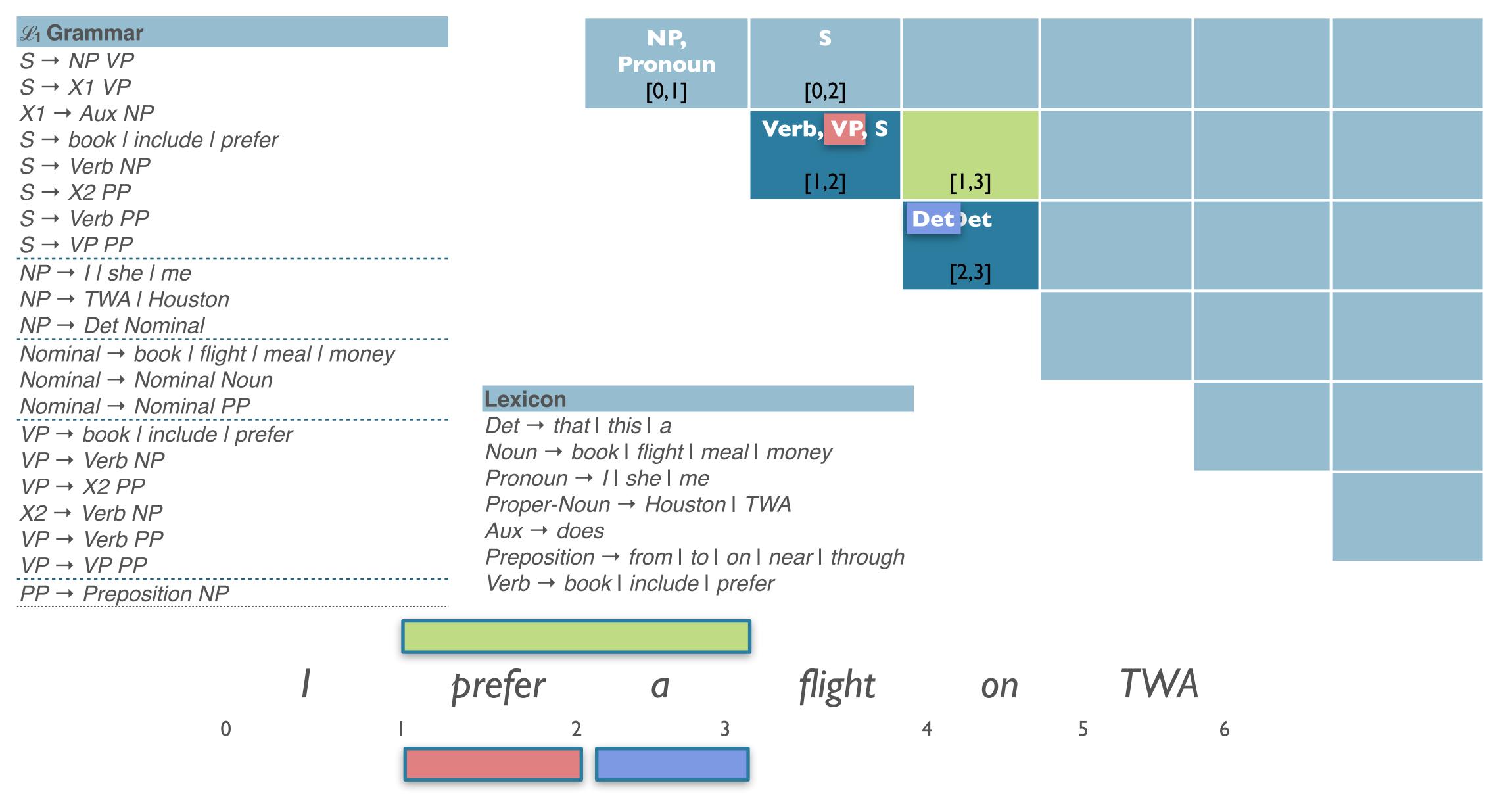


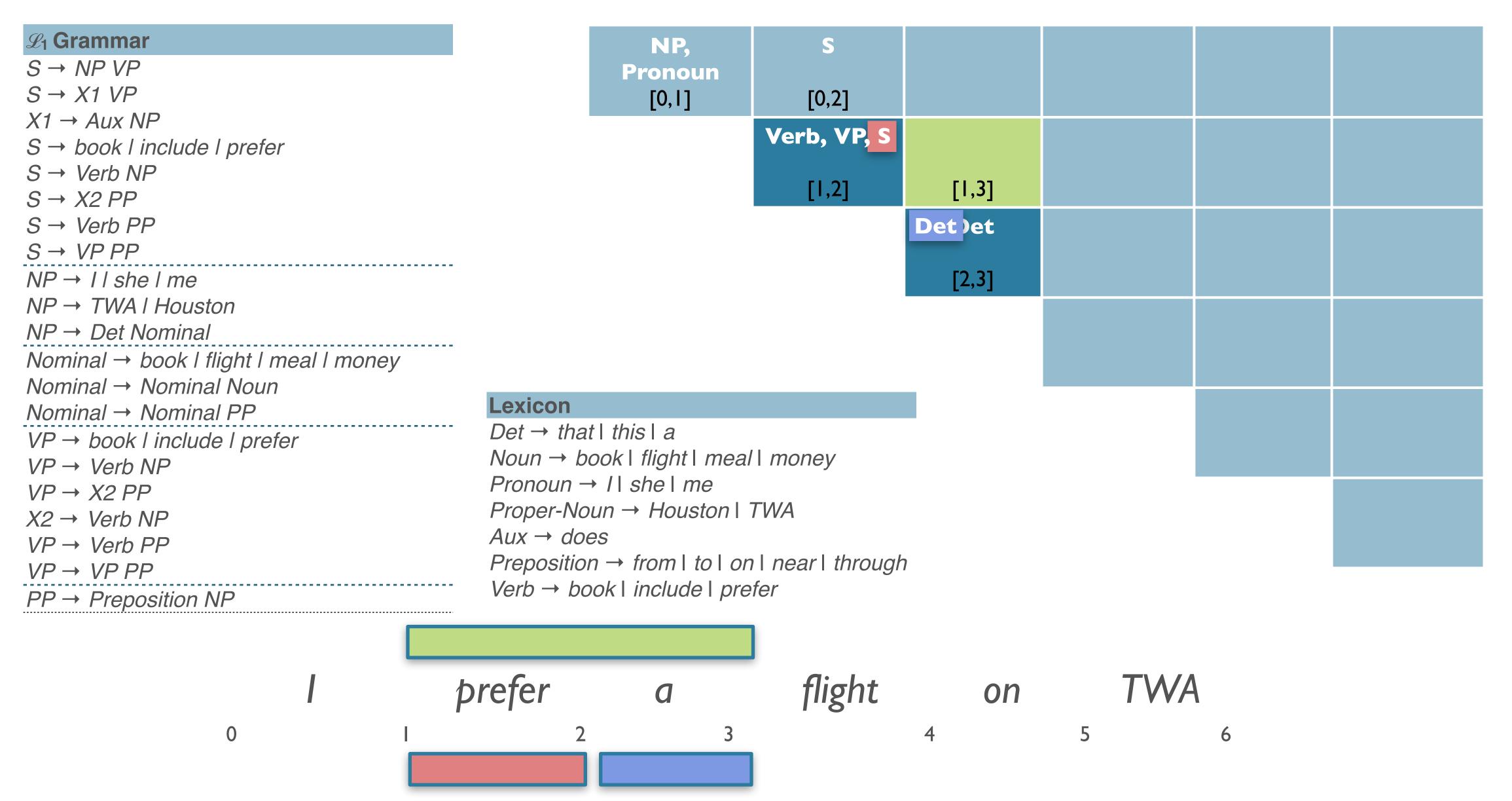


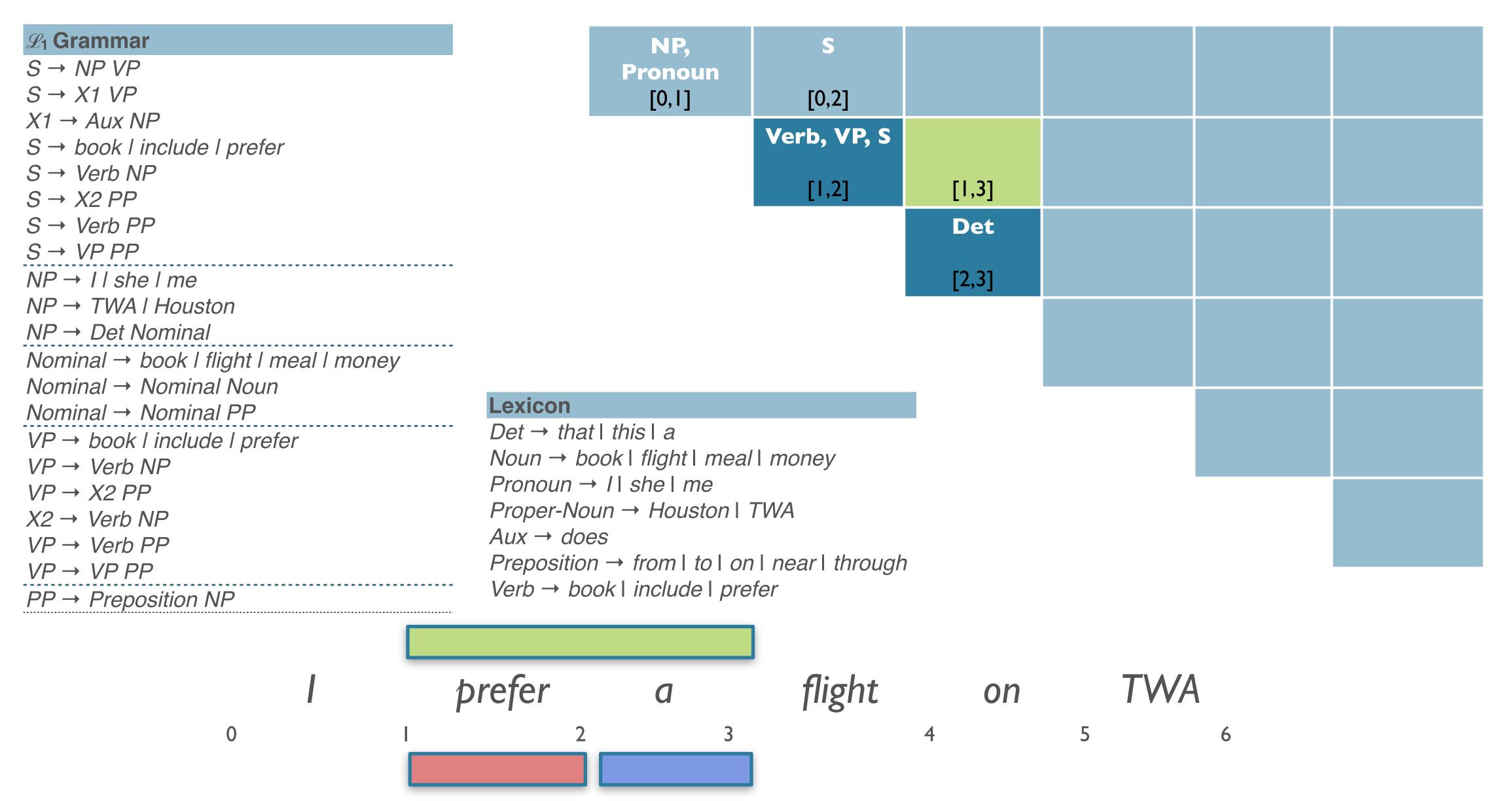


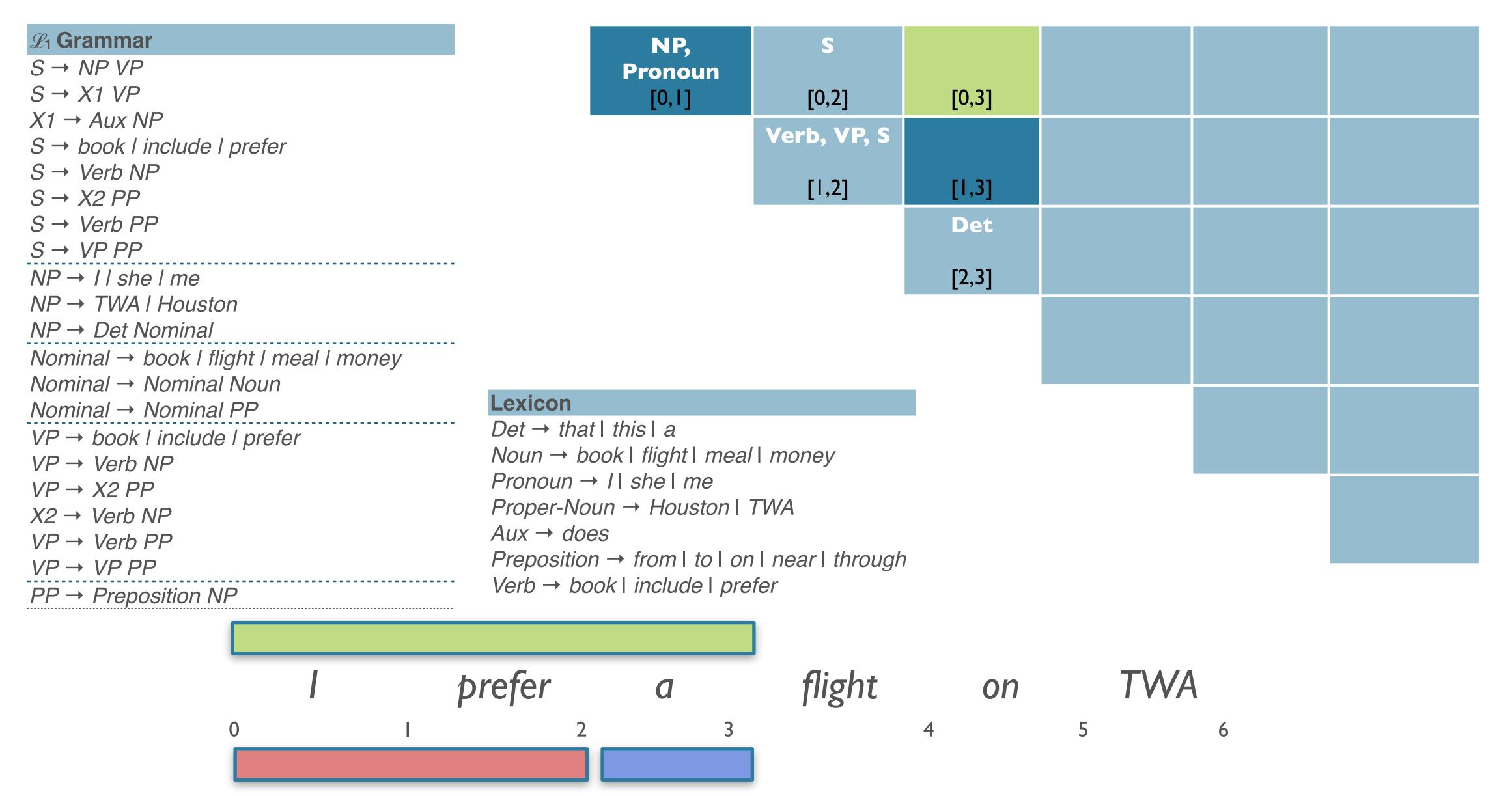


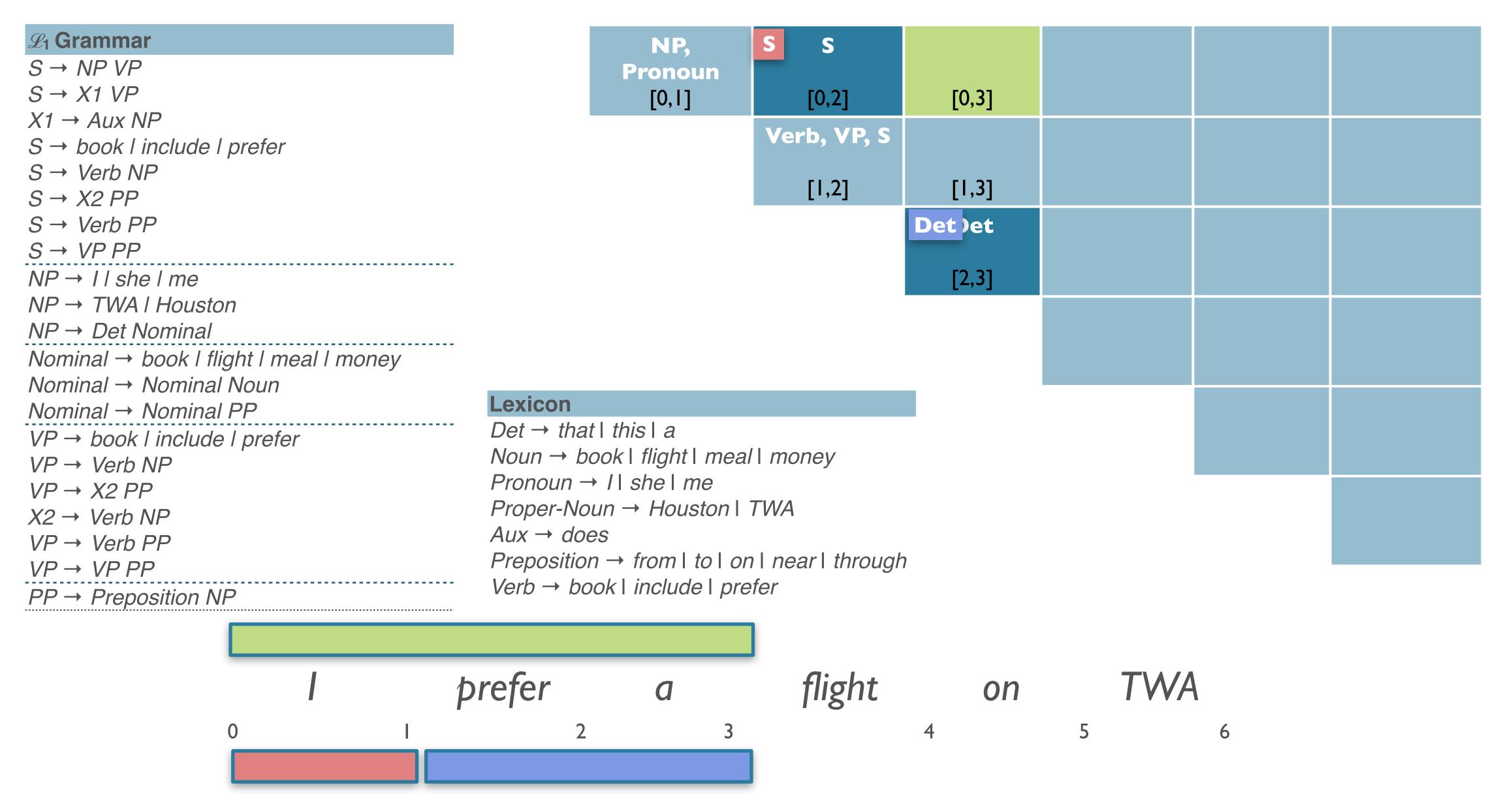


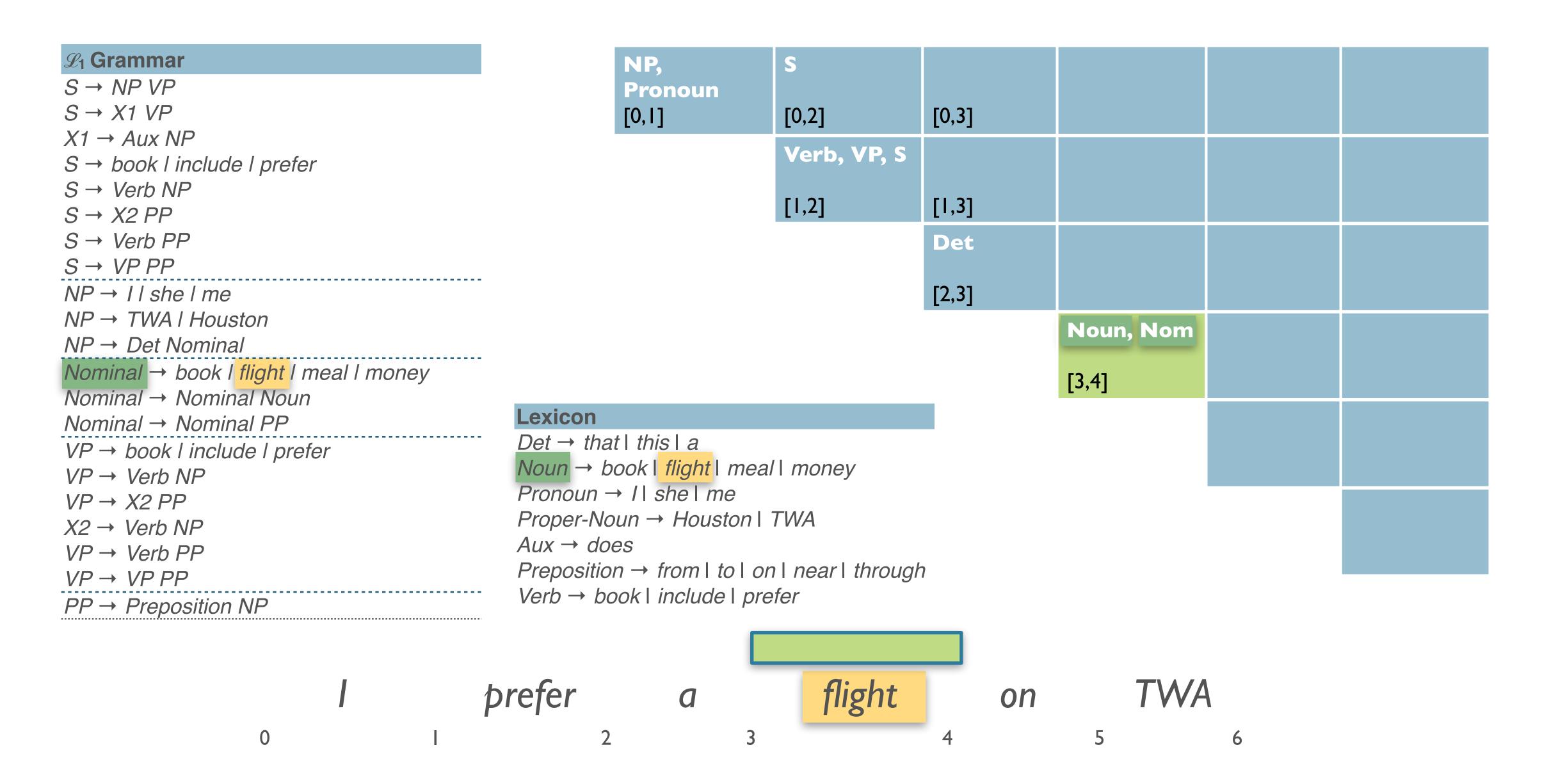


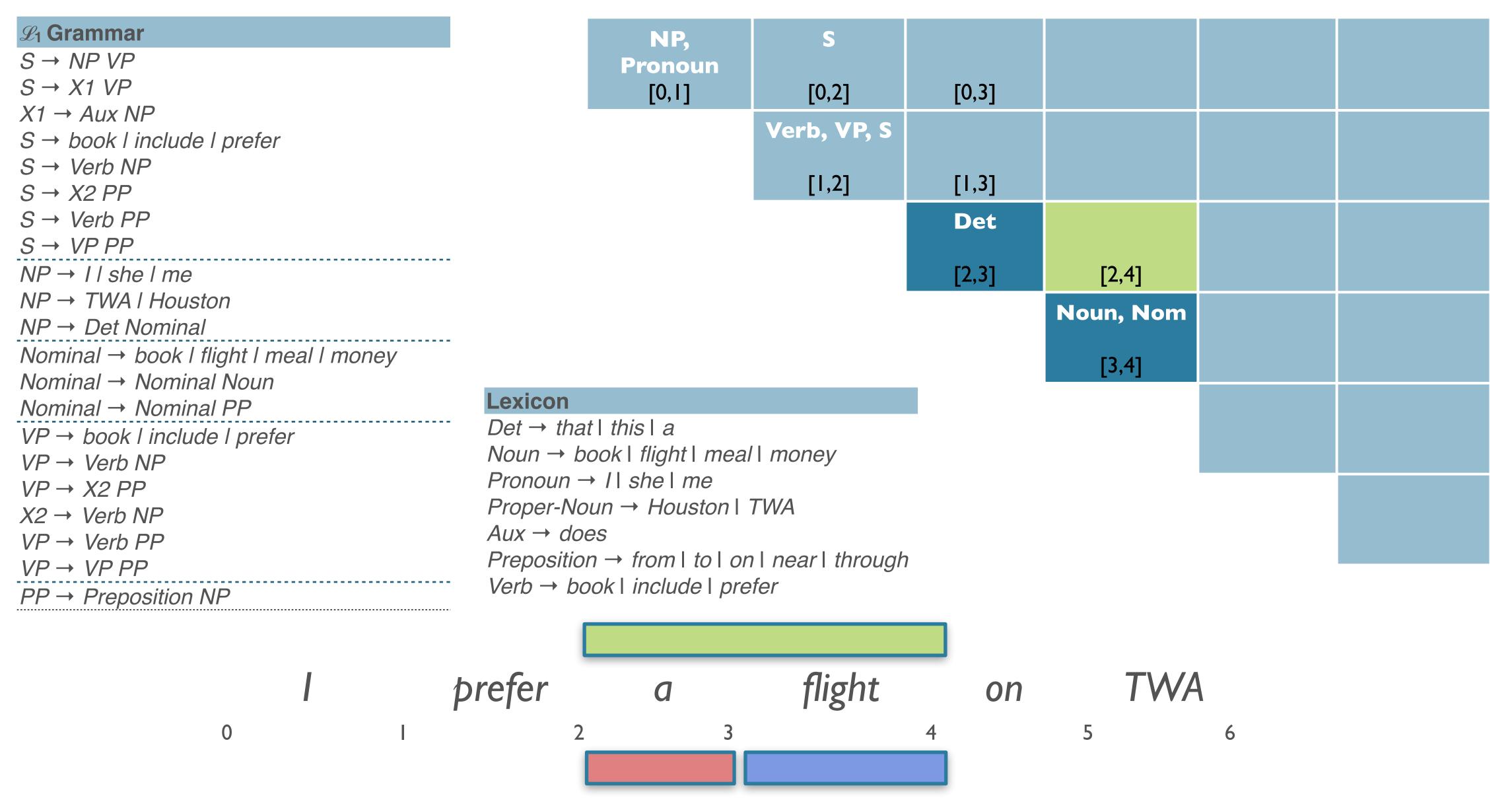


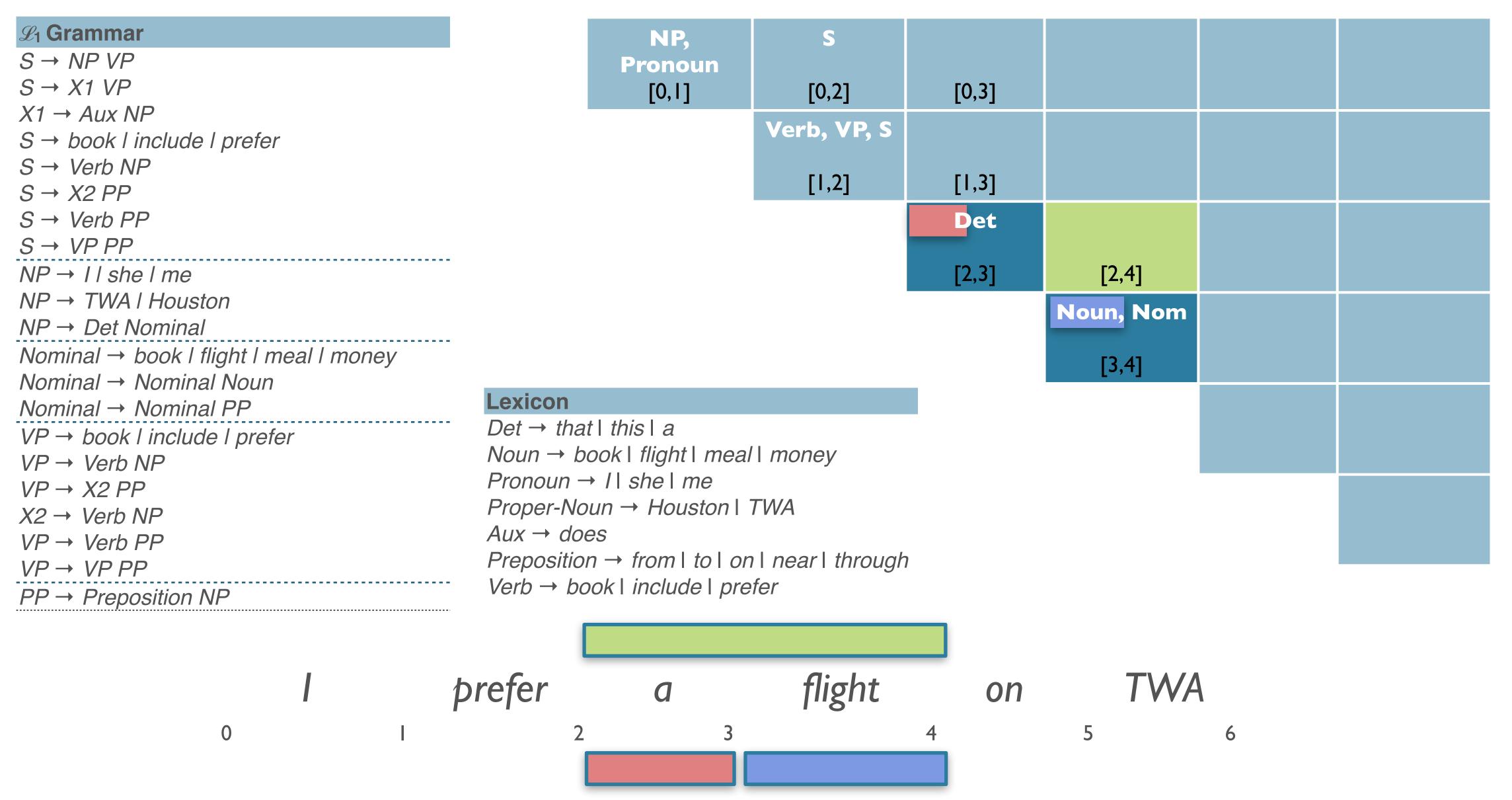


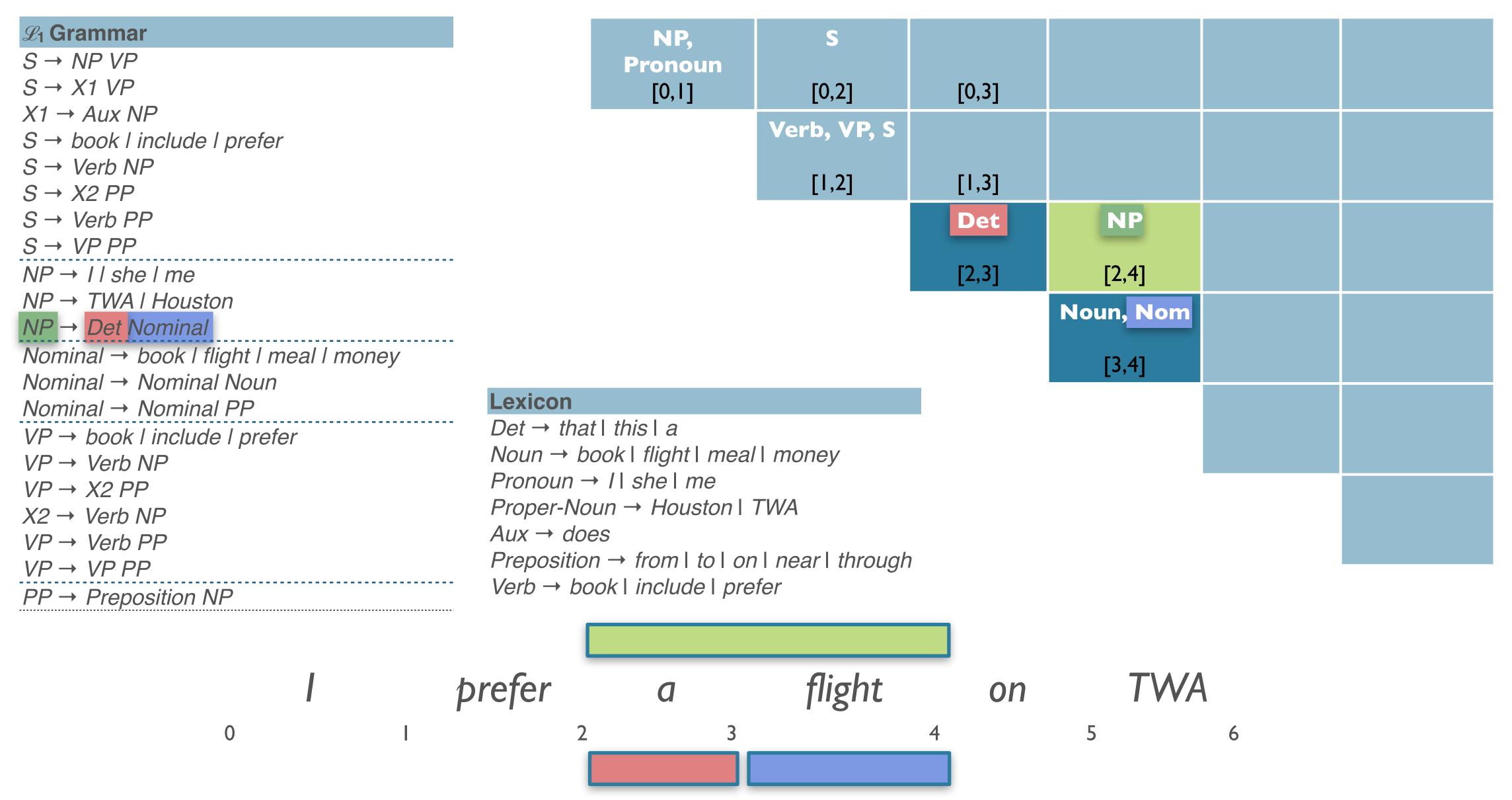


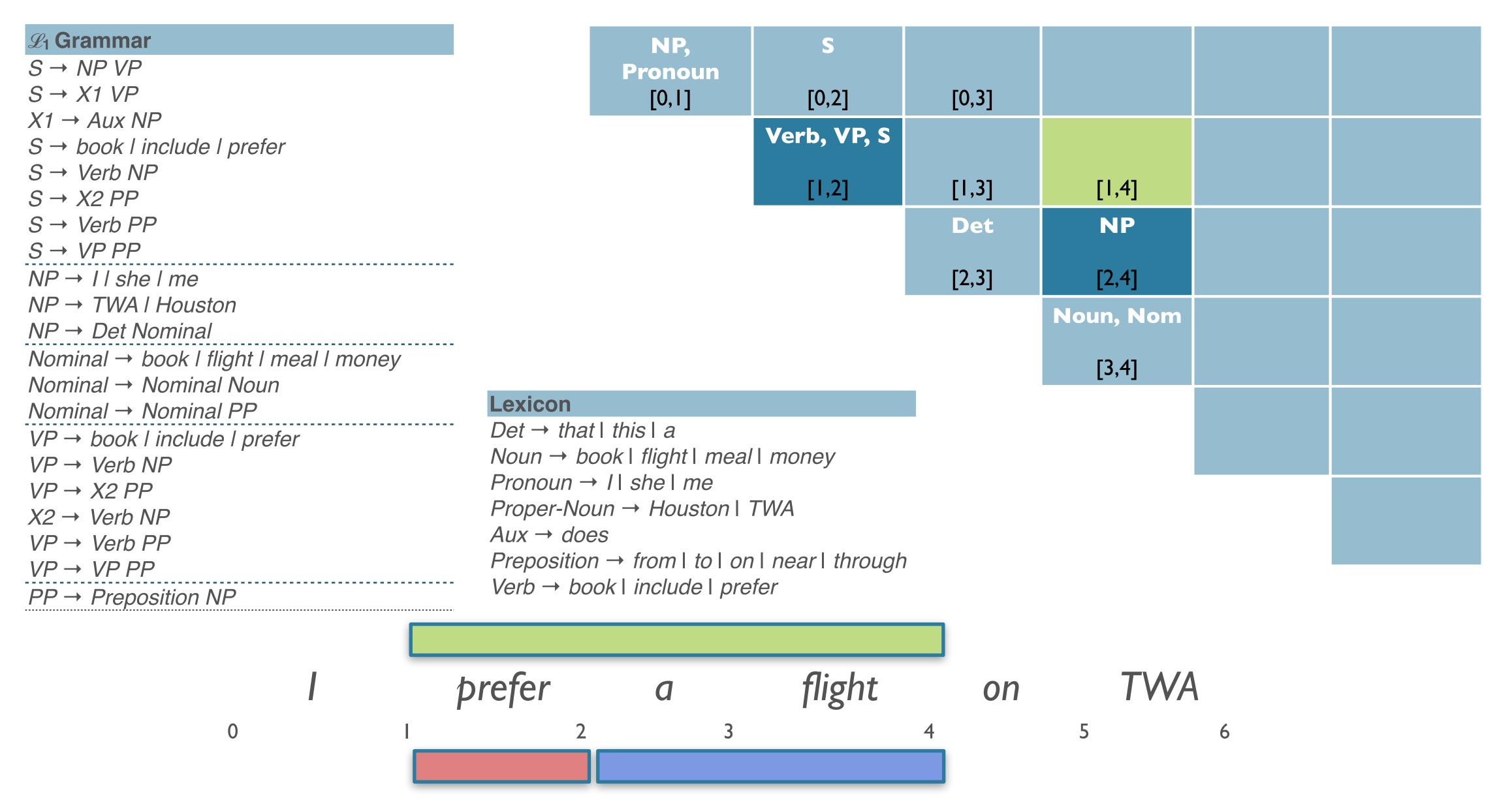


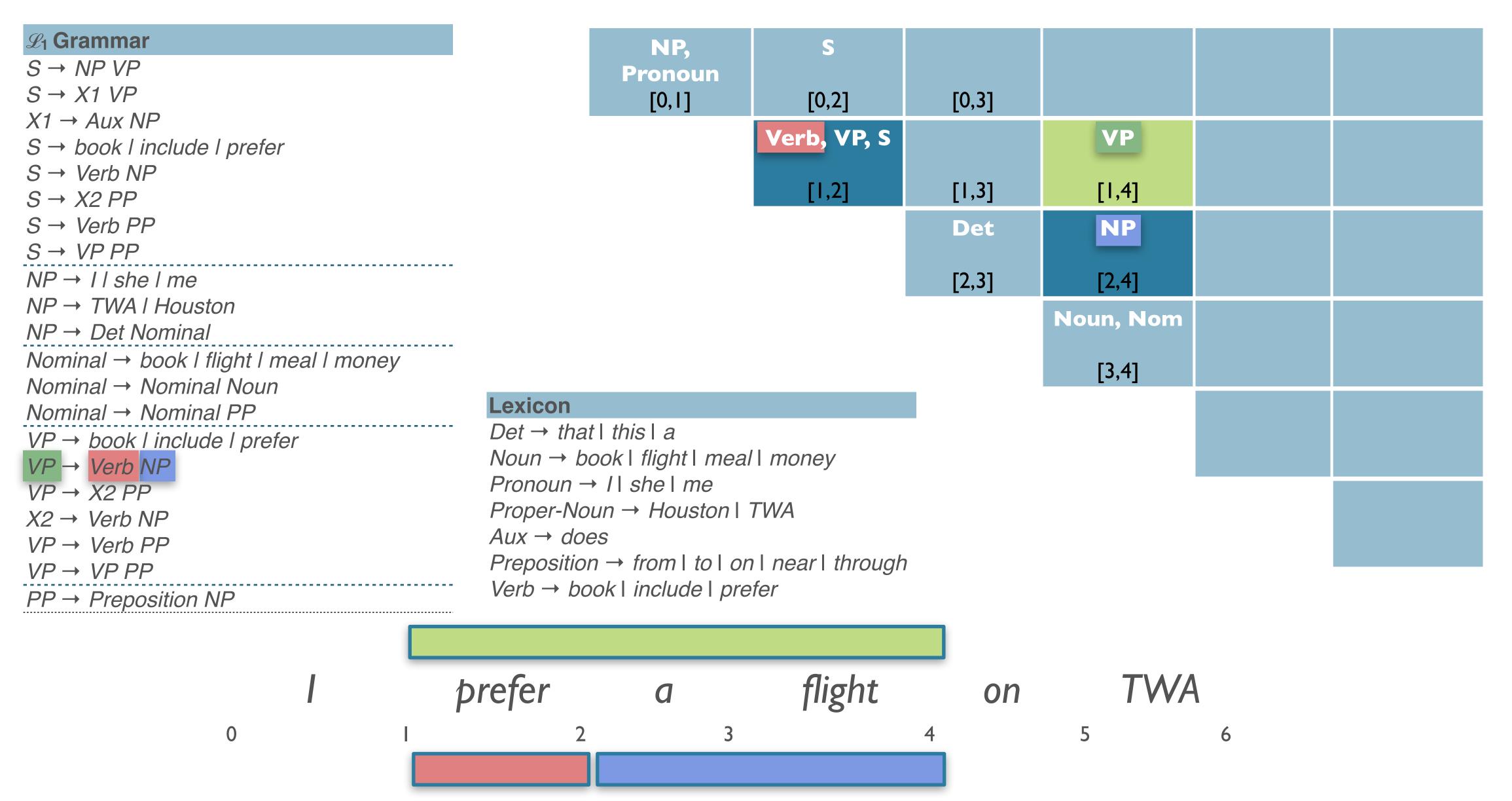


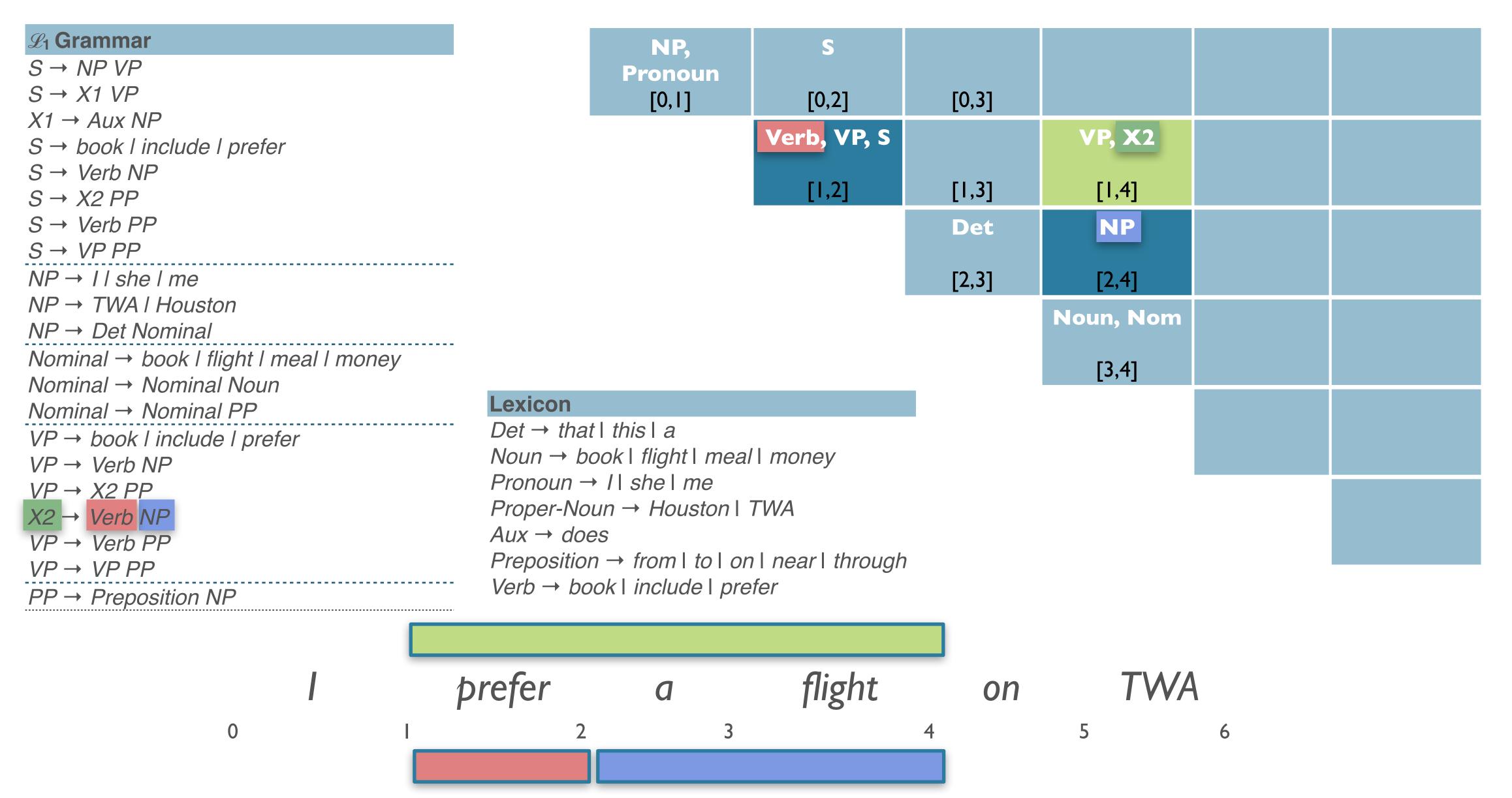


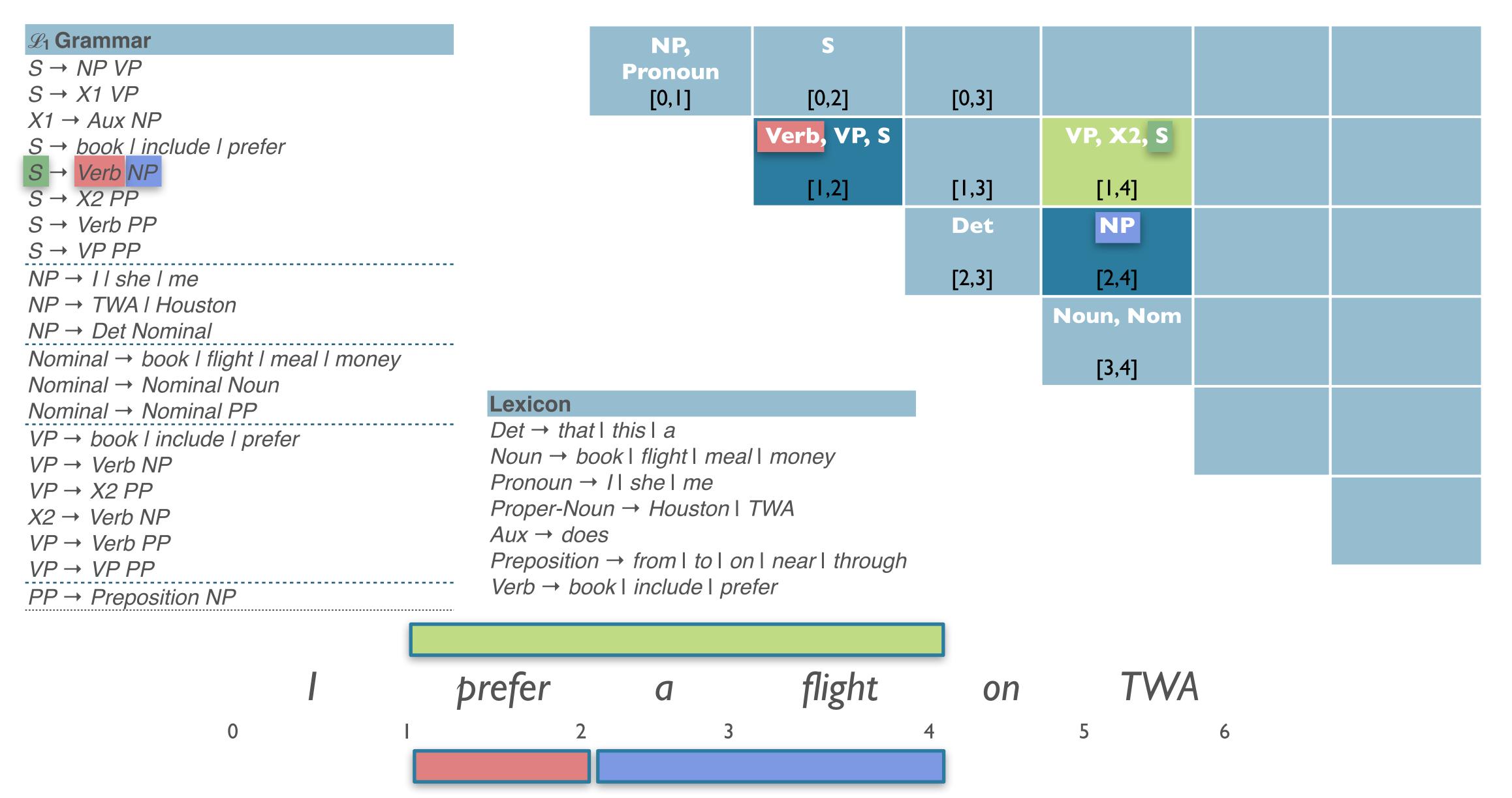


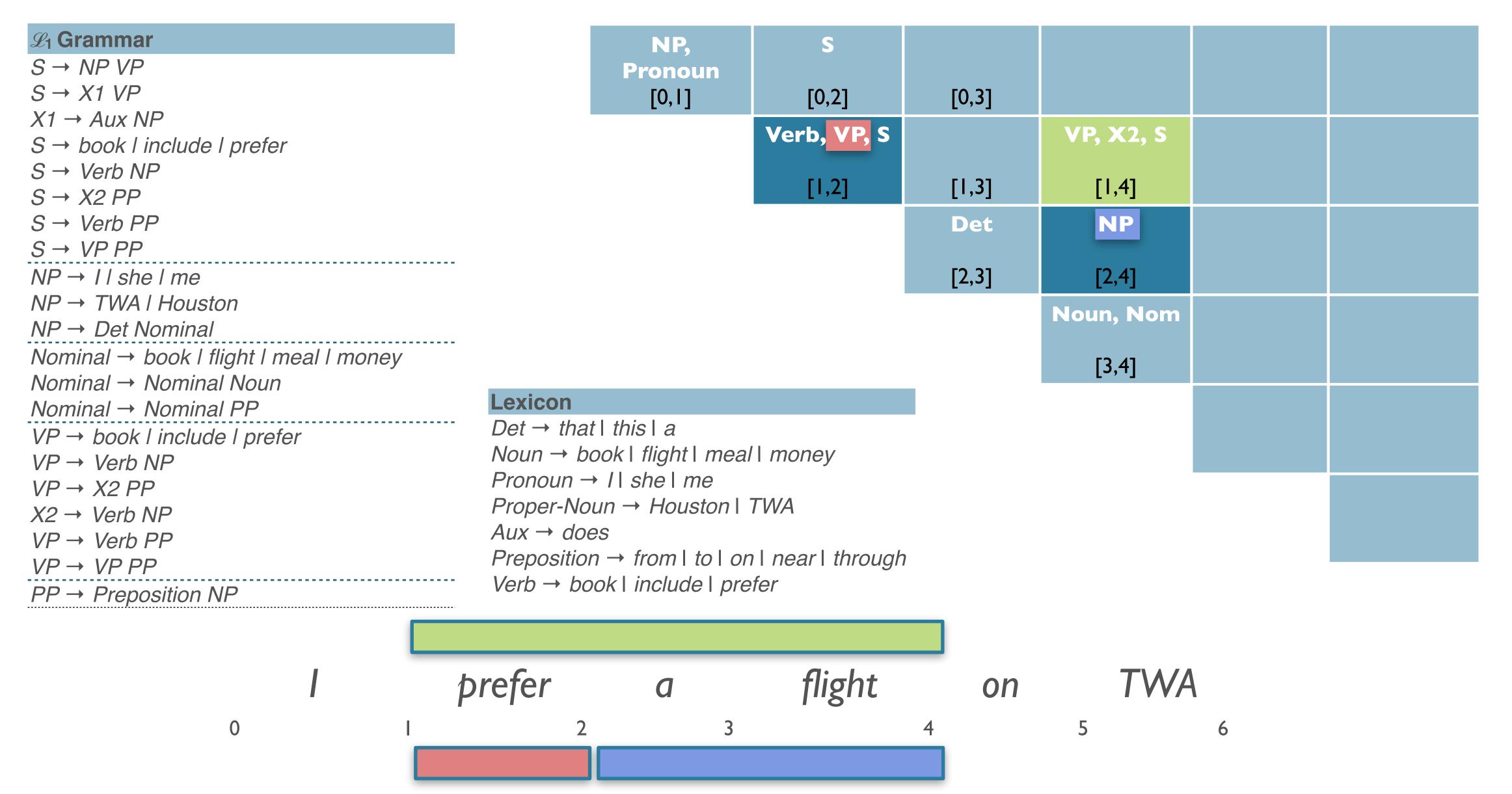


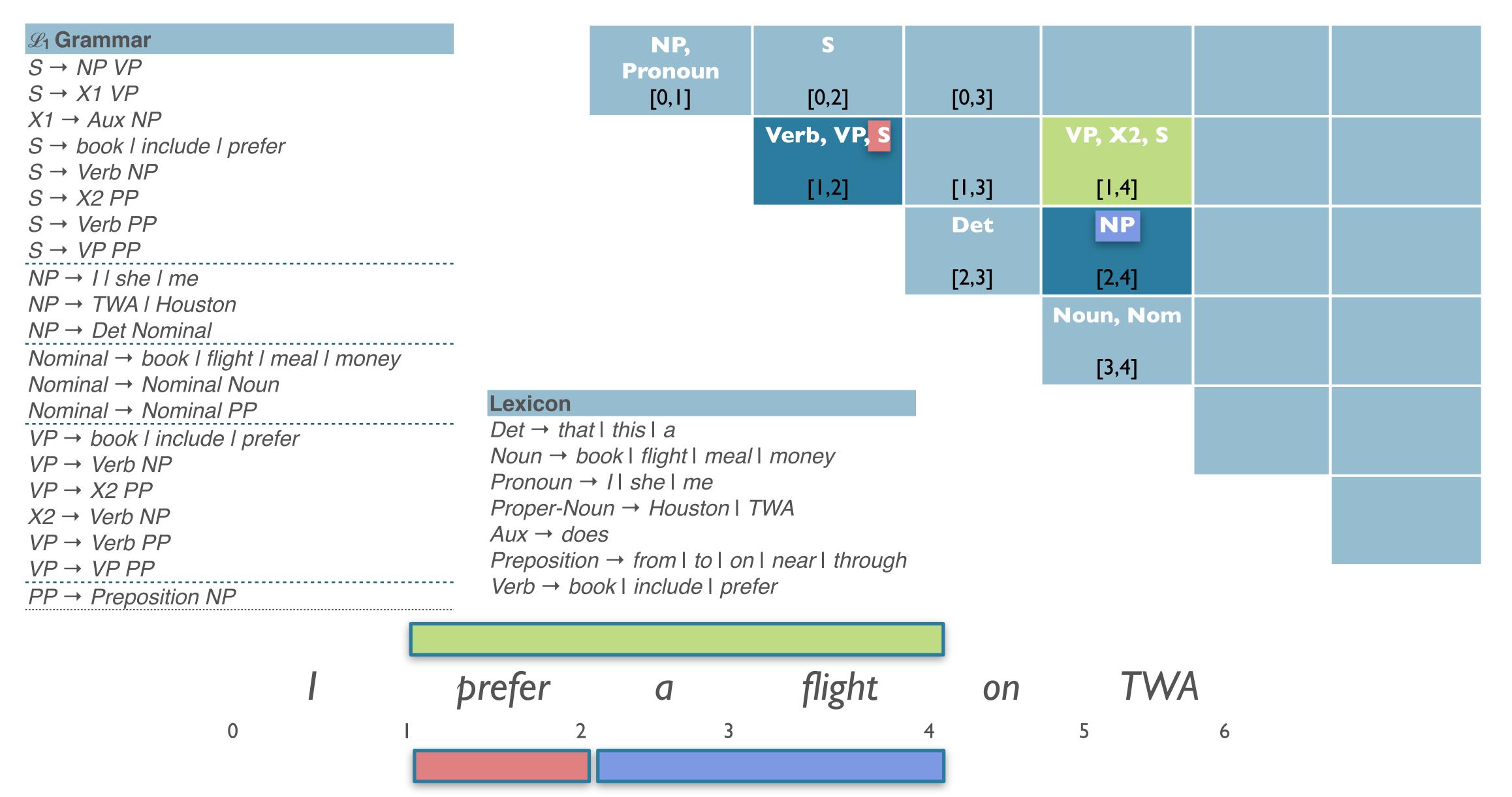


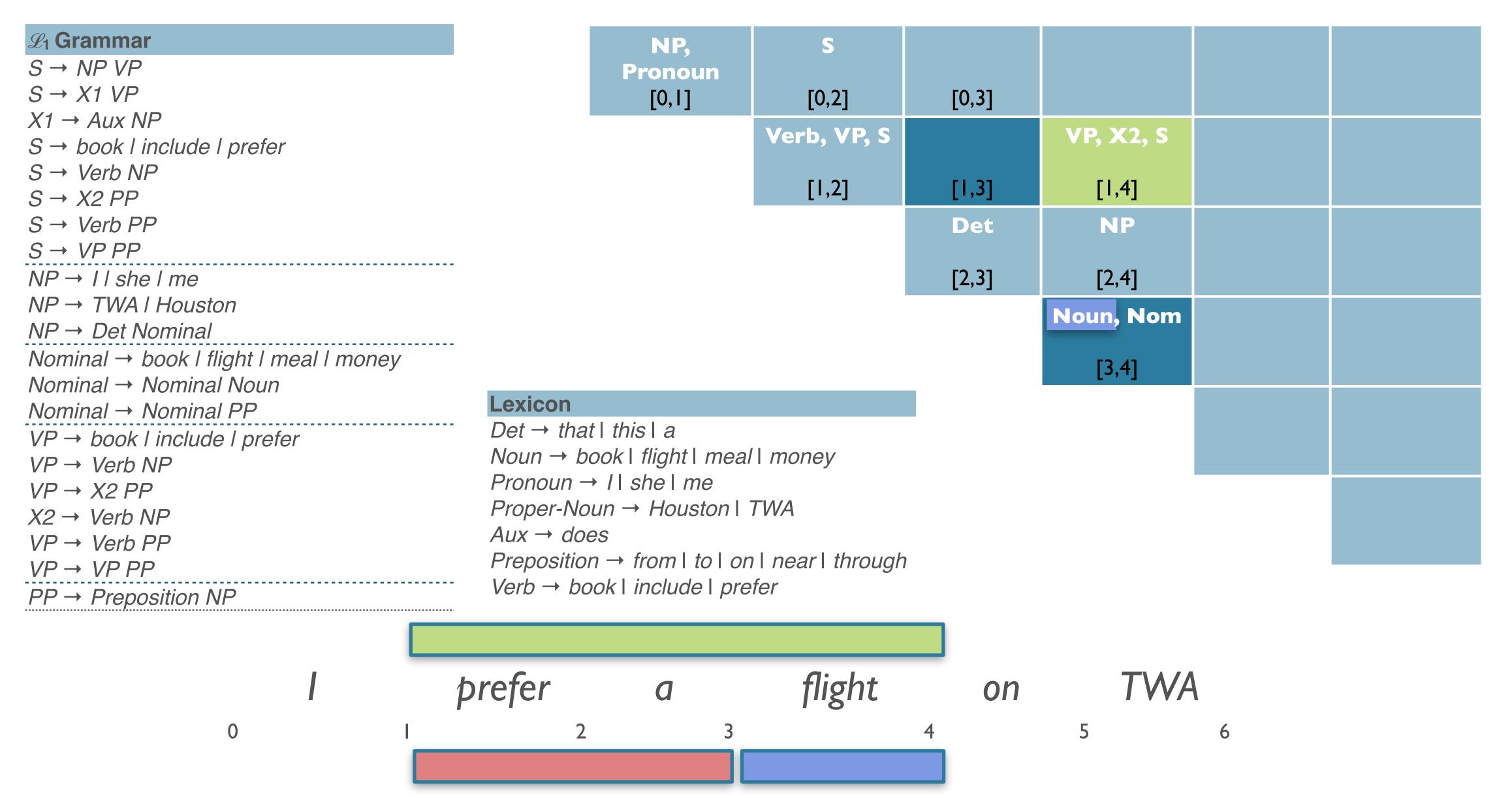


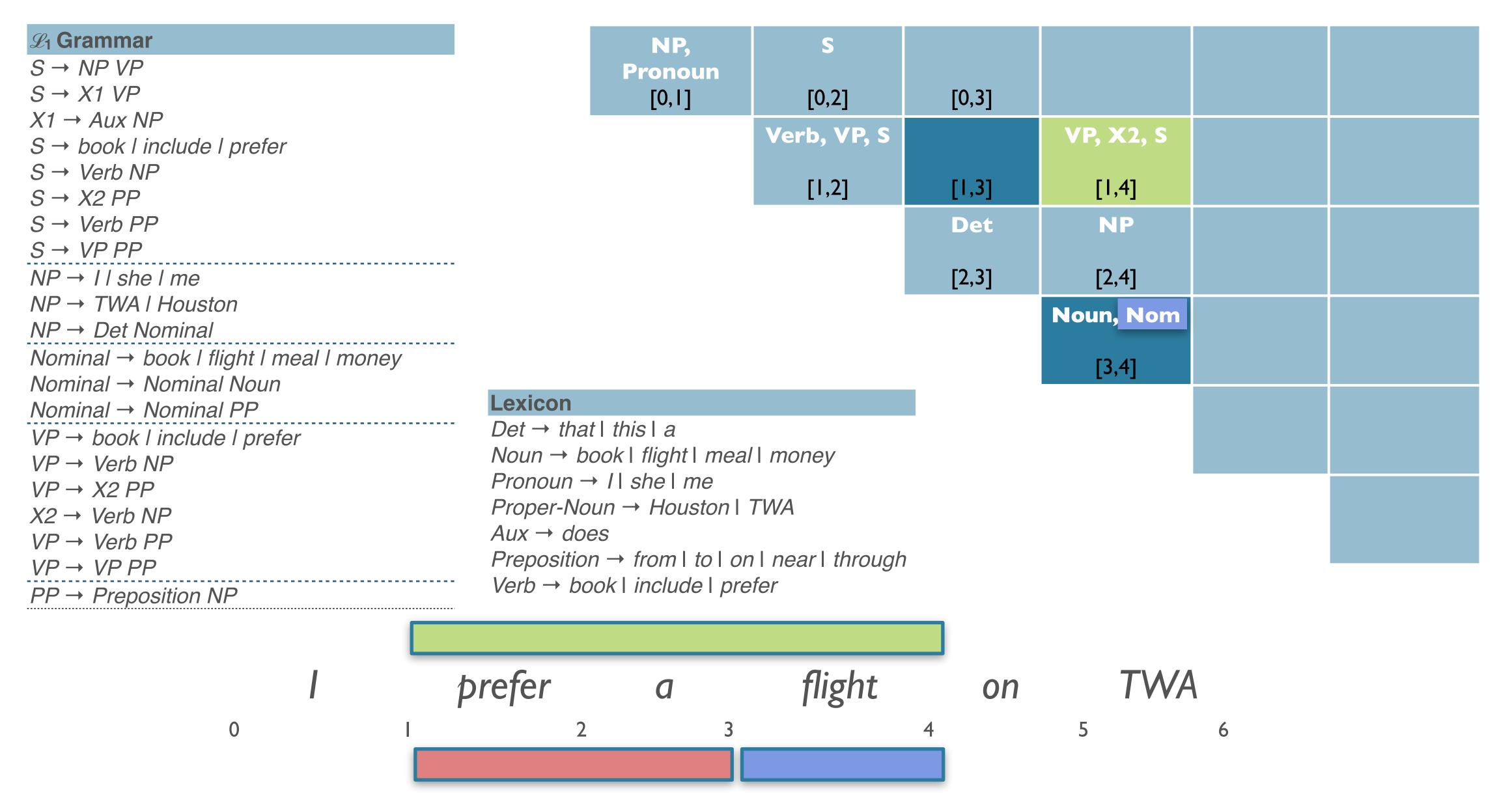


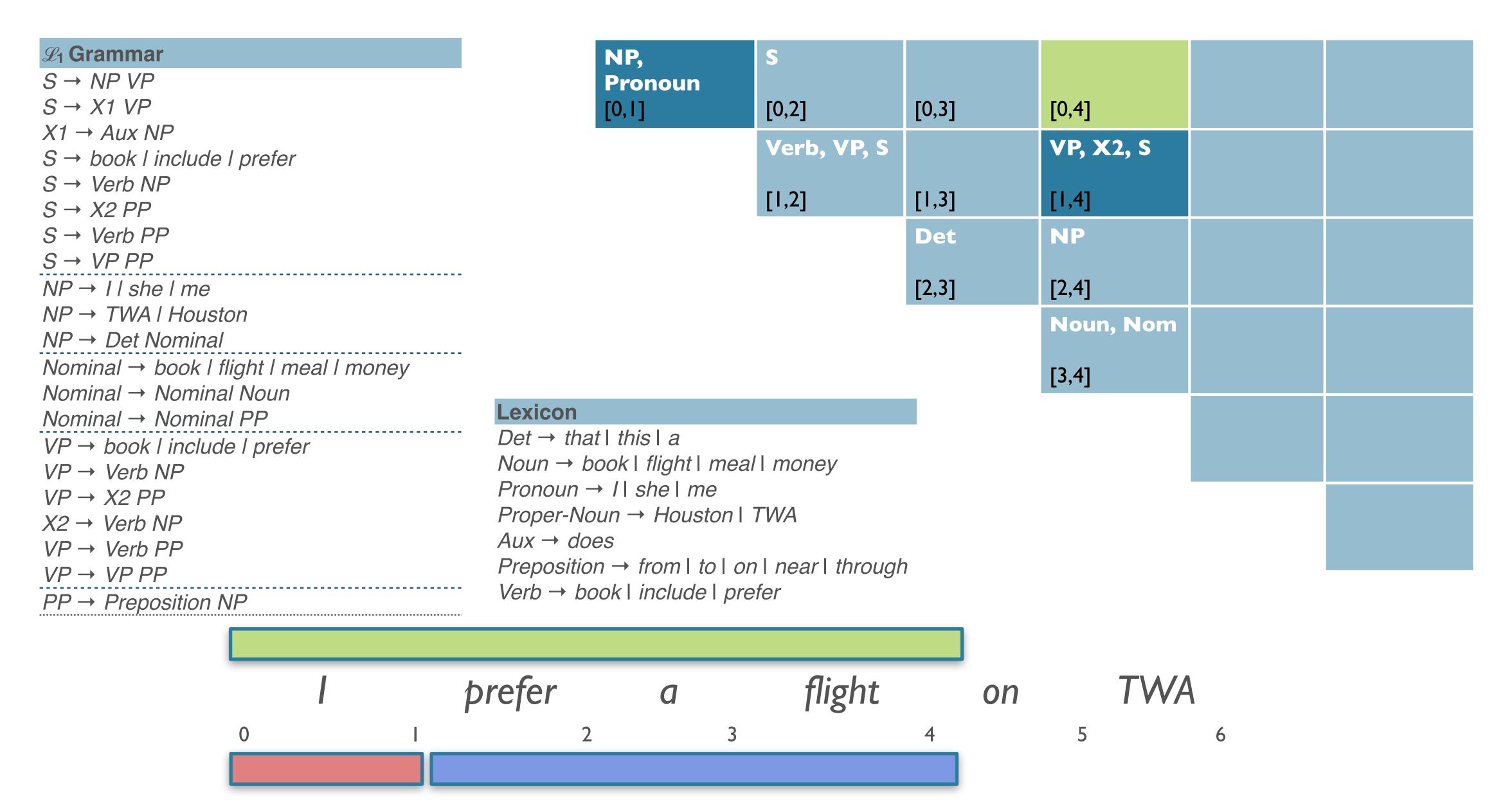


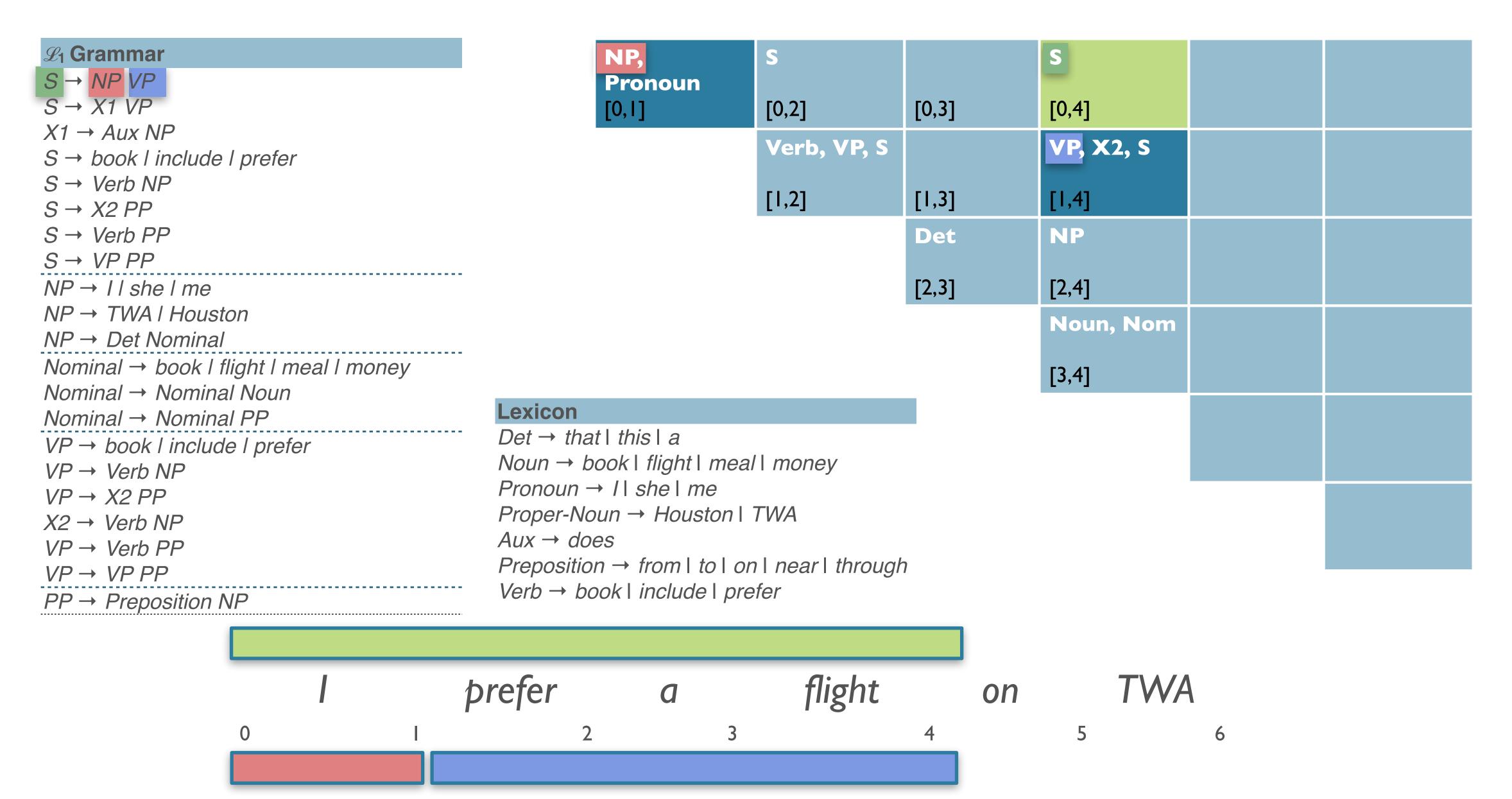


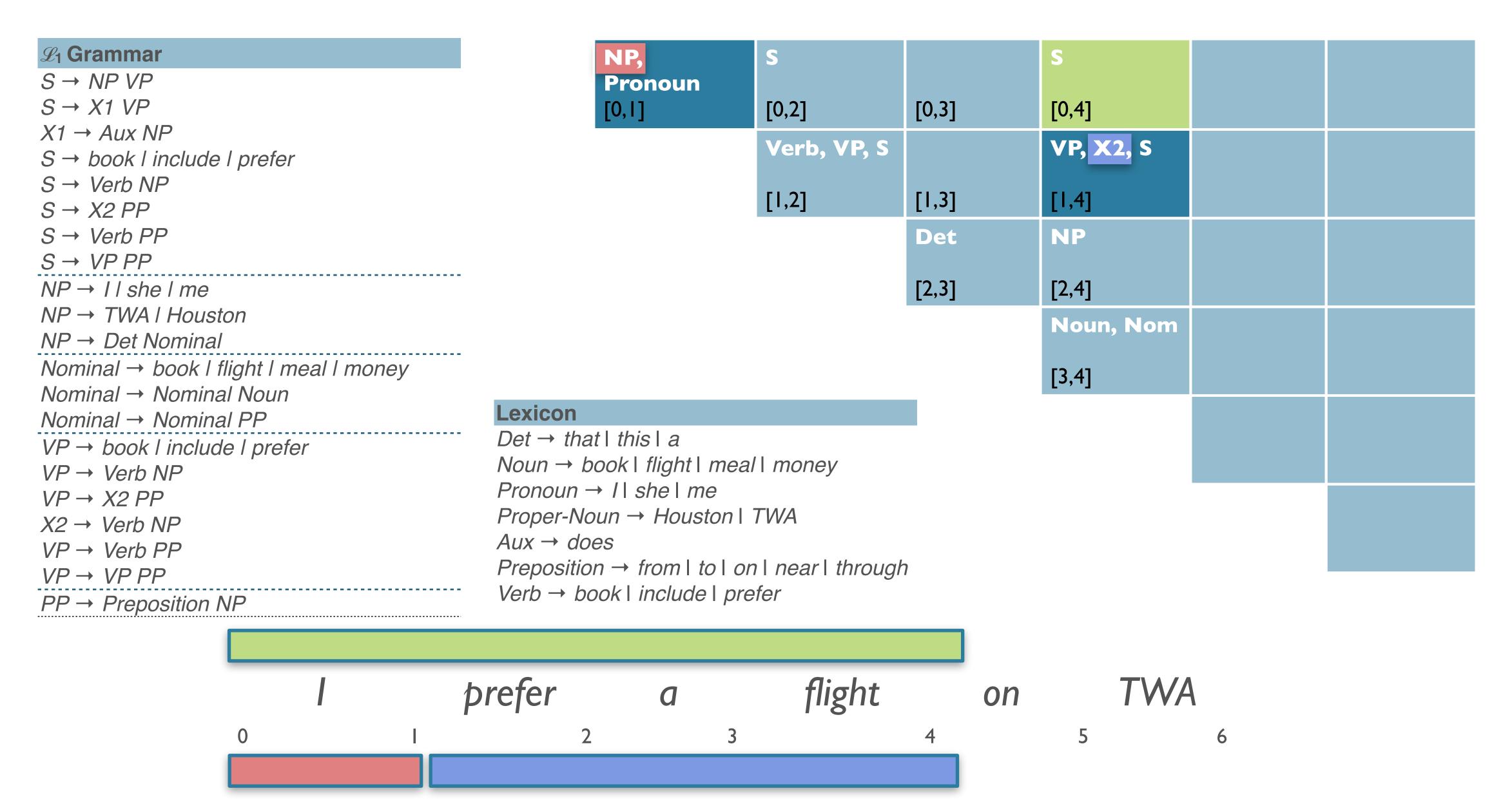


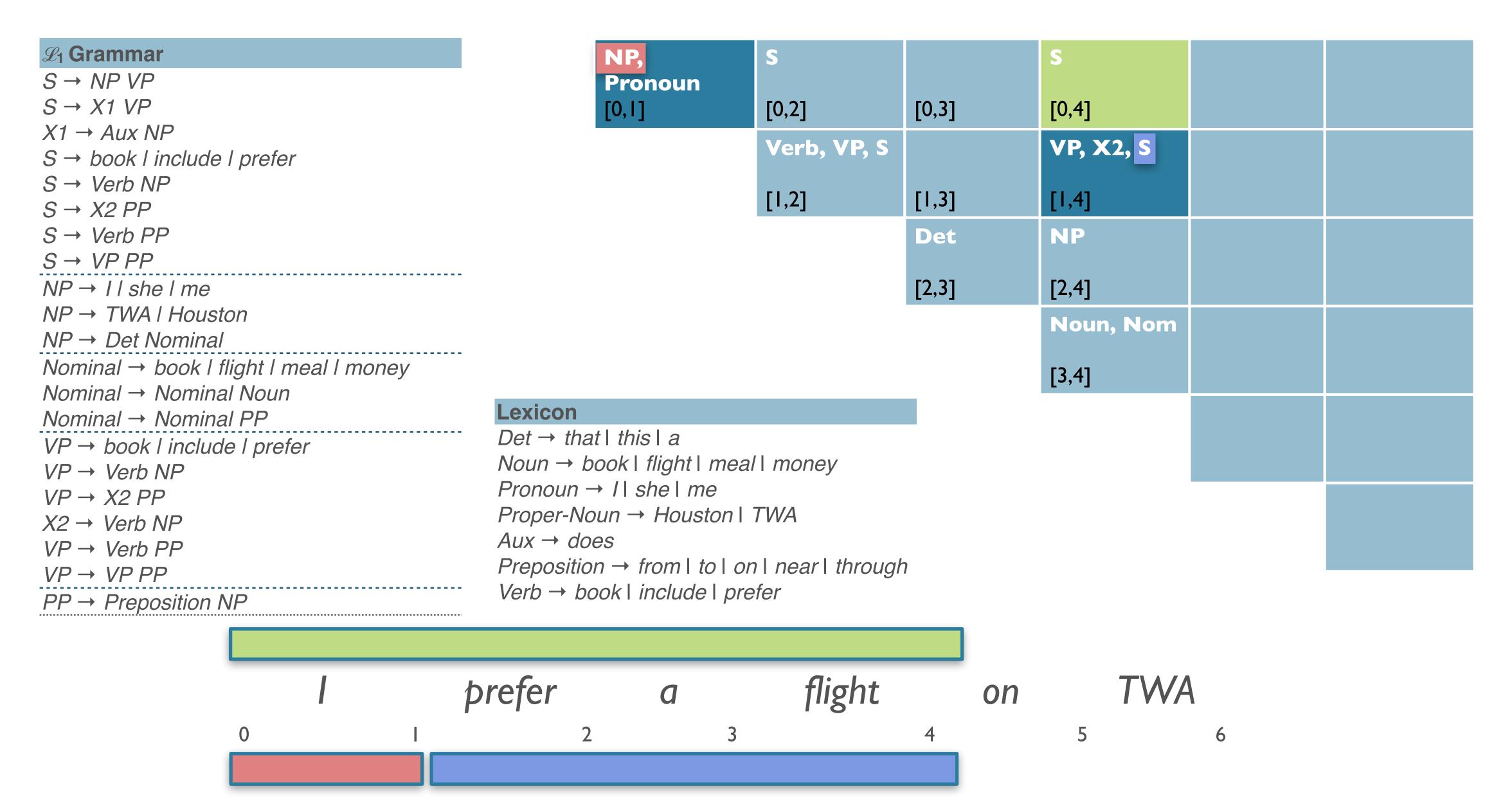


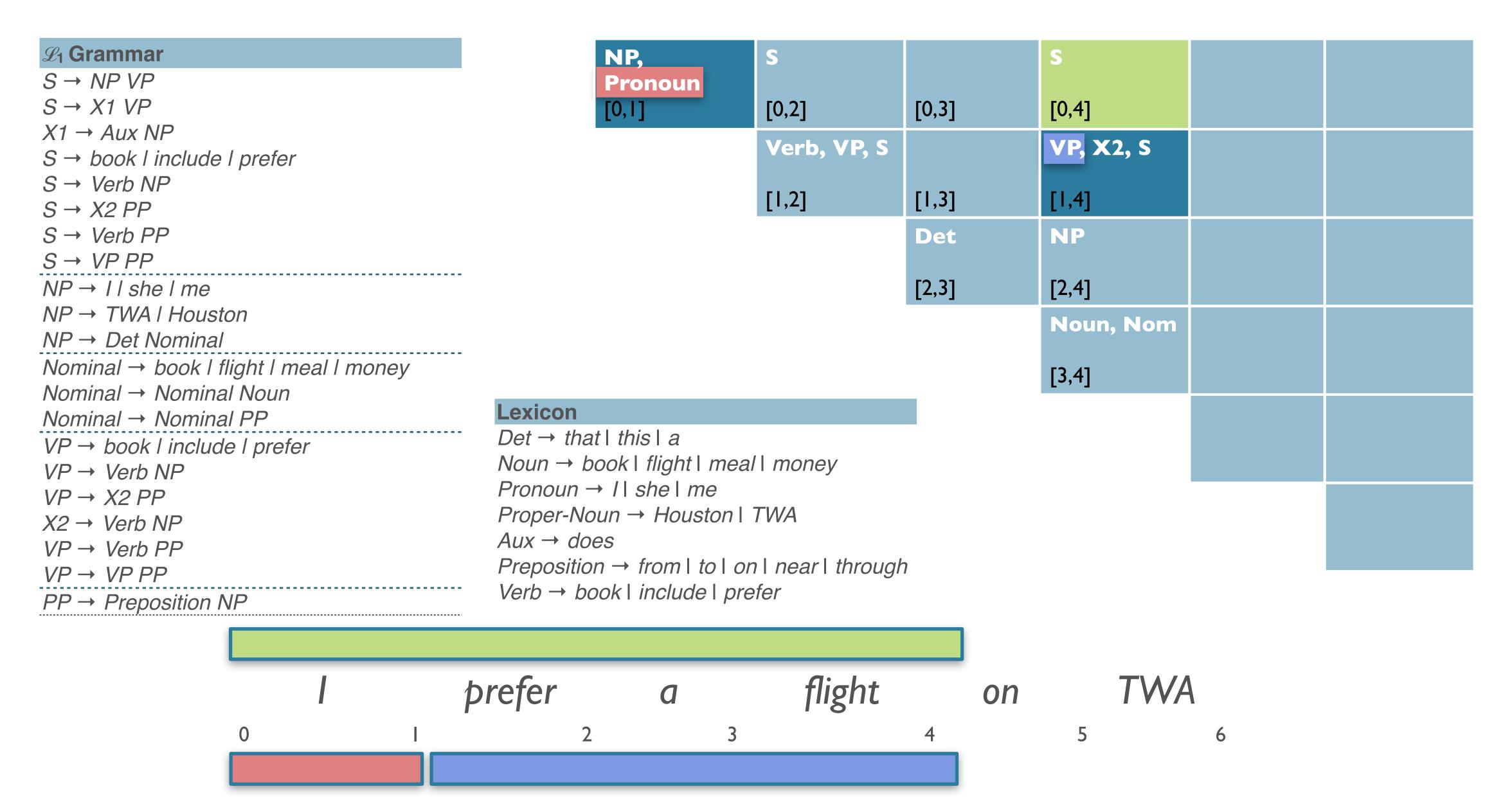


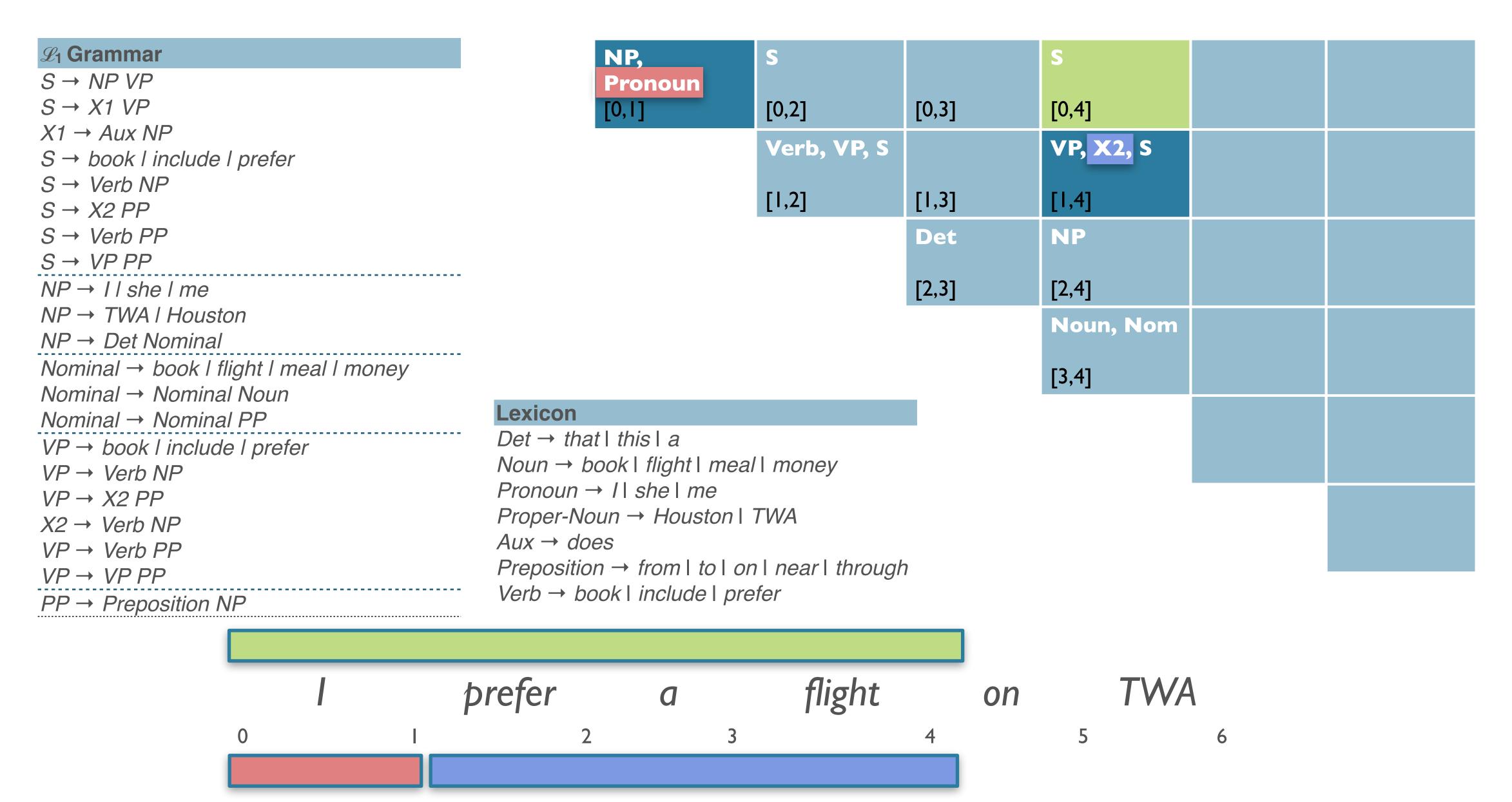


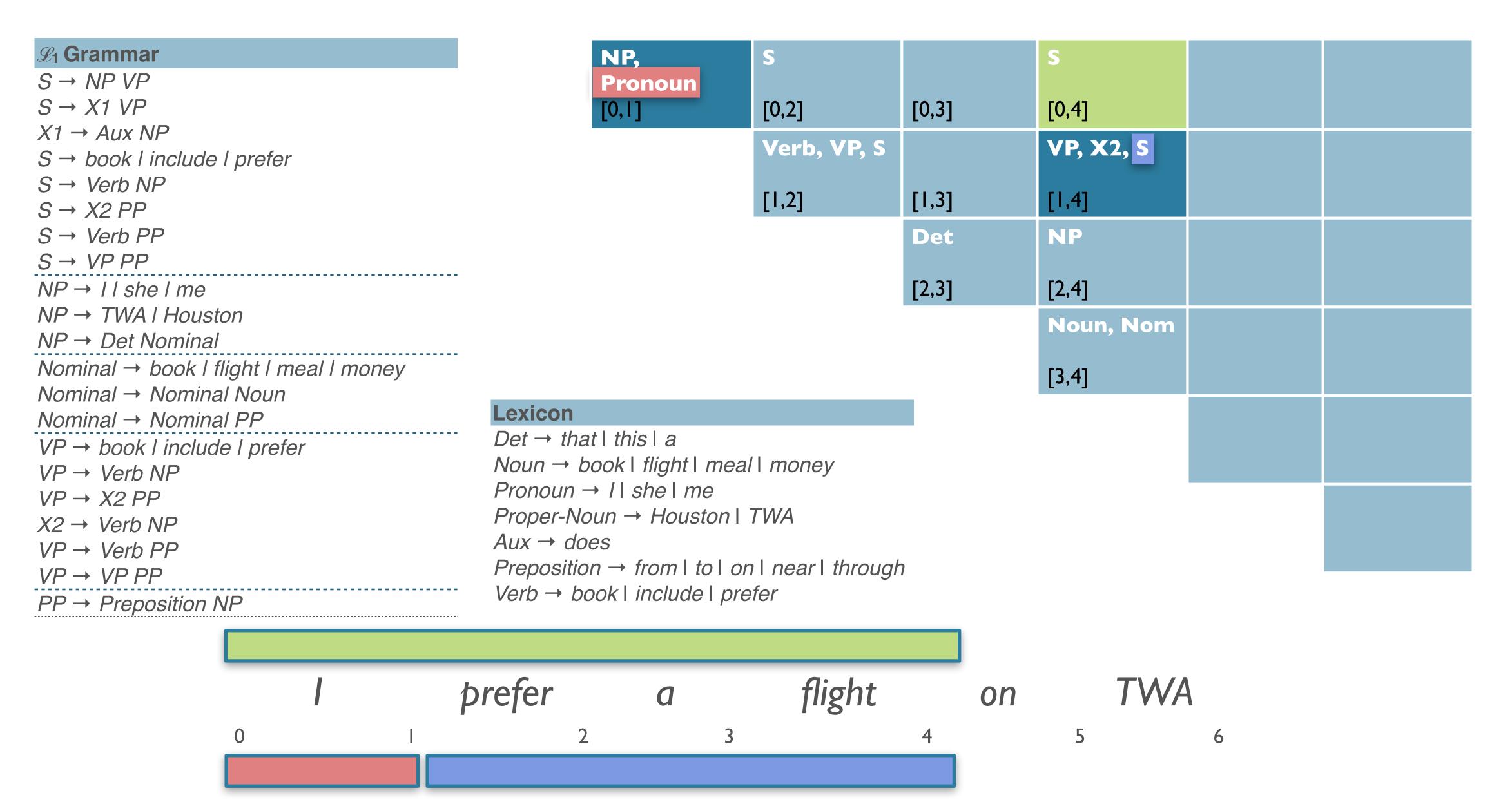


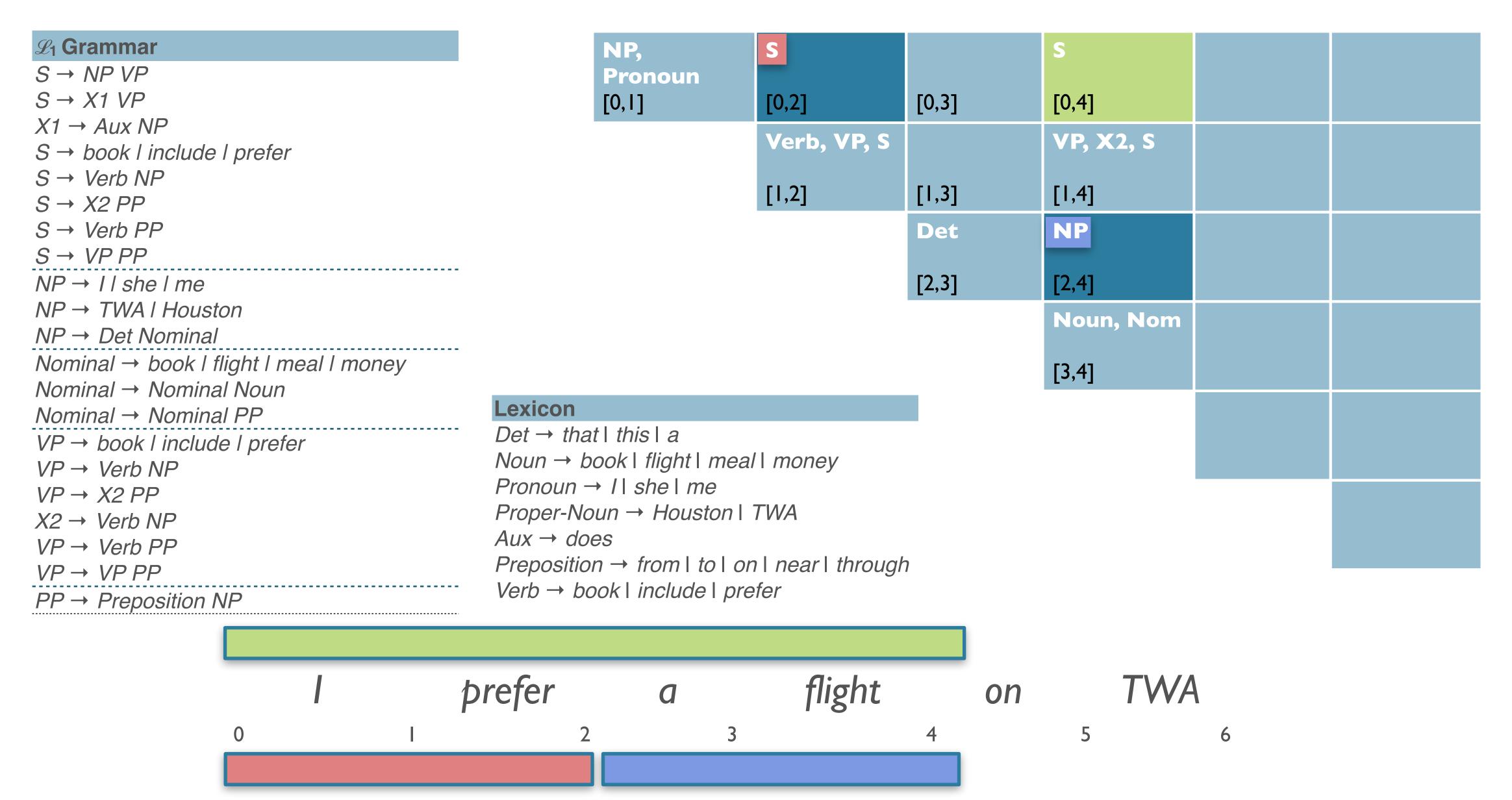












$\mathscr{L}_1$ Grammar	NID	C		S	
$S \rightarrow NP VP$	NP, Pronoun	5		5	
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	
$X1 \rightarrow Aux NP$	L~,·J		[0,0]		
S → book I include I prefer		Verb, VP, S		<b>VP, X2, S</b>	
S → Verb NP		F1 21	FI 21	ΓΙ <b>Δ</b> Ί	
$S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	
S → Verb PP			Det	NP	
$S \rightarrow VP PP$					
NP → I I she I me			[2,3]	[2,4]	
NP → TWA I Houston				Noun, Nom	
NP → Det Nominal	. = = =				
Nominal → book   flight   meal   money				[3,4]	
Nominal → Nominal Noun	Lexicon			- · ·	
Nominal → Nominal PP	$Det \rightarrow that \mid this \mid a$				
VP → book I include I prefer	Noun → book   flight   meal	l money			
VP → Verb NP	Pronoun → 1  she   me	1 THOHEY			
$VP \rightarrow X2 PP$	Proper-Noun → Houston   7	ΓΙΛ/Α			
X2 → Verb NP	Aux → does				
VP → Verb PP	Preposition → from   to   on	near   through			
$VP \rightarrow VP PP$	Verb → book   include   pre				
PP → Preposition NP					
		a. I.		T\	
	breter a	flight	on	TWA	
	prefer a	flight	on	IVVA	

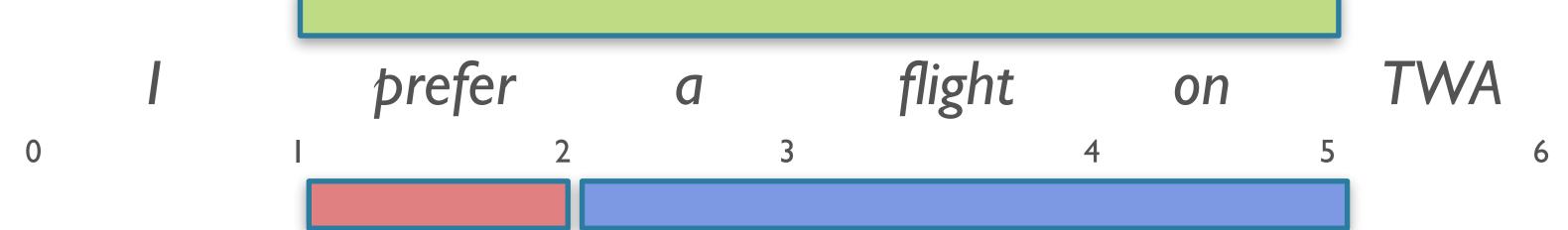
La Grammar	NP,	S		S		
$S \rightarrow NP VP$ $S \rightarrow X1 VP$	Pronoun	ΓΩ 21	LU 31	ΓΛ <i>4</i> 1		
$X1 \rightarrow Aux NP$	[0,1]	[0,2]	[0,3]	[0,4]		
$S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		<b>VP, X2, S</b>		
$S \rightarrow Verb NP$						
$S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]		
S → Verb PP			Det	NP		
$S \rightarrow VP PP$						
NP → I   she   me			[2,3]	[2,4]		
NP → TWA I Houston				Noun, Nom		
NP → Det Nominal				idodii, idoiii		
Nominal → book   flight   meal   money				[3,4]		
Nominal → Nominal Noun	The second second			Le, .]		
Nominal → Nominal PP	Lexicon  Det a that I think a				Prep	
VP → book I include I prefer	Det → that   this   a	all manay			F4 F3	
VP → Verb NP	Noun → book   flight   me Pronoun →     she   me	arrinoney			[4,5]	
$VP \rightarrow X2 PP$	Proper-Noun → Houston	ΙΤΙΛΙΔ				
X2 → Verb NP	Aux → does	1 / / / / (				
VP → Verb PP	Preposition → from   to   c	n   near   through	h			
$VP \rightarrow VP PP$	Verb → book   include   p		,			
PP → Preposition NP	·					
	h C	a: _l		T\ A /	Λ	
	prefer a	flight	OI	n TW/	4	

$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$		NP, Pronoun [0,1]	S [0,2]	[0,3]	<b>S</b> [0,4]		
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$ $S \rightarrow Verb \ NP$			Verb, VP, S		VP, X2, S		
$S \rightarrow X2PP$			[1,2]	[1,3]	[1,4]		
$S \rightarrow Verb PP$ $S \rightarrow VP PP$	_			Det	NP		
NP → II she I me				[2,3]	[2,4]		
NP → TWA I Houston NP → Det Nominal	· <b>-</b>				Noun, Nom		
Nominal → book   flight   meal   money Nominal → Nominal Noun					[3,4]	[3,5]	
Nominal → Nominal PP	Lexicon					Prep	
VP → book I include I prefer VP → Verb NP		ok   flight   mea	all money			[4,5]	
$VP \rightarrow X2 PP$	Pronoun →		TMA				
X2 → Verb NP	Aux → does	n → Houston I s	I VVA				
VP → Verb PP VP → VP PP			n I near I through				
PP → Preposition NP	Verb → boo	ok I include I pr	efer				
	prefer	a	flight	on	TWA		
	2	3		4	5	6	

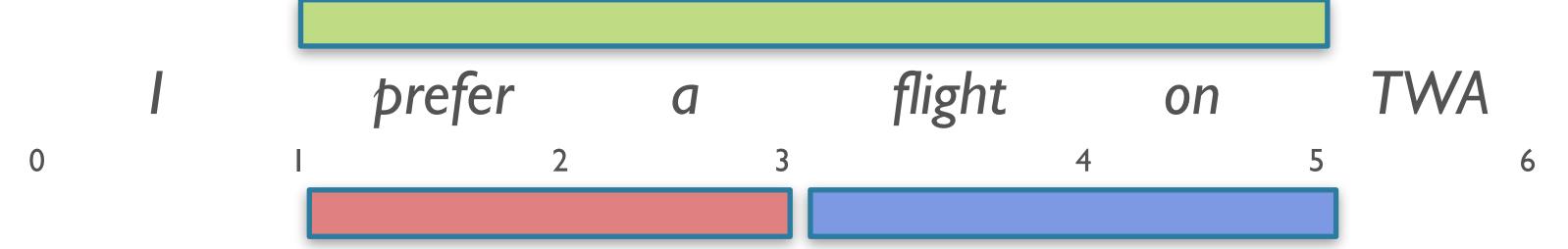
<b>Land Grammar</b>	NP,	S		S		
$S \rightarrow NP VP$	Pronoun					
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]		
$X1 \rightarrow Aux NP$		Verb, VP, S		VP, X2, S		
S → book I include I prefer				, ,		
S → Verb NP		[1,2]	[1,3]	[1,4]		
$S \rightarrow X2 PP$		[·,—]				
$S \rightarrow Verb PP$			Det	NP		
$S \rightarrow VP PP$			F0 07	FQ 47		
NP → I I she I me			[2,3]	[2,4]	[2,5]	
NP → TWA I Houston				Noun, Nom		
NP → Det Nominal	. <b>-</b>					
Nominal → book   flight   meal   money				[3,4]	[3,5]	
Nominal → Nominal Noun	Lexicon					
Nominal → Nominal PP	Det $\rightarrow$ that   this   a				Prep	
VP → book I include I prefer	Noun → book   flight   meal	II money			Γ <i>A</i> 51	
VP → Verb NP	Pronoun → 11 she 1 me	1 money			[4,5]	
$VP \rightarrow X2 PP$	Proper-Noun → Houston	ΤΙΛ/Δ				
X2 → Verb NP	$Aux \rightarrow does$					
VP → Verb PP	Preposition → from   to   or	n   near   through	)			
$VP \rightarrow VP PP$	Verb → book   include   pre					
PP → Preposition NP		ICI				
	prefer a	flight	on	TWA		
	P. 0101	1"8"			•	
0	2		4	5	6	

$\mathscr{L}_1$ Grammar	NID	C		C		
$S \rightarrow NP VP$	NP, Pronoun	S		S		
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]		
$X1 \rightarrow Aux NP$	[·,·]		[0,5]			
S → book I include I prefer		Verb, VP, S		<b>VP, X2, S</b>		
S → Verb NP		FL 21	ΓI 21	ΓΙ <b>Δ</b> Ί		
$S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]		
$S \rightarrow Verb PP$			Det	NP		
$S \rightarrow VP PP$			F2 21	FO 41	FO F1	
NP → I I she I me			[2,3]	[2,4]	[2,5]	
NP → TWA I Houston NP → Det Nominal				Noun, Nom		
Nominal → book   flight   meal   money				4-		
Nominal → Nominal Noun				[3,4]	[3,5]	
Nominal → Nominal PP	Lexicon				Prep	
VP → book I include I prefer	Det → that   this   a					
VP → Verb NP	Noun → book   flight   meal	l money			[4,5]	
$VP \rightarrow X2 PP$	Pronoun → II she I me	T 1 A / A				
X2 → Verb NP	Proper-Noun → Houston   `Aux → does	IVVA				
VP → Verb PP	Preposition → from   to   or	l near l through				
$VP \rightarrow VP PP$	<ul><li>Verb → book   include   pre</li></ul>					
PP → Preposition NP						
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,		<i>a</i> . 1				
	prefer a	flight	on	TWA		
$\cap$	7 3	-	4	5	6	

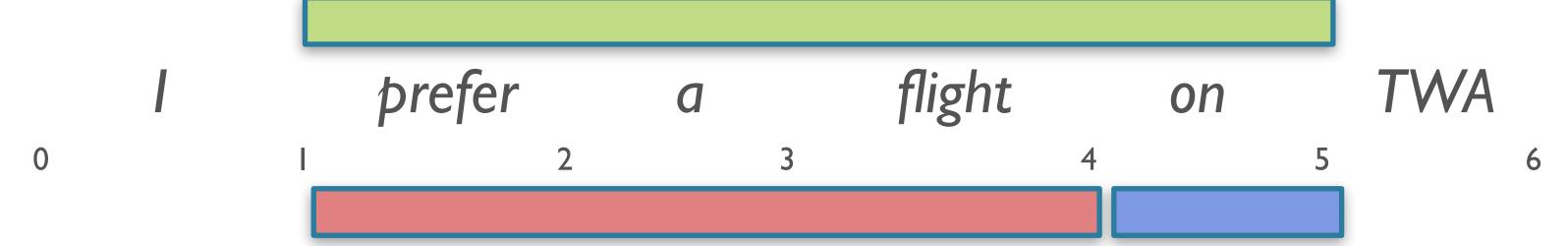
$\mathcal{L}_1$ Grammar $S \to NP VP$ $S \to X1 VP$	NP, Pronoun	S [0 2]	ΓΛ 21	S [0 4]			
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$	[0,1]	[0,2] <b>Verb, VP, S</b>	[0,3]	[0,4] VP, X2, S			
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]		
$S \rightarrow Verb PP$ $S \rightarrow VP PP$			Det	NP			
NP → I I she I me			[2,3]	[2,4]	[2,5]		
NP → TWA   Houston NP → Det Nominal				Noun, Nom			
Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]		
Nominal → Nominal PP	<b>Lexicon</b> Det → that   this   a				Prep		
VP → book I include I prefer VP → Verb NP	Noun → book   flight   me	all money			[4,5]		
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$	Pronoun →     she   me Proper-Noun → Houston   TWA Aux → does						
$VP \rightarrow VP PP$ $PP \rightarrow Preposition NP$	Preposition → from   to   o Verb → book   include   pr		h				



$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$ $S \rightarrow Verb \ NP$		Verb, VP, S [1,2]	[] 21	VP, X2, S [1,4]	[1,5]	
$S \rightarrow X2 PP$ $S \rightarrow Verb PP$ $S \rightarrow VP PP$		[1,4]	[1,3] <b>Det</b>	NP	[ו,ט]	
NP → I I she I me NP → TWA I Houston NP → Det Nominal			[2,3]	[2,4] Noun, Nom	[2,5]	
Nominal → book I flight I meal I money Nominal → Nominal Noun Nominal → Nominal PP	Lexicon			[3,4]	[3,5] Prep	
VP → book I include I prefer VP → Verb NP VP → X2 PP	Det → that   this   a Noun → book   flight   mean Pronoun →     she   me Proper-Noun → Houston				[4,5]	
$X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$ $PP \rightarrow Preposition NP$	Aux → does  Preposition → from   to   or  Verb → book   include   pre	n   near   through	7			



$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$ $S \rightarrow Verb \ NP$ $S \rightarrow X2 \ PP$		Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
$S \rightarrow Verb PP$ $S \rightarrow VP PP$			Det	NP Γ2 41	Γ2 <b>5</b> 1	
NP → I I she I me NP → TWA I Houston NP → Det Nominal			[2,3]	[2,4] Noun, Nom	[2,5]	
Nominal → book I flight I meal I money Nominal → Nominal Noun Nominal → Nominal PP	Lexicon  Dot - that I this I a			[3,4]	[3,5] Prep	
$VP \rightarrow book\ I \ include\ I \ prefer$ $VP \rightarrow Verb\ NP$ $VP \rightarrow X2\ PP$ $X2 \rightarrow Verb\ NP$	Det → that   this   a Noun → book   flight   mean Pronoun →     she   me Proper-Noun → Houston			[4,5]		
$VP \rightarrow Verb PP$ $VP \rightarrow VP PP$ $PP \rightarrow Preposition NP$	Aux → does Preposition → from   to   or Verb → book   include   pre	•	7			



$\mathcal{L}_1$ Grammar $S \to NP VP$ $S \to X1 VP$ $X1 \to Aux NP$ $S \to book \ l \ include \ l \ prefer$	NP, Pronoun [0,1]	S [0,2] Verb, VP, S	[0,3]	S [0,4] VP, X2, S	[0,5]	
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$ $S \rightarrow Verb PP$		[1,2]	[1,3] Det	[1,4] NP	[1,5]	
$S \rightarrow VP PP$ $NP \rightarrow I \mid she \mid me$ $NP \rightarrow TWA \mid Houston$ $NP \rightarrow Det Nominal$			[2,3]	[2,4] Noun, Nom	[2,5]	
Nominal → book   flight   meal   money Nominal → Nominal Noun Nominal → Nominal PP	Lexicon			[3,4]	[3,5] Prep	
$VP \rightarrow book\ I\ include\ I\ prefer$ $VP \rightarrow Verb\ NP$ $VP \rightarrow X2\ PP$ $X2 \rightarrow Verb\ NP$ $VP \rightarrow Verb\ PP$	Det → that   this   a  Noun → book   flight   me  Pronoun →     she   me  Proper-Noun → Houston  Aux → does  Preposition → from   to		[4,5]			
VP → VP PP PP → Preposition NP	Verb → book   include   p					
	prefer a	flight	on	TWA	A	

<b>2</b> ₁ Grammar	NP,	S		S		
$S \rightarrow NP VP$	Pronoun					
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
$X1 \rightarrow Aux NP$		Verb, VP, S		VP, X2, S		
S → book I include I prefer		verb, vr, 5		\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\		
S → Verb NP		[1,2]	[1,3]	[1,4]	[1,5]	
$S \rightarrow X2 PP$		[', <b>~</b> ]			[1,5]	
S → Verb PP			Det	NP		
$S \rightarrow VP PP$			FO 23	FO 47	FO F3	
NP → II she I me			[2,3]	[2,4]	[2,5]	
NP → TWA   Houston				Noun, Nom		
VP → Det Nominal						
Nominal → book I flight I meal I money Nominal → Nominal Noun				[3,4]	[3,5]	
Nominal → Nominal Nouri Nominal → Nominal PP	Lexicon				Prep	
VP → book I include I prefer	Det → that   this   a					
VP → Verb NP	Noun → book   flight   me	all money			[4,5]	
$VP \rightarrow X2 PP$	Pronoun → II she I me					
$X2 \rightarrow Verb NP$	Proper-Noun → Houston	TWA				
VP → Verb PP	Aux → does					
$VP \rightarrow VP PP$	$Preposition \rightarrow from \mid to \mid c$		7			
PP → Preposition NP	Verb → book   include   pi	refer				
	orefer a	flight	on	TW/	Λ	
	<i>11</i>	1110111				

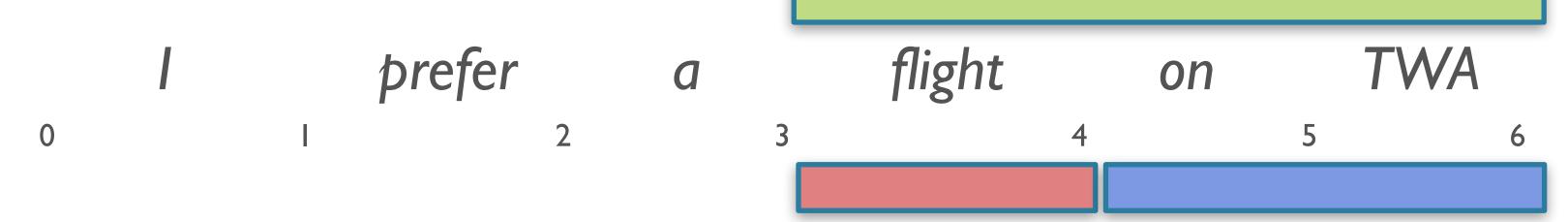
$\mathcal{L}_1$ Grammar $S \to NP VP$ $S \to X1 VP$ $X1 \to Aux NP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$S \rightarrow book\ l\ include\ l\ prefer$ $S \rightarrow Verb\ NP$ $S \rightarrow X2\ PP$		Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
$S \rightarrow Verb PP$ $S \rightarrow VP PP$ $NP \rightarrow II she I me$	- <u>-</u>		<b>Det</b> [2,3]	NP [2,4]	[2,5]	
NP → TWA   Houston NP → Det Nominal Nominal → book   flight   meal   money	- <del>-</del>			Noun, Nom [3,4]	[3,5]	
Nominal → Nominal Noun Nominal → Nominal PP  VP → book I include I prefer  VP → Verb NP	Lexicon  Det → that   this   a  Noun → book   flight   mea	d I money			Prep [4,5]	
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Pronoun →     she   me Proper-Noun → Houston   Aux → does Preposition → from   to   or		h			
PP → Preposition NP	Verb → book   include   pre					
	prefer a	flight	on	TWA	4	

$\mathcal{L}_1$ Grammar $S \rightarrow NP VP$ $S \rightarrow X1 VP$ $X1 \rightarrow Aux NP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]			
$S \rightarrow book\ I \ include\ I \ prefer$ $S \rightarrow Verb\ NP$ $S \rightarrow X2\ PP$		Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]			
$S \rightarrow Verb PP$ $S \rightarrow VP PP$ $NP \rightarrow II she I me$	- <del>-</del>		Det	NP	ro 51			
NP → TWA I Houston NP → Det Nominal			[2,3]	Noun, Nom	[2,5]			
Nominal → book I flight I meal I money Nominal → Nominal Noun Nominal → Nominal PP	Lexicon			[3,4]	[3,5] Prep			
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   mea Pronoun →     she   me	II money			[4,5]			
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$	Proper-Noun → Houston   Aux → does		2					
VP → VP PP PP → Preposition NP	Preposition → from   to   on   near   through  Verb → book   include   prefer							
	prefer a	flight	on	TWA	1			

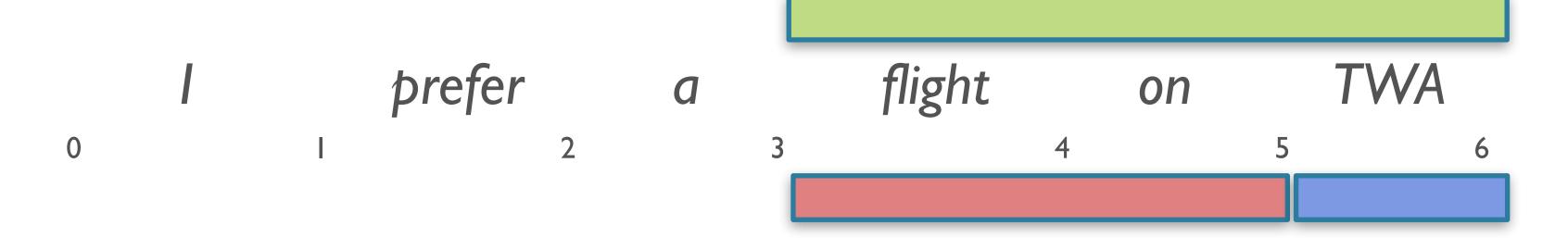
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$		NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$ $S \rightarrow Verb \ NP$ $S \rightarrow X2 \ PP$			Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
$S \rightarrow Verb PP$ $S \rightarrow VP PP$ $NP \rightarrow II she I me$				Det	NP	Γ <b>2</b> Ε1	
NP → TWA   Houston NP → Det Nominal				[2,3]	Noun, Nom	[2,5]	
Nominal → book I flight I meal I money Nominal → Nominal Noun Nominal → Nominal PP	Lexicon				[3,4]	[3,5] Prep	
VP → book I include I prefer VP → Verb NP VP → X2 PP	Det → that   this   a  Noun → book   flight   meal   money  Pronoun →     she   me						NNP, NP
$X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Proper-Noun → Houston   TWA  Aux → does  Preposition → from   to   on   near   through						[5,6]
PP → Preposition NP	Verb → book   include   prefer						
	prefer	prefer a flight on TWA					

<b>L</b> Grammar	NP,	S		S		
$S \rightarrow NP VP$	Pronoun					
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
$X1 \rightarrow Aux NP$		Verb, VP, S		VP, X2, S		
S → book I include I prefer		, , , , , ,		, , , , , , ,		
S → Verb NP		[1,2]	[1,3]	[1,4]	[1,5]	
$S \rightarrow X2 PP$		L - ,—J			[.,-]	
$S \rightarrow Verb PP$ $S \rightarrow VP PP$			Det	NP		
$NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	
NP → TWA I Houston			[_,•]		[-,-]	
NP → Det Nominal				Noun, Nom		
Nominal → book   flight   meal   money				<b>[2 √]</b>	[2 []	
Nominal → Nominal Noun				[3,4]	[3,5]	
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer	Det → that   this   a				- 4	T. 4. 4. 7
VP → Verb NP	Noun → book   flight   mea	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun →     she   me	$T$ I $\Lambda$ / $\Lambda$				NNP, NP
X2 → Verb NP	Proper-Noun → Houston   Aux → does	I VVA				
VP → Verb PP	Preposition → from   to   or	n I near I through	h			[5,6]
$VP \rightarrow VP PP$	Verb → book   include   pre	•				
PP → Preposition NP	vois soon i morado i pre	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,				
		<i>a.</i> 1		\ A # #		
	prefer a	flight	on	TWA	1	
	prefer a 3	flight	<b>on</b>	<b>TW</b> / <sub>5</sub>	6	

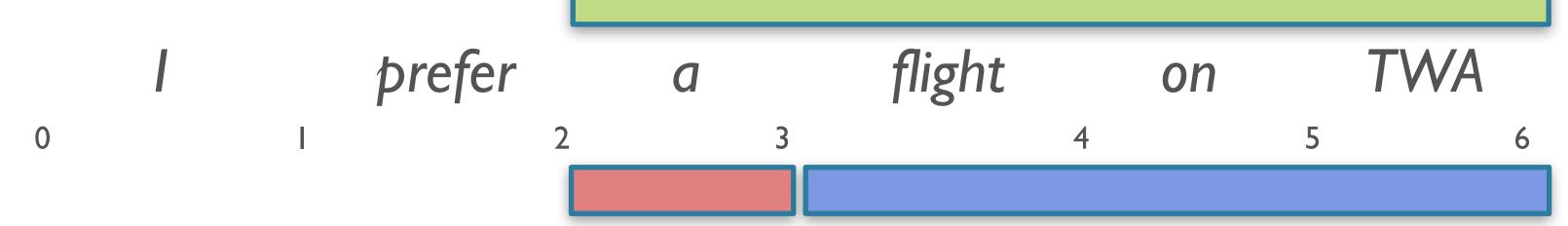
$\mathcal{L}_1$ Grammar $S \to NP VP$	NP, Pronoun	S		S		
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
$X1 \rightarrow Aux NP$		Verb, VP, S		VP, X2, S		
S → book   include   prefer						
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	
$S \rightarrow Verb PP$			Det	NP		
$S \rightarrow VPPP$						
$NP \rightarrow II sheIme$			[2,3]	[2,4]	[2,5]	
NP → TWA I Houston				Noun, Nom		Nom
NP → Det Nominal						
Nominal → book   flight   meal   money				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun	Lexicon					PP
Nominal → Nominal PP	Det → that   this   a				Prep	
VP → book I include I prefer VP → Verb NP	Noun → book   flight   mea	l I monev			[4,5]	[4,6]
$VP \rightarrow Verb NP$ $VP \rightarrow X2 PP$	Pronoun → II she I me				L ''	
$X2 \rightarrow Verb NP$	Proper-Noun → Houston I	TWA				NNP, NP
$VP \rightarrow Verb PP$	Aux → does					[5,6]
$VP \rightarrow VP PP$	Preposition → from   to   or		h			[5,0]
PP → Preposition NP	Verb → book   include   pre	efer				



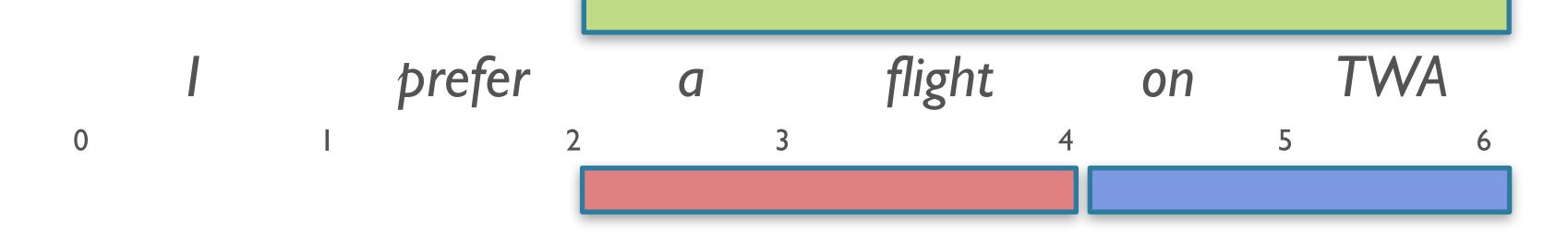
$\mathcal{L}_1$ Grammar $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$ $S \rightarrow Verb \ NP$ $S \rightarrow X2 \ PP$		Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
$S \rightarrow Verb PP$ $S \rightarrow VP PP$			Det	NP		
NP → I I she I me NP → TWA I Houston NP → Det Nominal			[2,3]	[2,4] Noun, Nom	[2,5]	Nom
Nominal → book I flight I meal I money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP VP → book l include l prefer	Lexicon  Det → that   this   a	/			Prep	PP
$VP \rightarrow Verb NP$ $VP \rightarrow X2 PP$	Noun $\rightarrow$ book   flight   meal   money Pronoun $\rightarrow$     she   me					[4,6] NNP, NP
X2 → Verb NP VP → Verb PP VP → VP PP PP → Preposition NP	Proper-Noun → Houston   TWA  Aux → does  Preposition → from   to   on   near   through  Verb → book   include   prefer					[5,6]



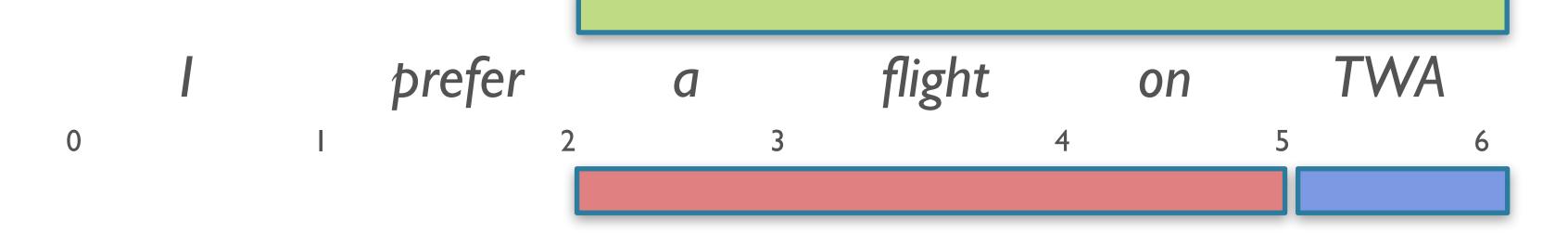
<b></b> Grammar	NP,	S		S		
$S \rightarrow NP VP$	Pronoun					
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
$X1 \rightarrow Aux NP$		Vorb VD C		VD V2 C		
S → book I include I prefer		Verb, VP, S		<b>VP, X2, S</b>		
S → Verb NP		[ ] O]	FL 23	F1 47	C1 C3	
$S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$						
NP → I I she I me			[2,3]	[2,4]	[2,5]	[2,6]
VP → TWA I Houston				Noun, Nom		Nom
NP → Det Nominal						
Nominal → book   flight   meal   money				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun				[2, .]		
Nominal → Nominal PP	Lexicon				Prep	PP
/P → book l include l prefer	Det → that   this   a					
/P → Verb NP	Noun → book   flight   med	all money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun → II she I me					NNP, NP
X2 → Verb NP	Proper-Noun → Houston	I IVVA				
/P → Verb PP	Aux → does					[5,6]
$VP \rightarrow VP PP$	Preposition → from   to   c		7			L / J
PP → Preposition NP	Verb → book   include   pi	reter				



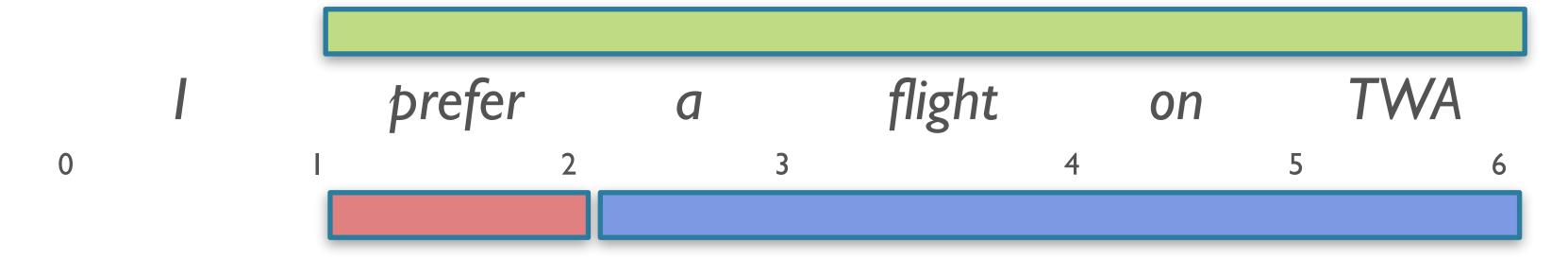
$\mathcal{L}_1$ Grammar $S \to NP VP$	NP, Pronoun	S		<b>S</b>			
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]		
$X1 \rightarrow Aux NP$		Verb, VP, S		VP, X2, S			
S → book I include I prefer							
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]		
$S \rightarrow Verb PP$			Det	NP		NP	
$S \rightarrow VPPP$							
NP → I I she I me	-		[2,3]	[2,4]	[2,5]	[2,6]	
NP → TWA   Houston						Nom	
NP → Det Nominal				Noun, Nom		Nom	
Nominal → book   flight   meal   money	-			[3,4]	[3,5]	[3,6]	
Nominal → Nominal Noun	Lavison			<b>L</b> / <b>J</b>			
Nominal → Nominal PP	Lexicon  Dot - that I thic I a				Prep	PP	
VP → book I include I prefer	Det → that   this   a	II monov			Γ <i>Λ</i> Ε1	Γ <i>Λ</i>	
VP → Verb NP	Noun → book \ flight \ mea  Pronoun → I \ she \ me	i i money			[4,5]	[4,6]	
$VP \rightarrow X2 PP$	Proper-Noun → Houston	$T \mathcal{U}/\Delta$				NNP, NP	
X2 → Verb NP	$Aux \rightarrow does$						
VP → Verb PP	Preposition → from   to   on   near   through					[5,6]	
$VP \rightarrow VP PP$	Verb → book   include   pre	·					
PP → Preposition NP		, i O i					



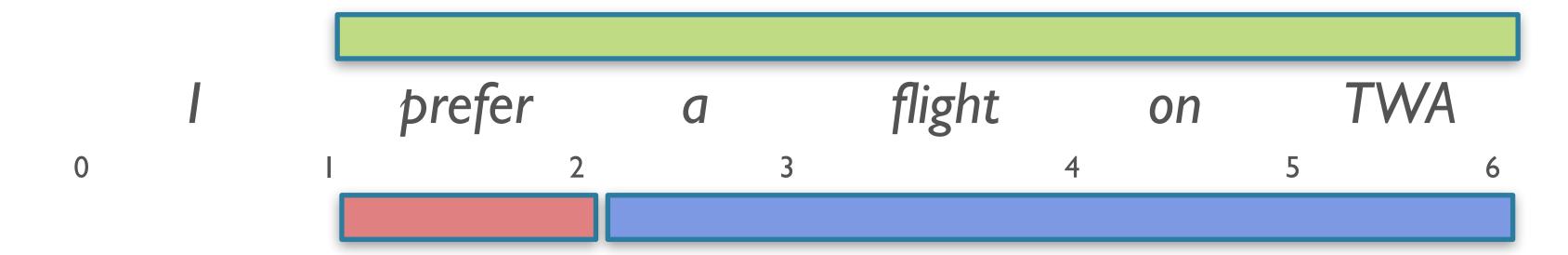
$\mathcal{L}_1$ Grammar $S \to NP VP$	NP, Pronoun	S		S		
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
X1 → Aux NP S → book I include I prefer		Verb, VP, S		VP, X2, S		
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	
$S \rightarrow Verb PP$ $S \rightarrow VP PP$			Det	NP		NP
NP → I I she I me			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston NP → Det Nominal				Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun  Nominal → Nominal PP	Lexicon  Det a the et la their land				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   mea	l money			[4,5]	[4,6]
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$	Pronoun →     she   me Proper-Noun → Houston			NNP, NP		
$VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does Preposition → from   to   or	7			[5,6]	
PP → Preposition NP	Verb → book   include   pre	•				



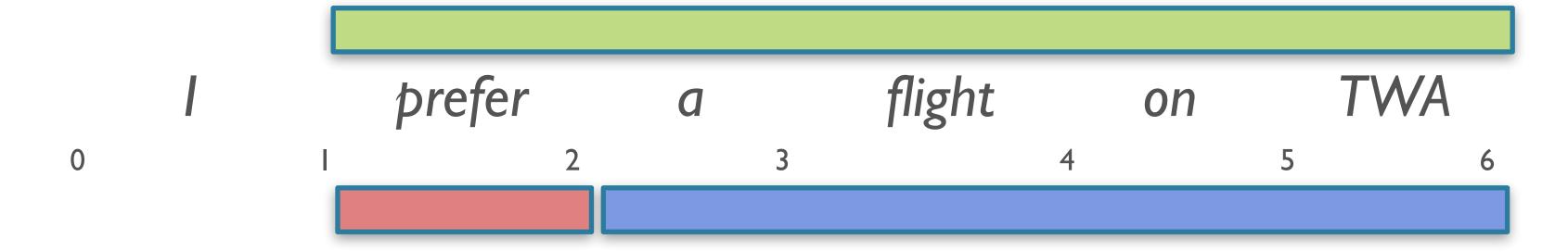
$\mathcal{L}_1$ Grammar $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
X1 → Aux NP S → book Linglado Lorofor		Verb, VP, S		VP, X2, S		
S → book I include I prefer S → Verb NP						
$S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$						
NP → I I she I me			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston				Noun, Nom		Nom
NP → Det Nominal						
Nominal → book   flight   meal   money				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun Nominal → Nominal PP	Lexicon				Prep	PP
$VP \rightarrow book \ linclude \ lprefer$	Det → that   this   a					
VP → DOOK I Include I prefer VP → Verb NP	Noun → book   flight   mea	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun → II she I me					NNP, NP
X2 → Verb NP	Proper-Noun → Houston	TWA				
VP → Verb PP	Aux → does					[5,6]
$VP \rightarrow VP PP$	Preposition → from   to   or					
PP → Preposition NP	Verb → book   include   pre	eier				



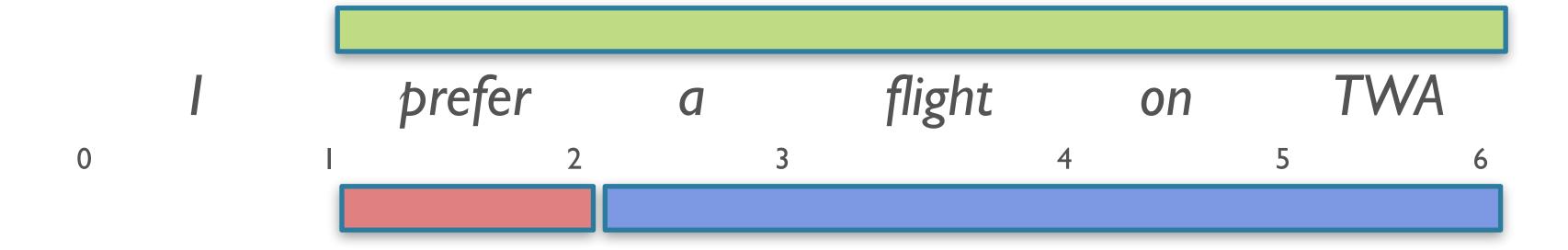
$\mathcal{L}_1$ Grammar $S \to NP VP$ $S \to X1 VP$		NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$			Verb, VP, S		VP, X2, S		VP
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$			[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP				Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow I  l  she  l  me$				[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston NP → Det Nominal					Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun					[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun	Lexicon					Prep	PP
VP → book I include I prefer  VP → Verb NP		ok   flight   meal	l money			[4,5]	[4,6]
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$	Pronoun → Proper-Nou	n → Houston l	TWA				NNP, NP
$VP \rightarrow Verb \ PP$ $VP \rightarrow VP \ PP$	Aux → does Preposition		n I near I through				[5,6]
PP → Preposition NP	Verb → boo	k   include   pre	fer				



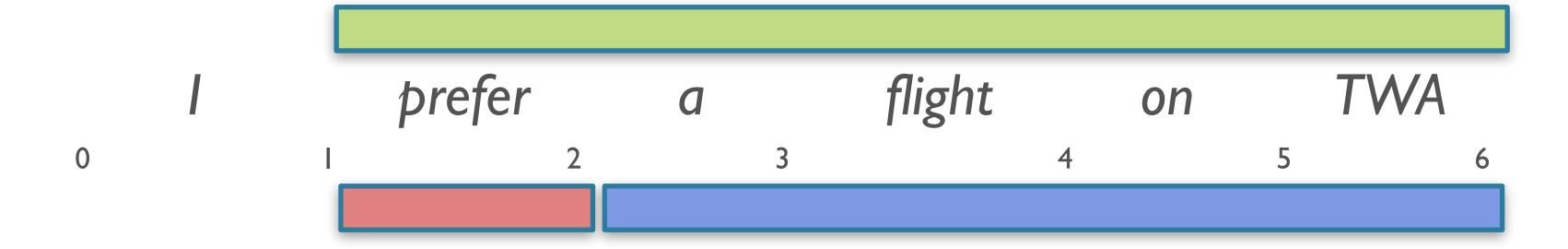
$\mathcal{L}_1$ Grammar $S \rightarrow NP VP$		NP,	S		S		
$S \rightarrow X1 VP$		Pronoun [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$			Verb, VP, S		<b>VP, X2, S</b>		VP, X2
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$			[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP				Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow I  l  she  l  me$				[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston				[2,5]	Noun, Nom	[2,3]	Nom
NP → Det Nominal Nominal → book   flight   meal   money							
Nominal → Nominal Noun					[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon	tl thial a				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that Noun → b	ook   flight   mear	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$		→     she   me	ΤΙΛ/Δ				NNP, NP
$X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$	Proper-Noun → Houston   TWA Aux → does						[5,6]
$VP \rightarrow VP PP$	•	n → from   to   or ok   include   pre					[3,0]
PP → Preposition NP	•	ok I include I pre					



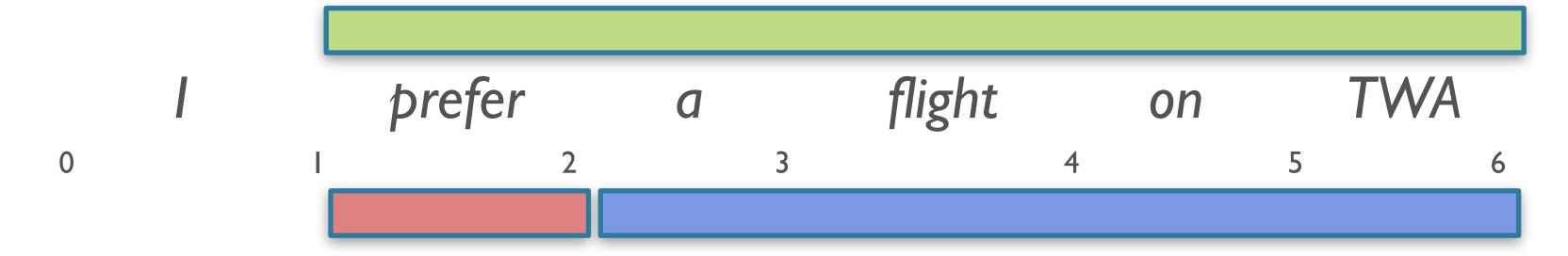
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
X1 → Aux NP S → book I include I prefer		Verb, VP, S		VP, X2, S		<b>VP, X2, S</b>
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston NP → Det Nominal				Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   mea	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun →     she   me Proper-Noun → Houston	TWA				NNP, NP
$X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does  Preposition → from   to   or				[5,6]	
PP → Preposition NP	Verb → book   include   pre	•				



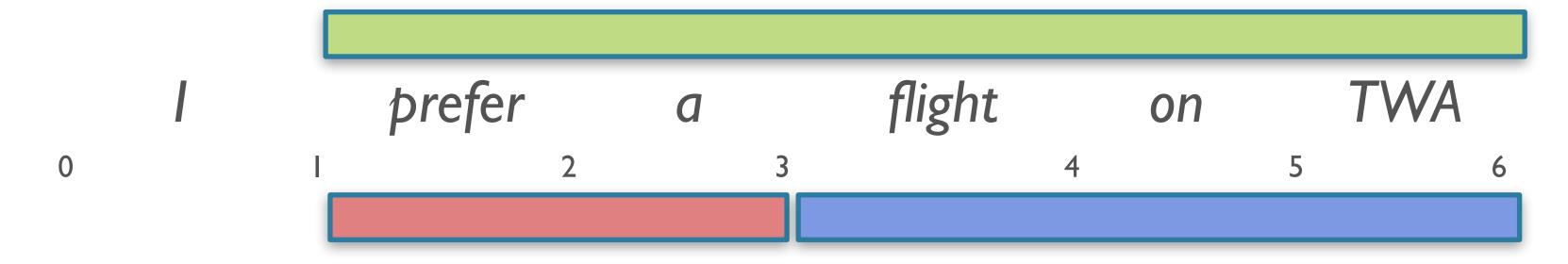
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$		NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$			Verb, VP, S		VP, X2, S		VP, X2, S
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$			[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
$S \rightarrow Verb PP$				Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$				[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston NP → Det Nominal					Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun					[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun  Nominal → Nominal PP	Lexicon					Prep	PP
VP → book I include I prefer VP → Verb NP		ook   flight   meal	I money			[4,5]	[4,6]
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$		>     she   me un → Houston   `	TWA				NNP, NP
$VP \rightarrow Verb NP$ $VP \rightarrow VP PP$	Aux → doe						[5,6]
PP → Preposition NP	Verb → bo	ok I include I pre	efer				



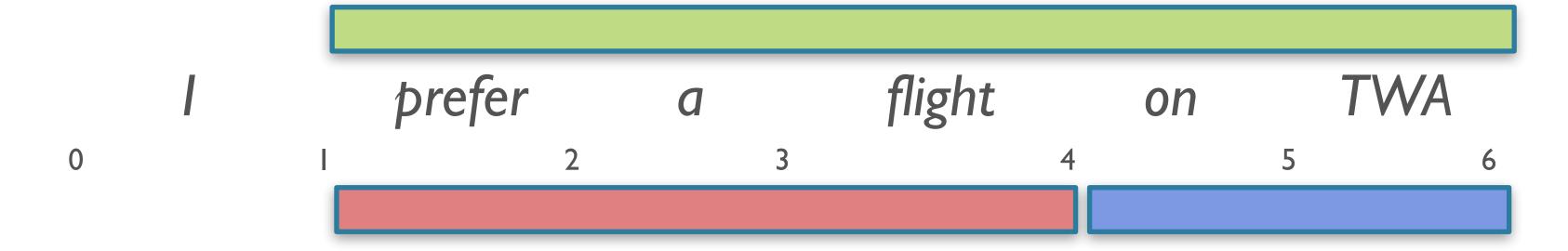
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	<b>S</b> [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		VP, X2, S		VP, X2, S
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
$S \rightarrow Verb PP$			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow I  l  she  l  me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston NP → Det Nominal				Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   meal	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$	Pronoun →     she   me Proper-Noun → Houston	TWA				NNP, NP
$VP \rightarrow Verb NI$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does  Preposition → from   to   or	n   near   through	)			[5,6]
PP → Preposition NP	Verb → book   include   pre					



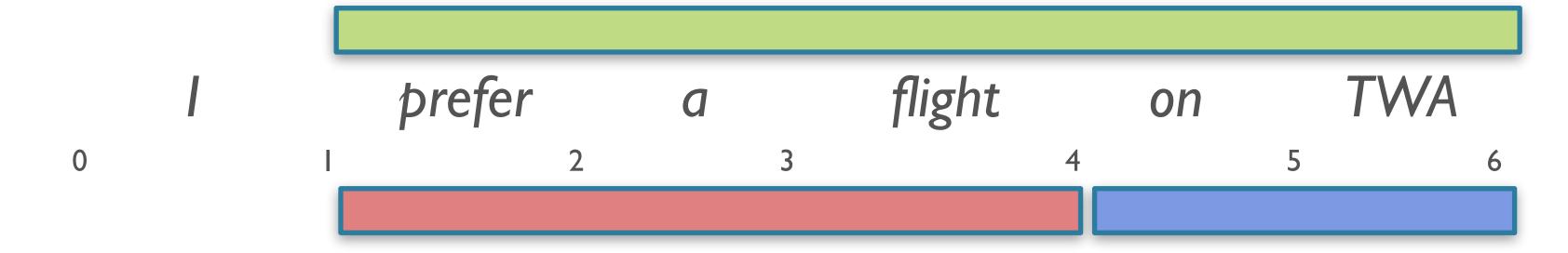
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		VP, X2, S		VP, X2, S
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston NP → Det Nominal				Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   mea	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun →     she   me Proper-Noun → Houston				NNP, NP	
$X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does  Preposition → from   to   on   near   through					[5,6]
PP → Preposition NP	Verb → book   include   pre	efer				



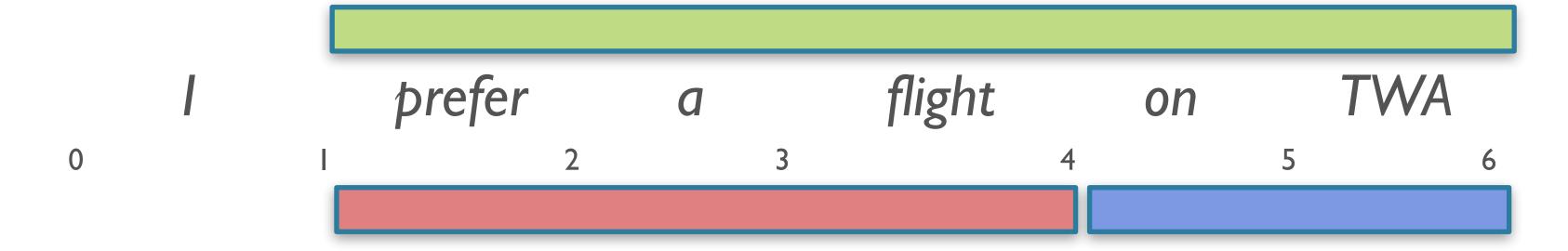
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		<b>VP, X2, S</b>		<b>VP, X2, S</b>
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston NP → Det Nominal				Noun, Nom		Nom
NP → Det Nominal Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   meal	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun →     she   me Proper-Noun → Houston				NNP, NP	
$X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does Preposition → from   to   or		7			[5,6]
PP → Preposition NP	Verb → book   include   pre	efer				



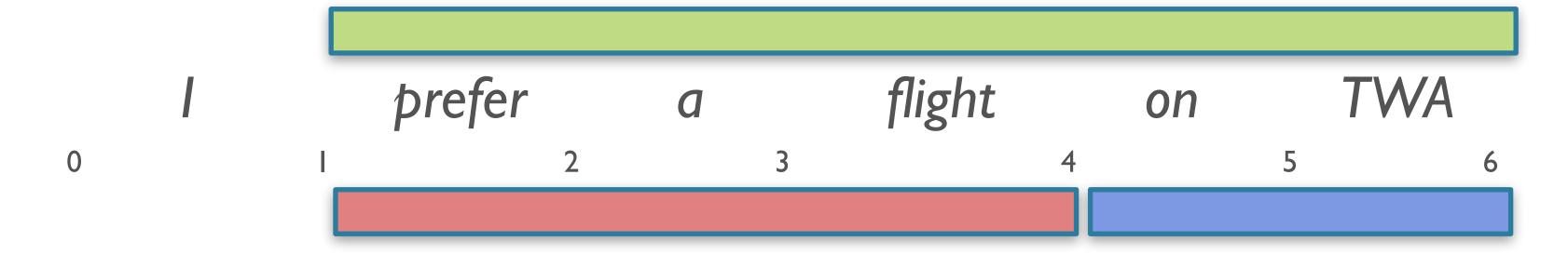
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	<b>S</b> [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		VP, X2, S		VP, X2, S
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston NP → Det Nominal				Noun, Nom		Nom
NP → Det Normhai Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   meal	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun →     she   me Proper-Noun → Houston   TWA					NNP, NP
$X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does Preposition → from   to   or		7			[5,6]
PP → Preposition NP	Verb → book   include   pre	efer				



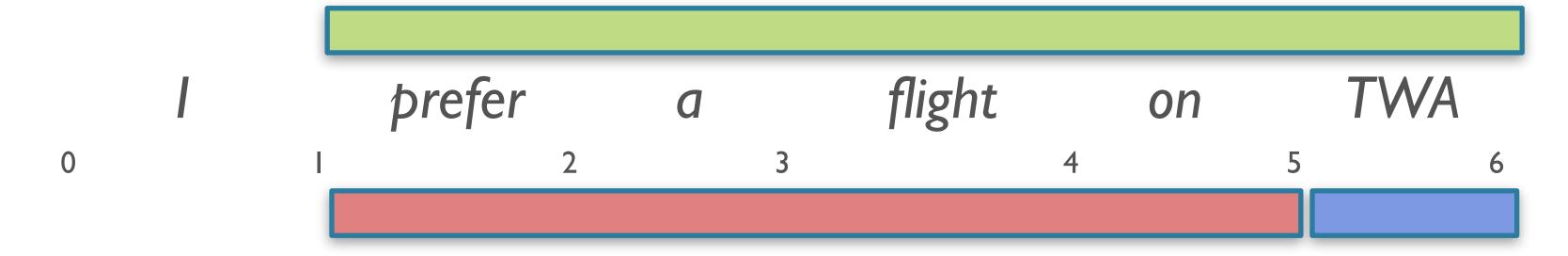
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
X1 → Aux NP S → book I include I prefer		Verb, VP, S		VP, X2, S		<b>VP</b> , <b>X</b> 2, <b>S</b>
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston NP → Det Nominal				Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   meal	I money			[4,5]	[4,6]
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$	Pronoun →     she   me Proper-Noun → Houston				NNP, NP	
$VP \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does  Preposition → from   to   on   near   through					[5,6]
PP → Preposition NP	Verb → book   include   pre	fer				



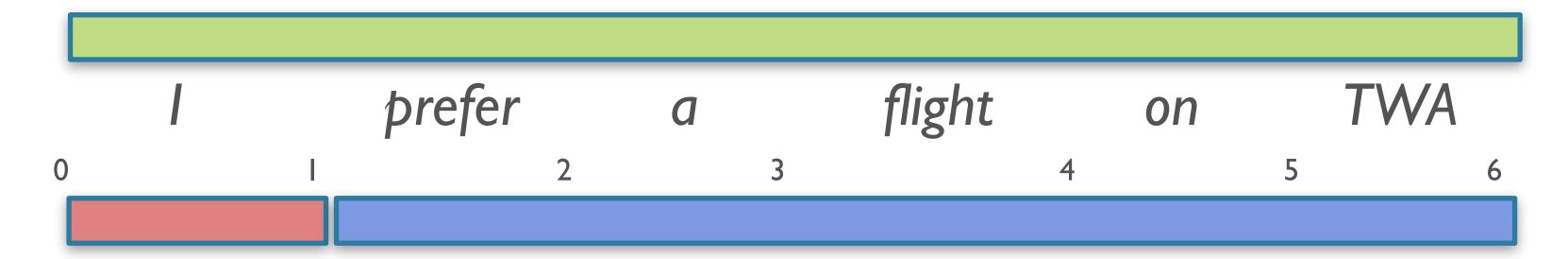
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		VP, X2, S		VP, X2, S
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston				Noun, Nom		Nom
NP → Det Nominal Nominal → book I flight I meal I money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   meal	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun →     she   me Proper-Noun → Houston   TWA					NNP, NP
$X2 \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does  Preposition → from   to   on   near   through					[5,6]
PP → Preposition NP	Verb → book   include   pre	efer				



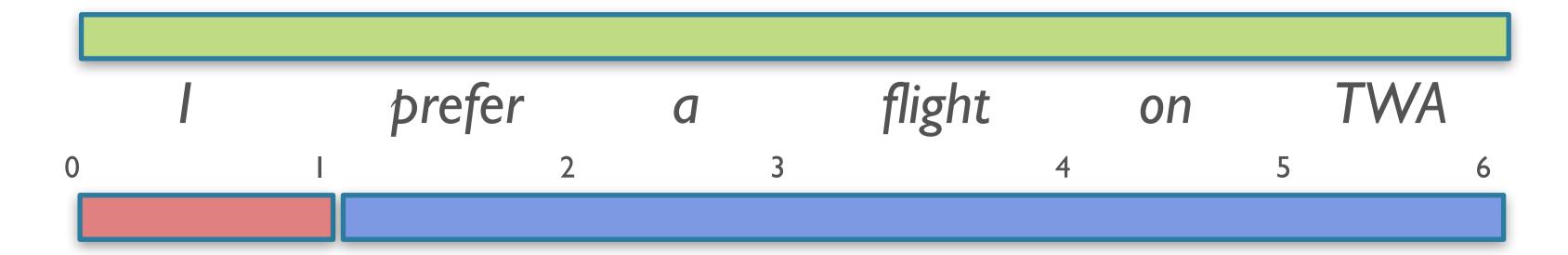
$\mathcal{L}_1$ Grammar $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		VP, X2, S		VP, X2, S
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston NP → Det Nominal				Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   mea	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$	Pronoun →     she   me Proper-Noun → Houston				NNP, NP	
VP → Verb PP	Aux → does Preposition → from   to   or	n   near   through	7			[5,6]
VP → VP PP PP → Preposition NP	Verb → book   include   pre		•			



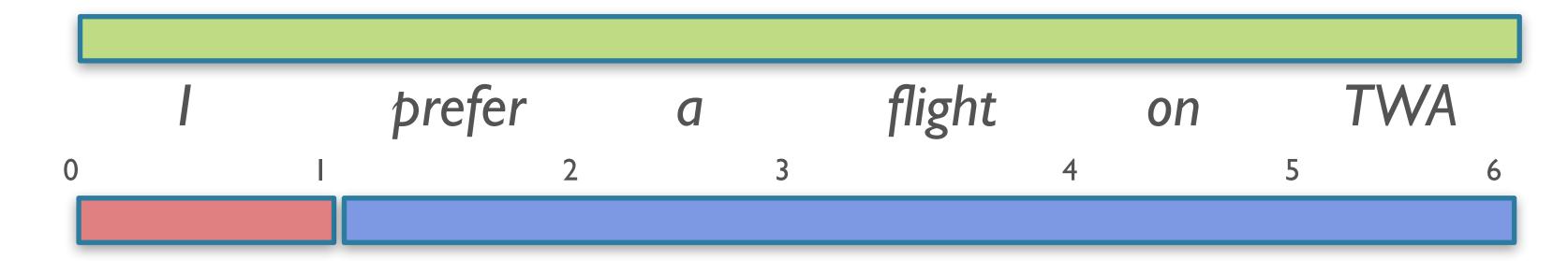
$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun	S [0.21	<b>Γ</b> Λ 21	S [0.4]	ΓΛ ΕΊ	ΓΟ (1
$X1 \rightarrow Aux NP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	[0,6]
S → book I include I prefer		Verb, VP, S		<b>VP, X2, S</b>		<b>VP, X2, S</b>
S → Verb NP		F1 21	FL 21	ΓΙ <i>Α</i> Ί	F1 F3	Γ1 <b>/</b> 3
$S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$						
NP → I I she I me			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston				Noun, Nom		Nom
NP → Det Nominal				,		
Nominal → book   flight   meal   money				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun	Lexicon				Prep	PP
Nominal → Nominal PP	Det → that   this   a				ПСР	
VP → book I include I prefer VP → Verb NP	Noun → book   flight   meal	II monev			[4,5]	[4,6]
$VP \rightarrow Verb NP$ $VP \rightarrow X2 PP$	Pronoun → II she I me	<i>y</i>			[ .,• ]	
$X2 \rightarrow Verb NP$	Proper-Noun → Houston   TWA					NNP, NP
$VP \rightarrow Verb PP$	Aux → does					Γ <b>5</b>
$VP \rightarrow VP PP$	Preposition → from   to   on   near   through					[5,6]
PP → Preposition NP	Verb → book   include   pre	efer				



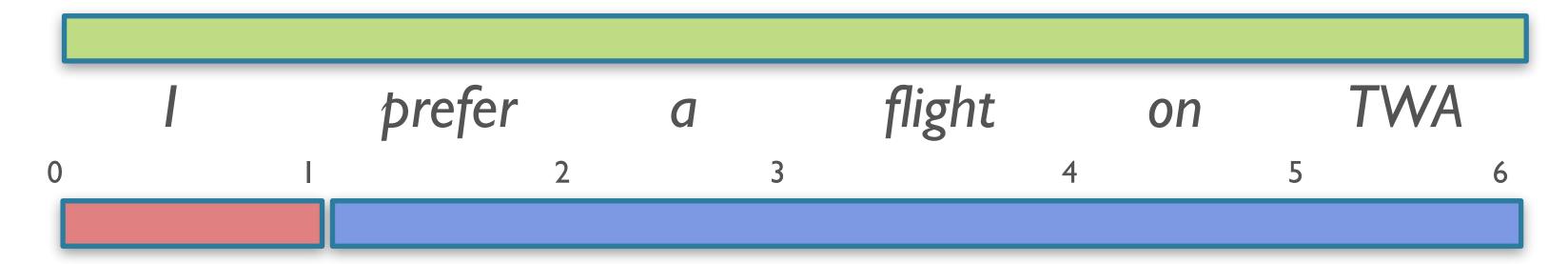
$\mathcal{L}_1$ Grammar $S \rightarrow NP VP$ $S \rightarrow X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	[0,6]
$X1 \rightarrow Aux NP$	[~, .]	Verb, VP, S	[0,0]	VP, X2, S	[0,0]	VP, X2, S
S → book I include I prefer		VCI 15, VI, 5		<b>VI, A2, S</b>		, AZ, S
$S \rightarrow Verb NP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
$S \rightarrow X2 PP$		[.,_]			[.,5]	
$S \rightarrow Verb PP$			Det	NP		NP
$S \rightarrow VP PP$						
<i>NP</i> → <i>I I she I me</i>			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston				Noun, Nom		Nom
NP → Det Nominal						
Nominal → book   flight   meal   money				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun	Lexicon				Prep	PP
Nominal → Nominal PP	Det → that   this   a				Prep	
VP → book l include l prefer	Noun → book   flight   meal	II monev			Γ <b>4</b> 51	Γ <u>4</u>
VP → Verb NP	Pronoun → 11 she 1 me	Tillolley			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Proper-Noun → Houston I	ΤΙΛ/Δ				NNP, NP
X2 → Verb NP	$Aux \rightarrow does$					
VP → Verb PP	Preposition → from   to   or				[5,6]	
$VP \rightarrow VP PP$	Verb → book   include   pre					
PP → Preposition NP	verb book i include i pre					



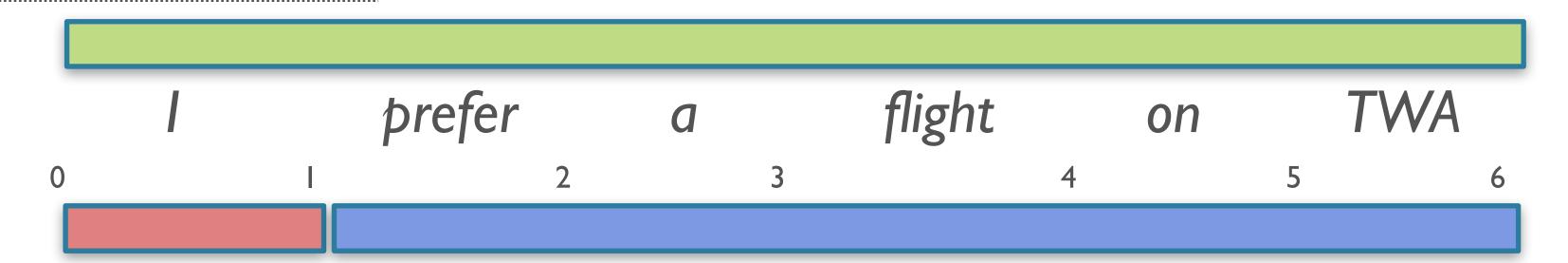
$\mathcal{L}_1$ Grammar $S \rightarrow NP VP$	NP, Pronoun	S		S		S
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	[0,6]
X1 → Aux NP S → book Lipstude Lorefor		Verb, VP, S		VP, X2, S		VP, X2, S
S → book I include I prefer S → Verb NP						
$S \rightarrow X2PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$						
NP → I I she I me			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston				Noun, Nom		Nom
NP → Det Nominal						
Nominal → book   flight   meal   money				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun	Lexicon				Prep	PP
Nominal → Nominal PP	Det → that   this   a				· · · · ·	
VP → book I include I prefer VP → Verb NP	Noun → book   flight   mea	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun → II she I me	-			<b>L</b> ' <b>J</b>	NNP, NP
$X2 \rightarrow Verb NP$	Proper-Noun → Houston I	TWA				NINP, INP
VP → Verb PP	Aux → does					[5,6]
$VP \rightarrow VP PP$	Preposition → from   to   or				[5,5]	
PP → Preposition NP	Verb → book   include   pre	eter				



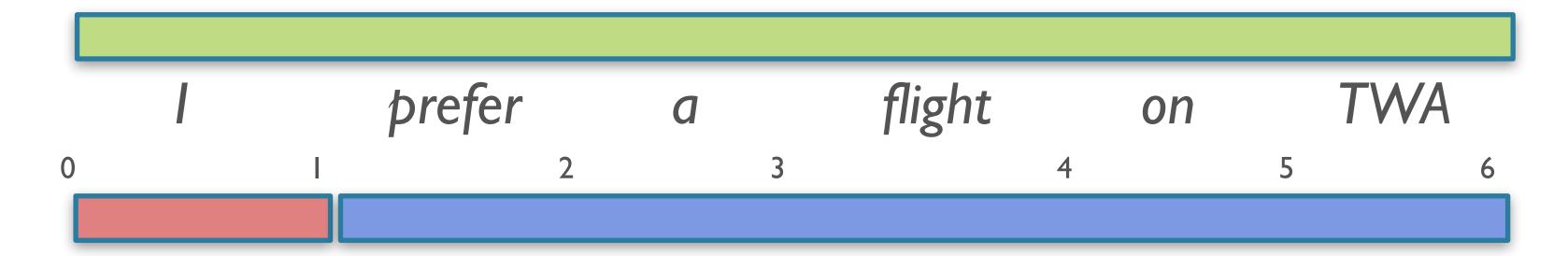
$\mathcal{L}_1$ Grammar $S \rightarrow NP VP$	NP, Pronoun	S		S		S
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	[0,6]
$X1 \rightarrow Aux NP$		Verb, VP, S		VP, X2, S		VP, X2, S
S → book I include I prefer						<b>,,</b> -
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
$S \rightarrow Verb PP$			Det	NP		NP
$S \rightarrow VPPP$						
NP → I I she I me			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA   Houston				Noun, Nom		Nom
NP → Det Nominal				idouii, idoiii		
Nominal → book   flight   meal   money				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun	Lexicon				Prep	PP
Nominal → Nominal PP	Det → that   this   a				Prep	
VP → book I include I prefer	Noun → book   flight   meal	II monev			[4,5]	[4,6]
$VP \rightarrow Verb NP$ $VP \rightarrow X2 PP$	Pronoun → II she I me				L "," ]	
$X2 \rightarrow Verb NP$	Proper-Noun → Houston   TWA					NNP, NP
$VP \rightarrow Verb PP$	Aux → does					[5 6]
$VP \rightarrow VP PP$	Preposition → from   to   on   near   through					[5,6]
PP → Preposition NP	Verb → book   include   pre	efer				



$\mathcal{L}_1$ Grammar $S \rightarrow NP VP$	NP, Pronoun	S		S		S
$S \rightarrow X1 VP$	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	[0,6]
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		VP, X2, S		VP, X2, S
S → Verb NP		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
$S \rightarrow X2PP$		[1,4]			[1,5]	
$S \rightarrow Verb PP$ $S \rightarrow VP PP$			Det	NP		NP
$NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston			Noun, Nom		Nom	
NP → Det Nominal				raduii, radiii		
Nominal → book   flight   meal   money				[3,4]	[3,5]	[3,6]
Nominal → Nominal Noun Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer	Det → that   this   a				_,	
VP → Verb NP	Noun → book   flight   m	eal I money			[4,5]	[4,6]
$VP \rightarrow X2 PP$	Pronoun → II she I me Proper-Noun → Houston					NNP, NP
X2 → Verb NP	$Aux \rightarrow does$					
VP → Verb PP	Preposition → from   to   on   near   through					[5,6]
VP → VP PP PP → Preposition NP	Verb → book   include   p					



$\mathcal{L}_{1} \mathbf{Grammar}$ $S \to NP VP$ $S \to X1 VP$	NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
$X1 \rightarrow Aux NP$ $S \rightarrow book \ l \ include \ l \ prefer$		Verb, VP, S		VP, X2, S		VP, X2, S
$S \rightarrow Verb NP$ $S \rightarrow X2 PP$		[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
S → Verb PP			Det	NP		NP
$S \rightarrow VP PP$ $NP \rightarrow II she I me$			[2,3]	[2,4]	[2,5]	[2,6]
NP → TWA I Houston NP → Det Nominal				Noun, Nom		Nom
Nominal → book   flight   meal   money Nominal → Nominal Noun				[3,4]	[3,5]	[3,6]
Nominal → Nominal PP	Lexicon				Prep	PP
VP → book I include I prefer VP → Verb NP	Det → that   this   a Noun → book   flight   mea Pronoun →     she   me	II money			[4,5]	[4,6]
$VP \rightarrow X2 PP$ $X2 \rightarrow Verb NP$	Proper-Noun → Houston	TWA				NNP, NP
$VP \rightarrow Verb NP$ $VP \rightarrow Verb PP$ $VP \rightarrow VP PP$	Aux → does  Preposition → from   to   or	[5,6]				
PP → Preposition NP	Verb → book   include   pre	erer				



# Current CKY Algorithm

#### Limitations:

Only stores non-terminals in cell

Not rules or cells corresponding to RHS

Stores SETS of non-terminals

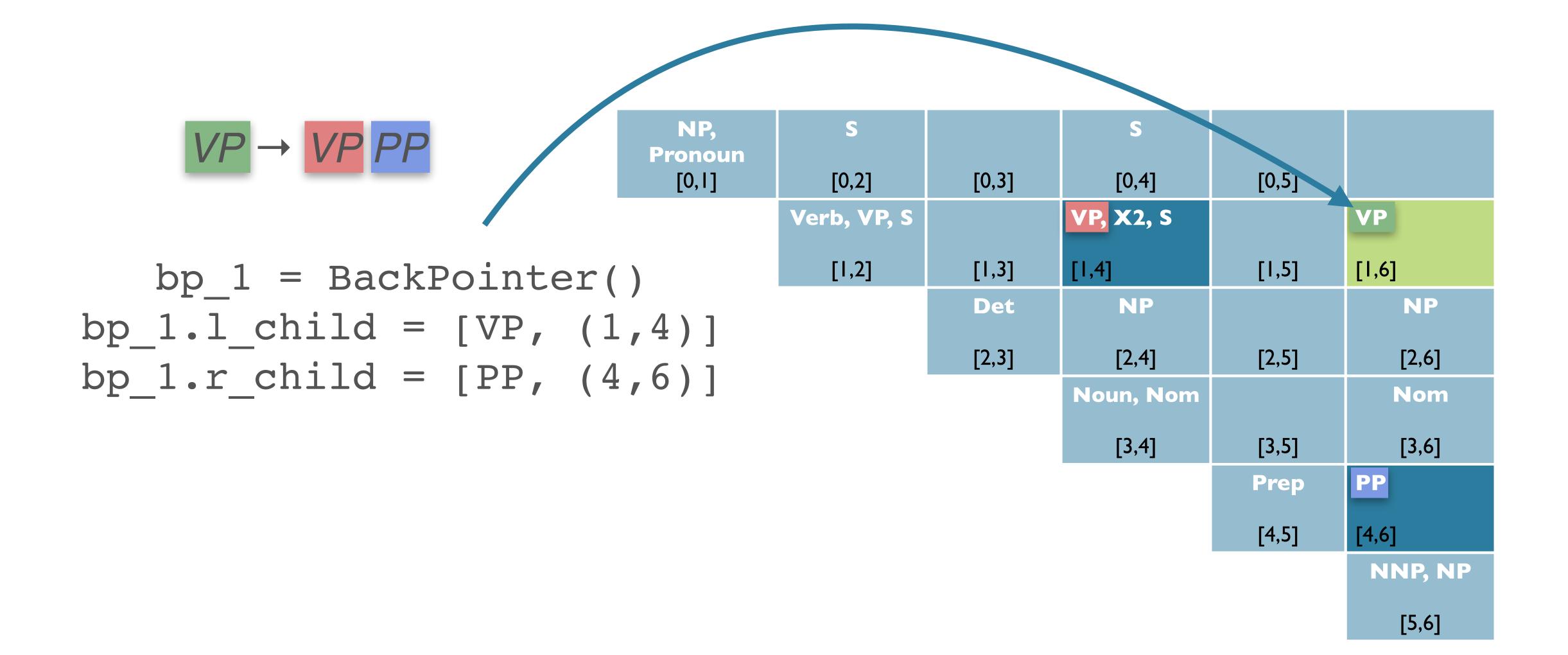
Multiple rules with same LHS collide

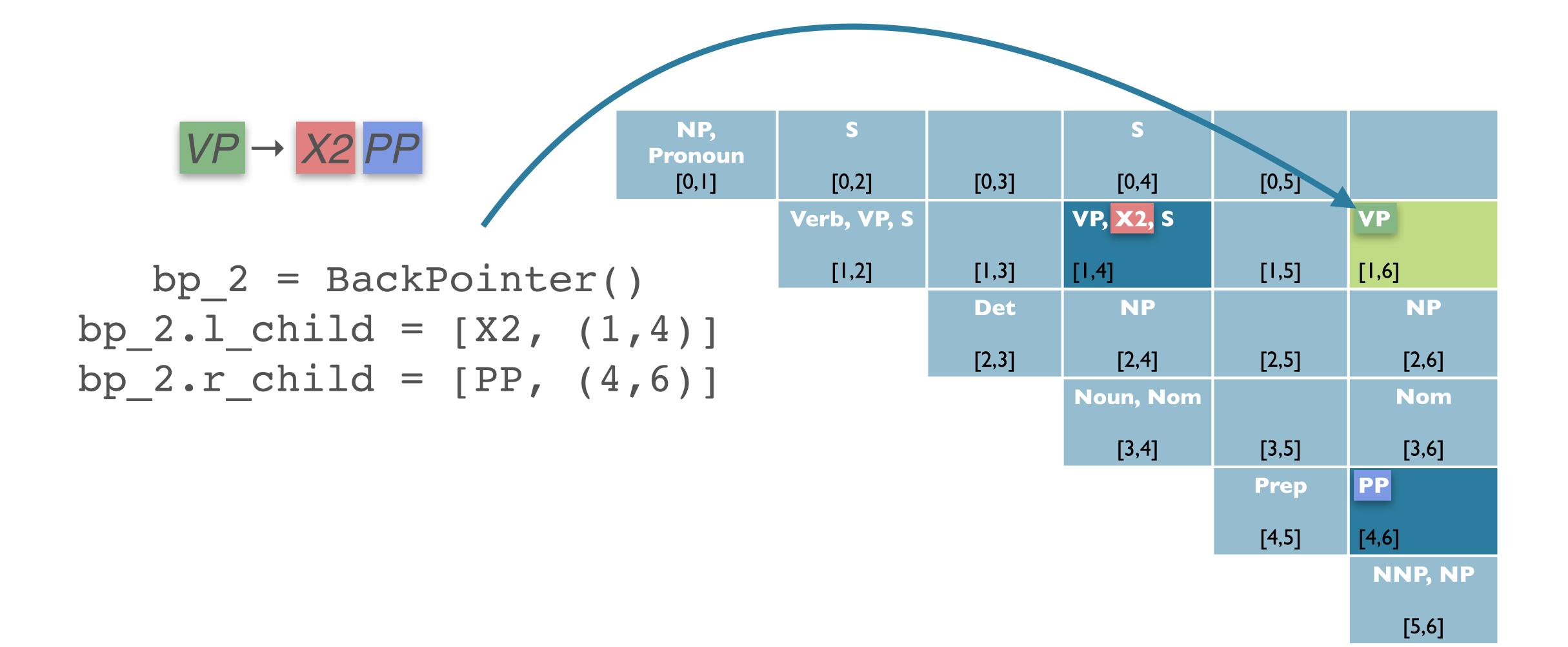
Currently only acceptance/recognition

## CKY Parser

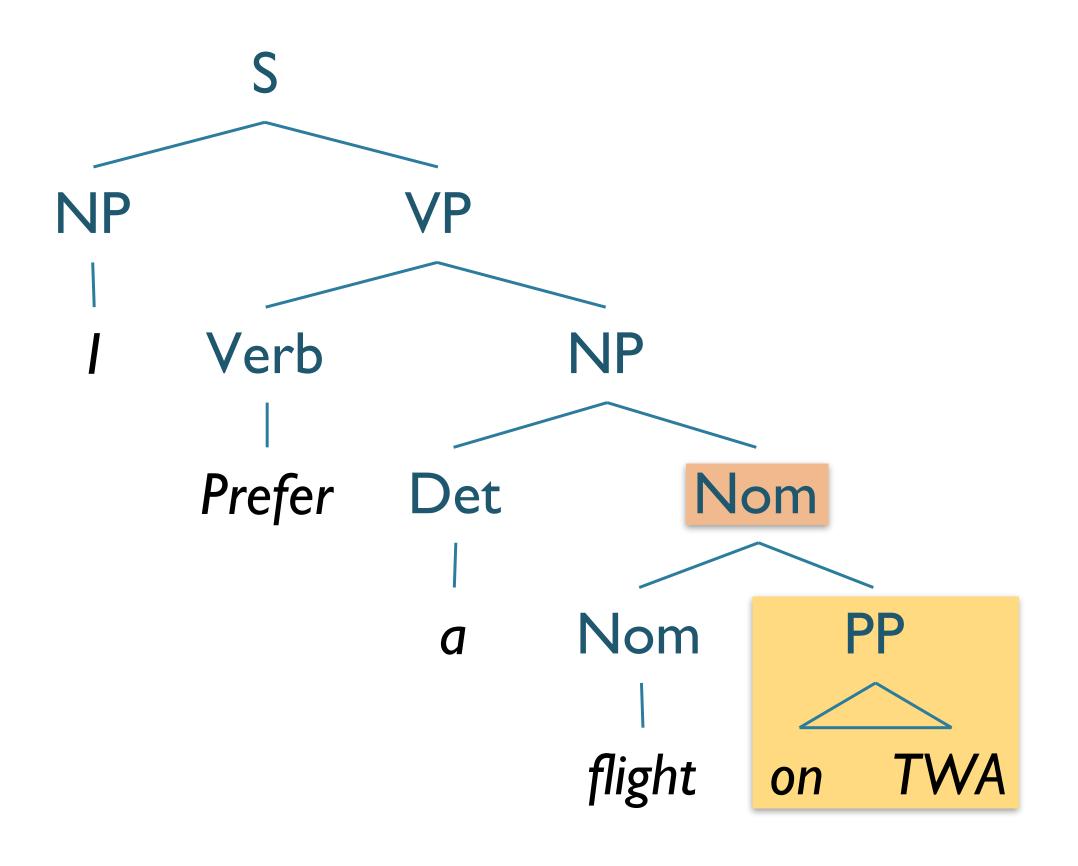
- Pair each nonterminal with back-pointer to cells from which it was derived
- Last step:
  - construct trees from back-pointers in [0, n]

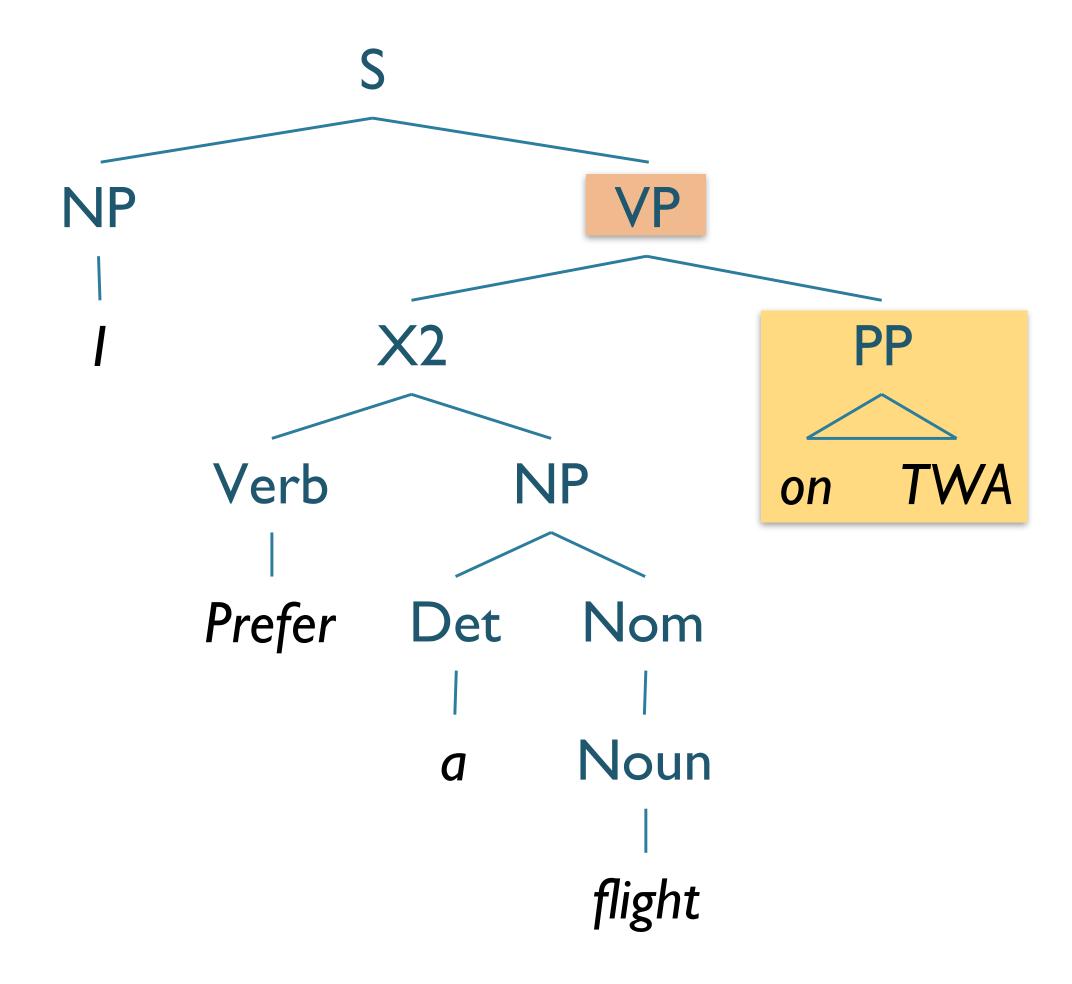
NP, Pronoun	S		S		
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Verb, VP, S		VP, X2, S		VP
	[1,2]	[1,3]	[1,4]	[1,5]	[1,6]
		Det	NP		NP
		[2,3]	[2,4]	[2,5]	[2,6]
			Noun, Nom		Nom
			[3,4]	[3,5]	[3,6]
				Prep	PP
				[4,5]	[4,6]
					NNP, NP
					[5,6]





# Resulting Parses





### CKY Discussion

### Running time:

 $O(n^3)$  where n is the length of the input string Inner loop grows as square of # of non-terminals

### Expressiveness:

- As implemented, requires CNF
- Weak equivalence to original grammar
- Doesn't capture full original structure
- Back-conversion?
  - Can do binarization, terminal conversion
  - Unit productions requires change in CKY

# Unit Testing

# Unit Testing

- Strategy of testing individual pieces of code in isolation
- Helps ensure:
  - Basic functionality in isolation
  - Complex functionality when individual components are combined
- In many industry jobs, you can't commit code without unit tests!

# Unit Testing in Python

- Many good tutorials on the web [here's one]
- In a nutshell:

# Unit Testing in Python

"nose" is a test running framework that will detect all test\_\* method functions
 python3 -m nose hw2.py

# Unit Testing in Python

"nose" is a test running framework that will detect all test\_\* method functions
 python3 -m nose hw2.py

# Unit Testing

- Good practice:
  - Save input that crashes your program for a unit test