

CKY Parsing

LING 571 — Deep Processing Techniques for NLP

October 7, 2019

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(h/t Ryan Georgi)

Announcements

- HW1: grades posted
 - In general, we will aim for Monday release of grades.
- Distance students:
 - Let us know in chat if/when audio cuts out.
 - Trying one thing today; will try something more drastic if the issues persist.

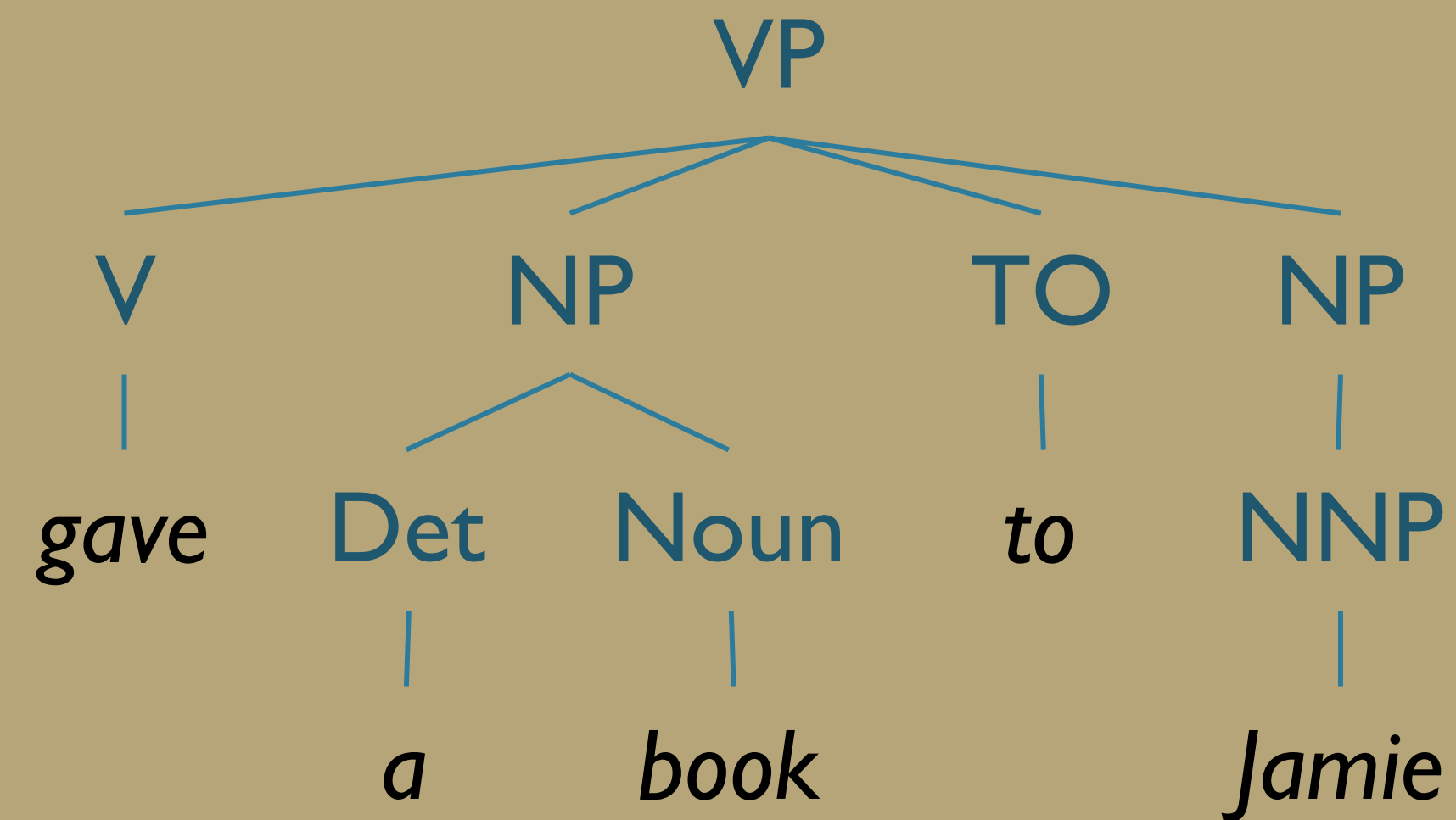
Announcements: HW #2

- Due Wednesday, 11:00pm Pacific
- Grammar output format:
 1. Terminals quoted (single or double)
 2. One production per line
 - *Optionally*, RHS alternates with vertical bar/pipe character: |
 3. Start symbol is first nonterminal encountered in file

Variation in CNF: Binarization

Original Rule

$VP \rightarrow V NP TO NP$



Variation in CNF: Binarization

Original Rule

$VP \rightarrow V NP TO NP$

Left to Right Reduction

$VP \rightarrow X1 TO NP$

$X1 \rightarrow V NP$

$VP \rightarrow X2 NP$

$X2 \rightarrow X1 TO$

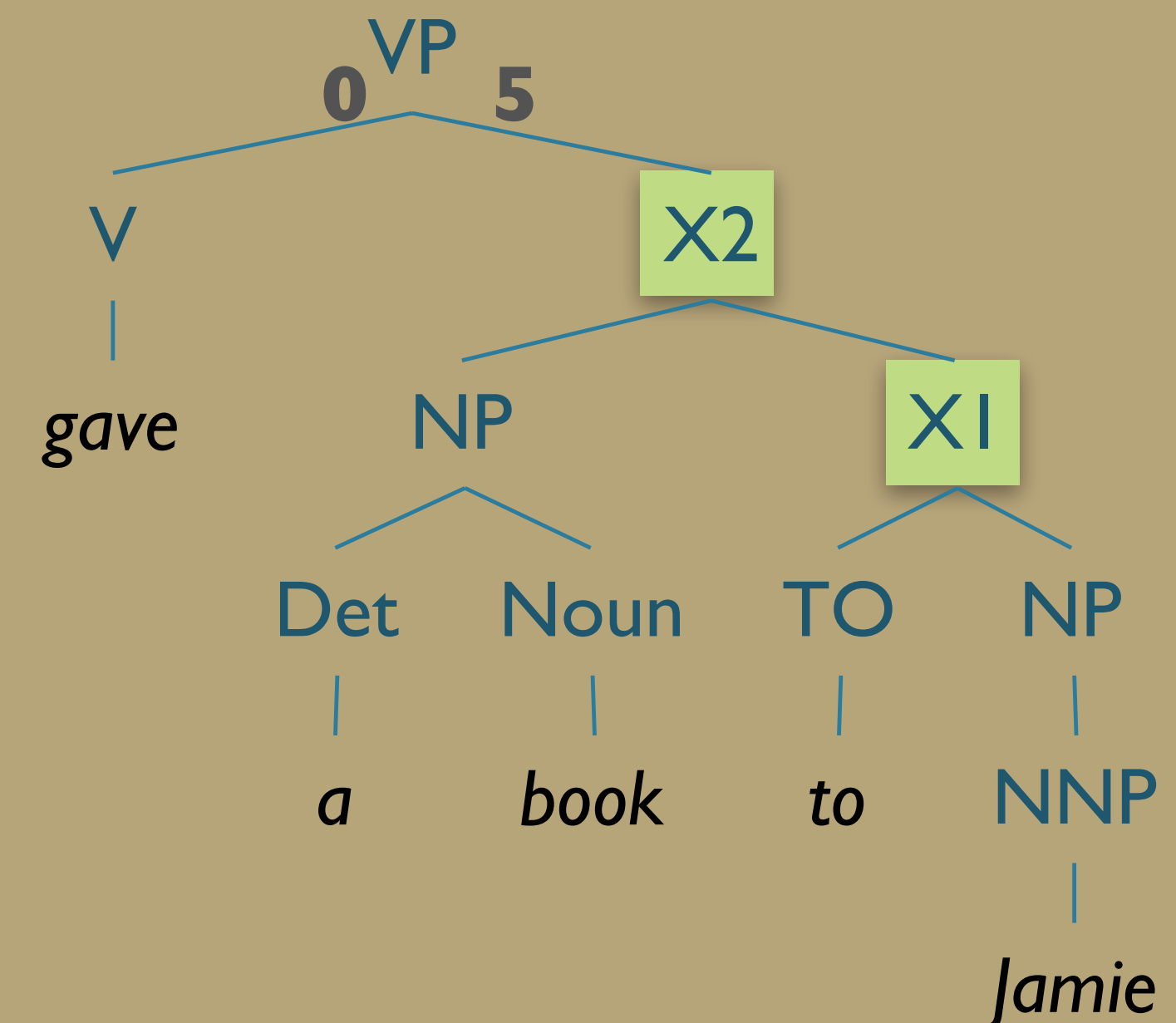
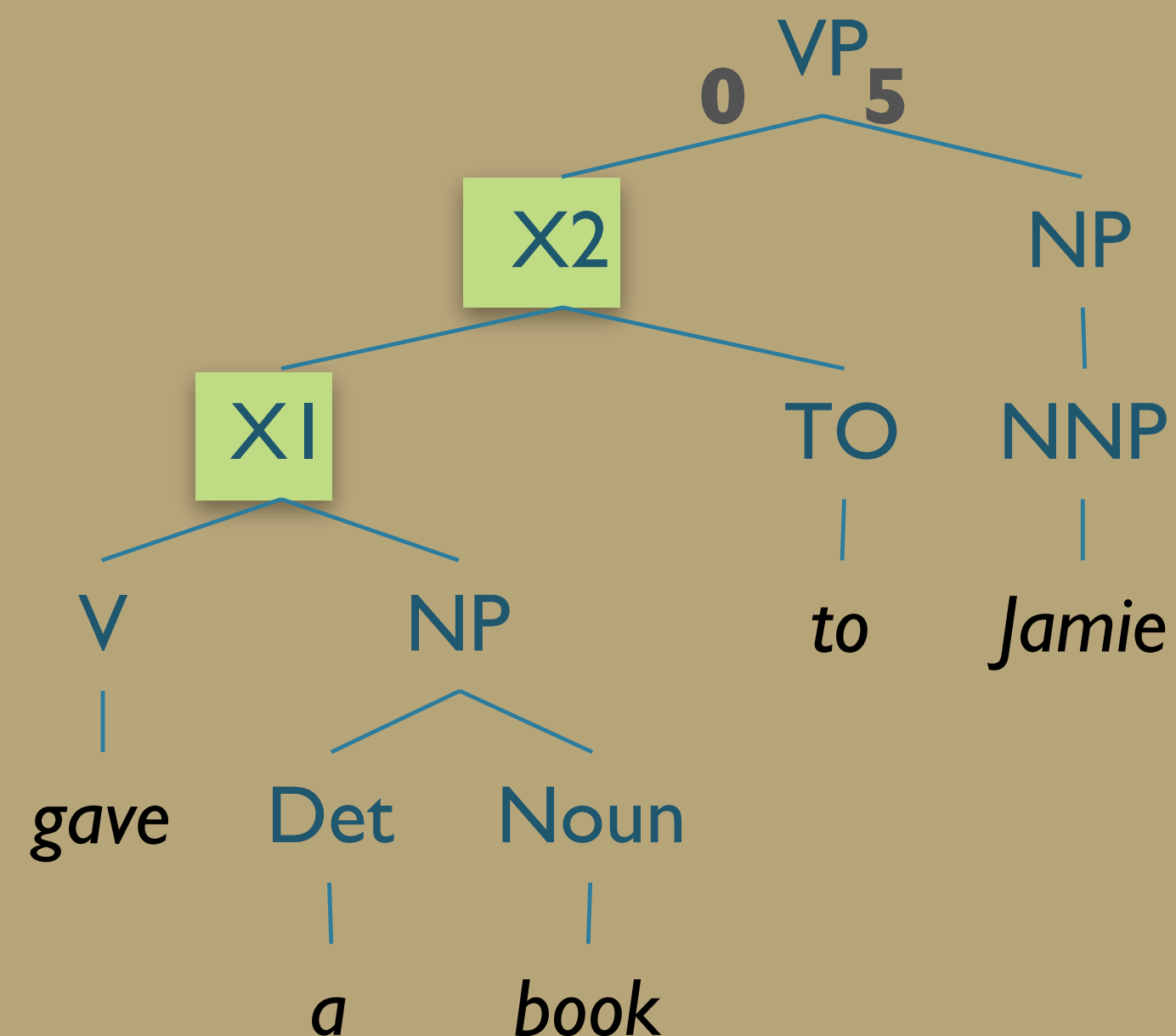
Right to Left Reduction

$VP \rightarrow V NP X1$

$X1 \rightarrow TO NP$

$VP \rightarrow V X2$

$X2 \rightarrow NP X1$



Roadmap

- CKY Parsing Algorithm
 - Dynamic programming structure
 - Algorithm and example
 - Analysis & Discussion
- Probabilistic CFGs
 - Motivation: Ambiguity
 - Augmenting CFGs with probabilities

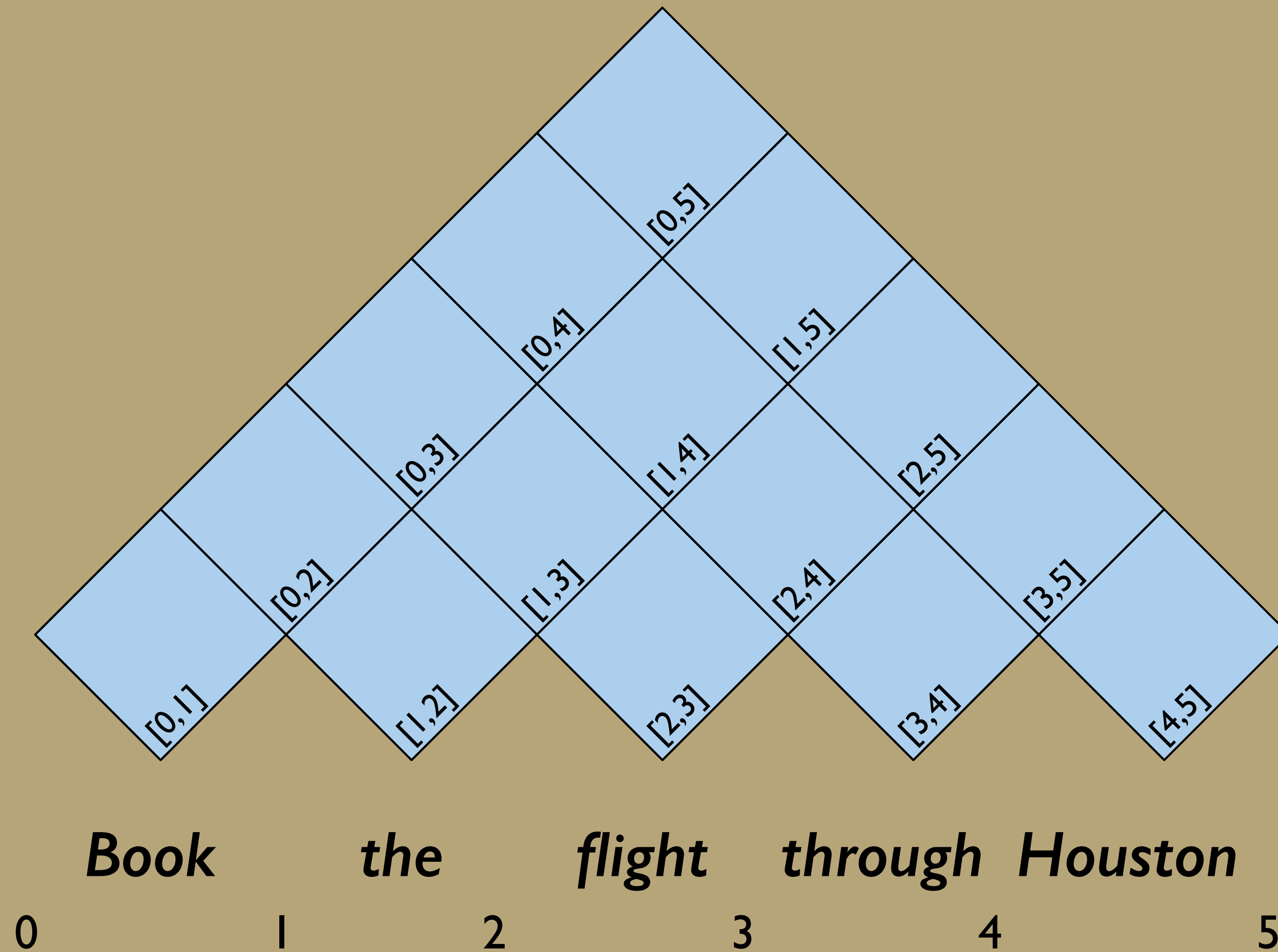
CKY

- Given an input string S of length n
- Build table $(n + 1) \times (n + 1)$
 - Indexes correspond to inter-word positions
 - $_0$ Book $_1$ that $_2$ flight $_3$
- Each cell $[i, j]$ contains SETS $\{ \}$ of non-terminals with derivations spanning i, j
 - $[j - 1, j]$ contains pre-terminals
 - if $[0, n]$ contains the start symbol, the sentence is ***accepted***

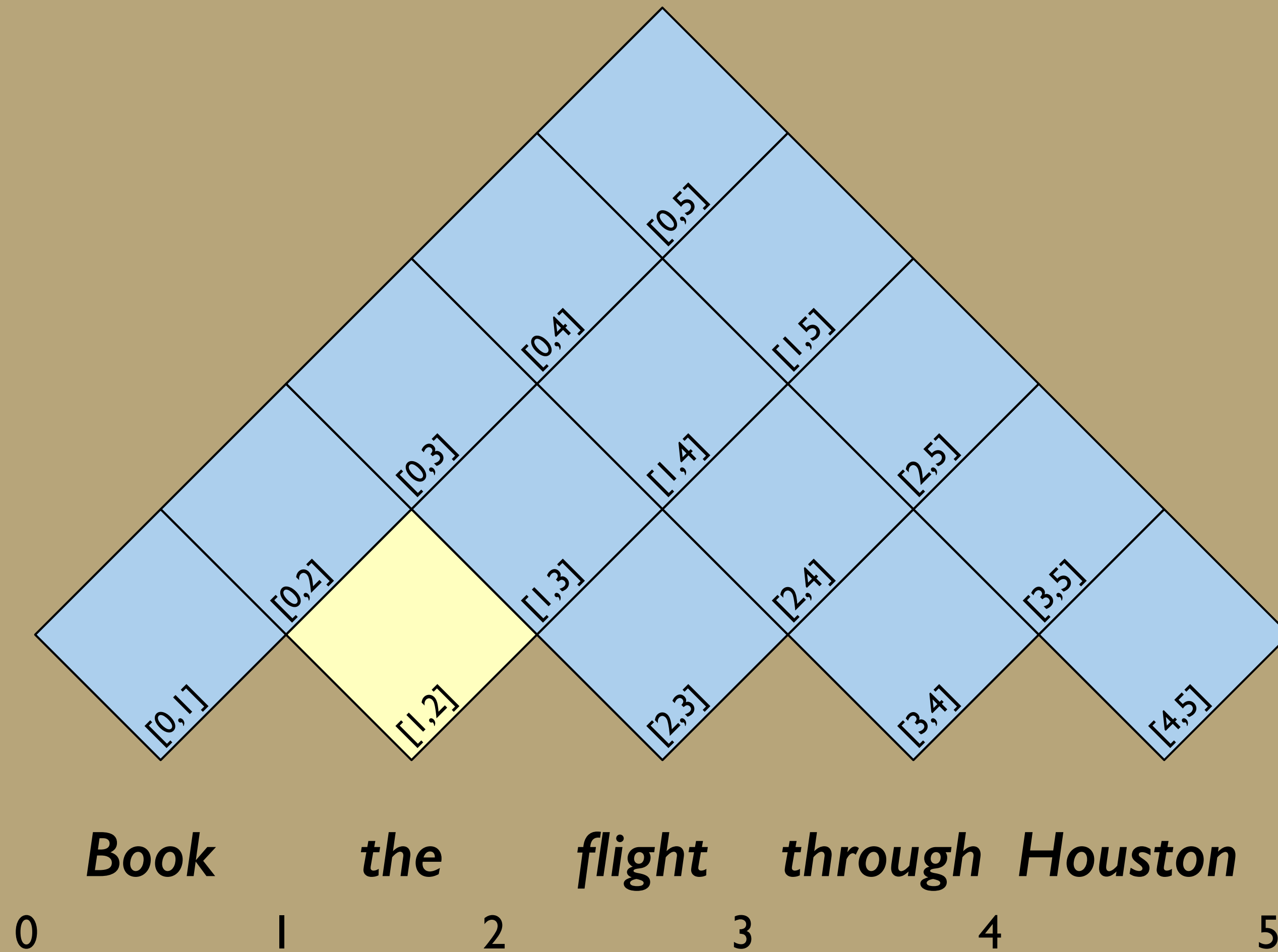
CKY Matrix

<i>Book</i>	<i>the</i>	<i>flight</i>	<i>through</i>	<i>Houston</i>
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	[1,2]	[1,3]	[1,4]	[1,5]
		[2,3]	[2,4]	[2,5]
			[3,4]	[3,5]
				[4,5]

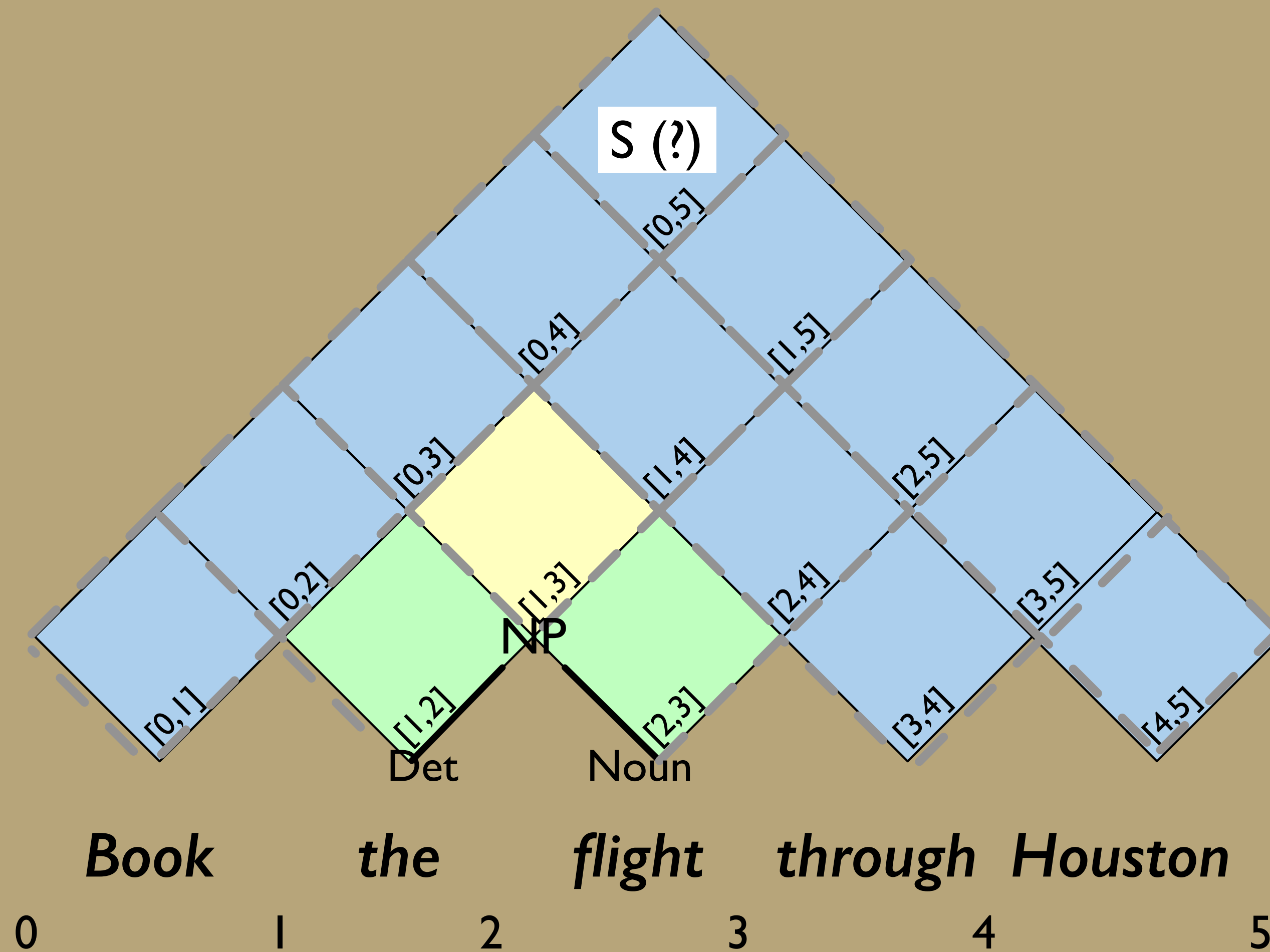
CKY Matrix



CKY Matrix



CKY Matrix

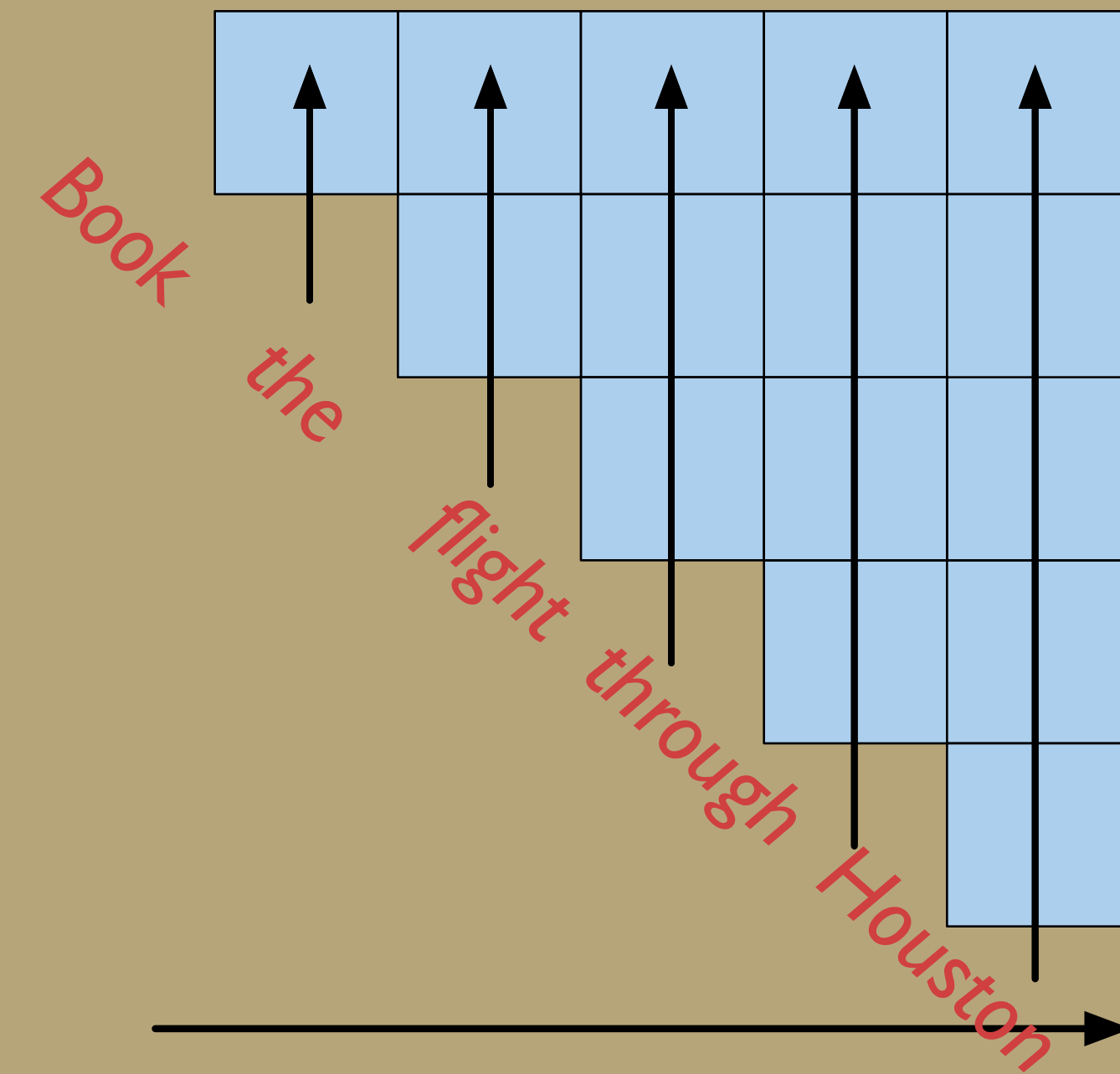


Dynamic Programming in CKY

- Key idea:
 - for $i < k < j$
 - ...and a parse spanning substring $[i, j]$
 - There is a k such that there are parses spanning $[i, k]$ and $[k, j]$
 - We can construct parses for whole sentences by building from these partial parses
- So to have a rule $A \rightarrow B \ C$ in $[i, j]$
 - Must have B in $[i, k]$ and C in $[k, j]$ for some $i < k < j$
 - CNF forces this for all $j > i + 1$

CKY

- Table fills:
 - Column-by-column, left-to-right
 - bottom-to-top
- Why?
 - To fill each cell, must have access to all sub-spans
 - For cell $[i, j]$, must be able to access all cells $[i, k]$ and $[k, j]$ where $i < k < j$
 - Allows online sentence analysis
 - Works across input string as it arrives

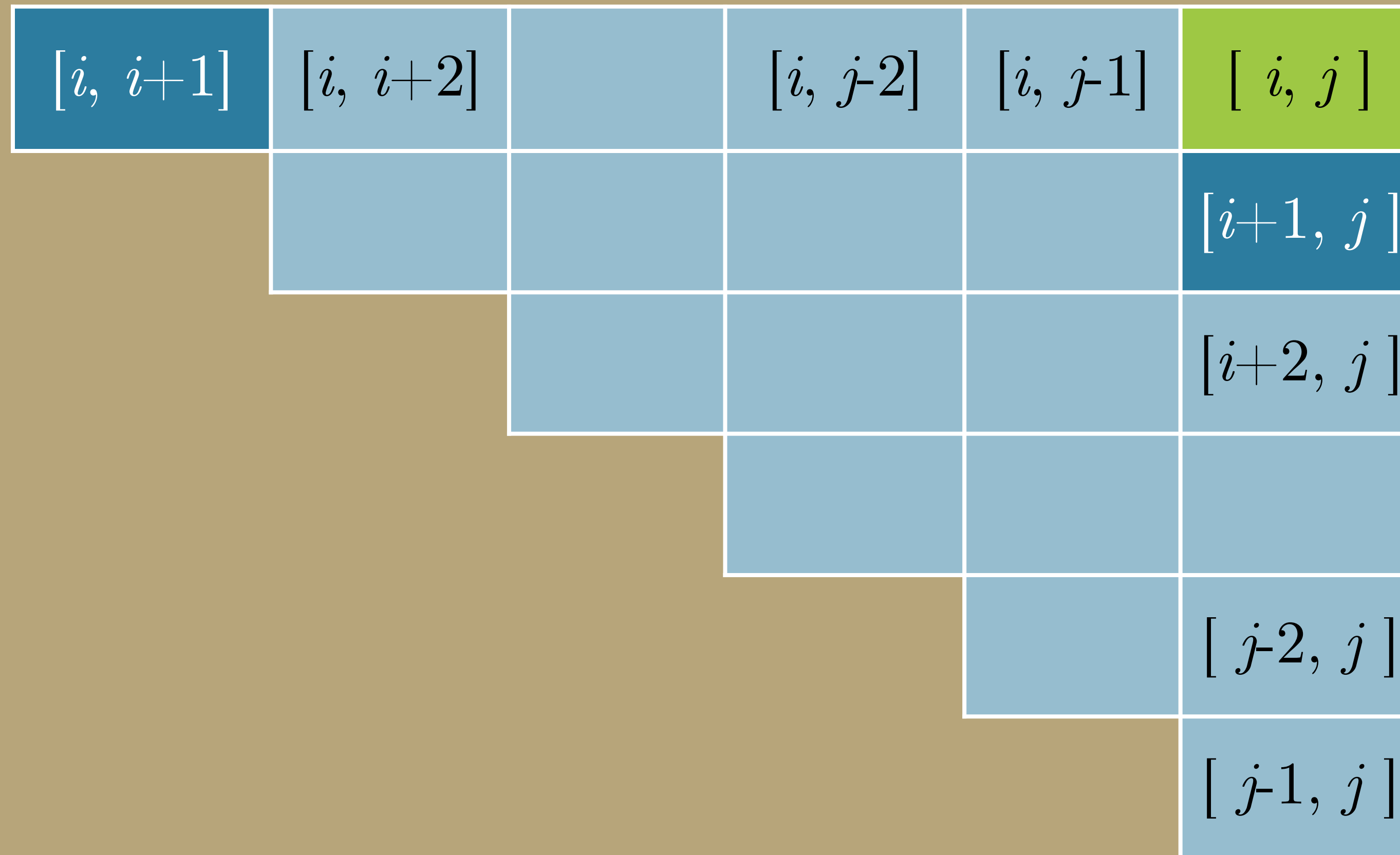


CKY Algorithm

```
function CKY-PARSE(words, grammar) returns table
for j ← from 1 to LENGTH(words) do
  table[j - 1, j] ← { A | A → words[j] ∈ grammar }
  for i ← from j - 2 downto 0 do
    for k ← i + 1 to j - 1 do
      table[i, j] ← table[i, j] ∪
        { A | A → BC ∈ grammar
          B ∈ table[i, k],
          C ∈ table[k, j] }
```

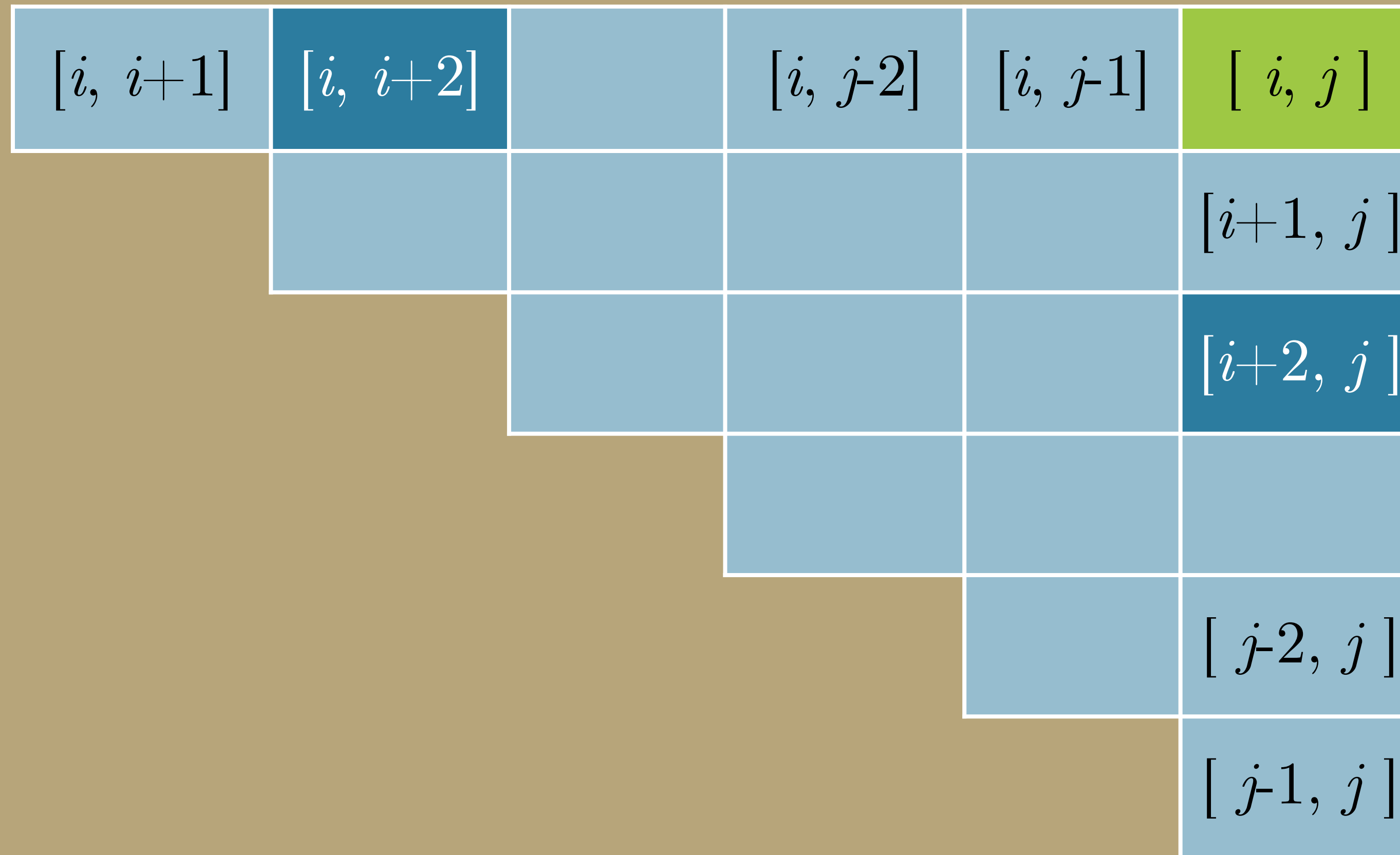
- Left-to-right, bottom-to-top
- Sliding *k* as the pivot for child nodes
- Filling the cell with set of applicable rules

Filling Cell $[i, j]$



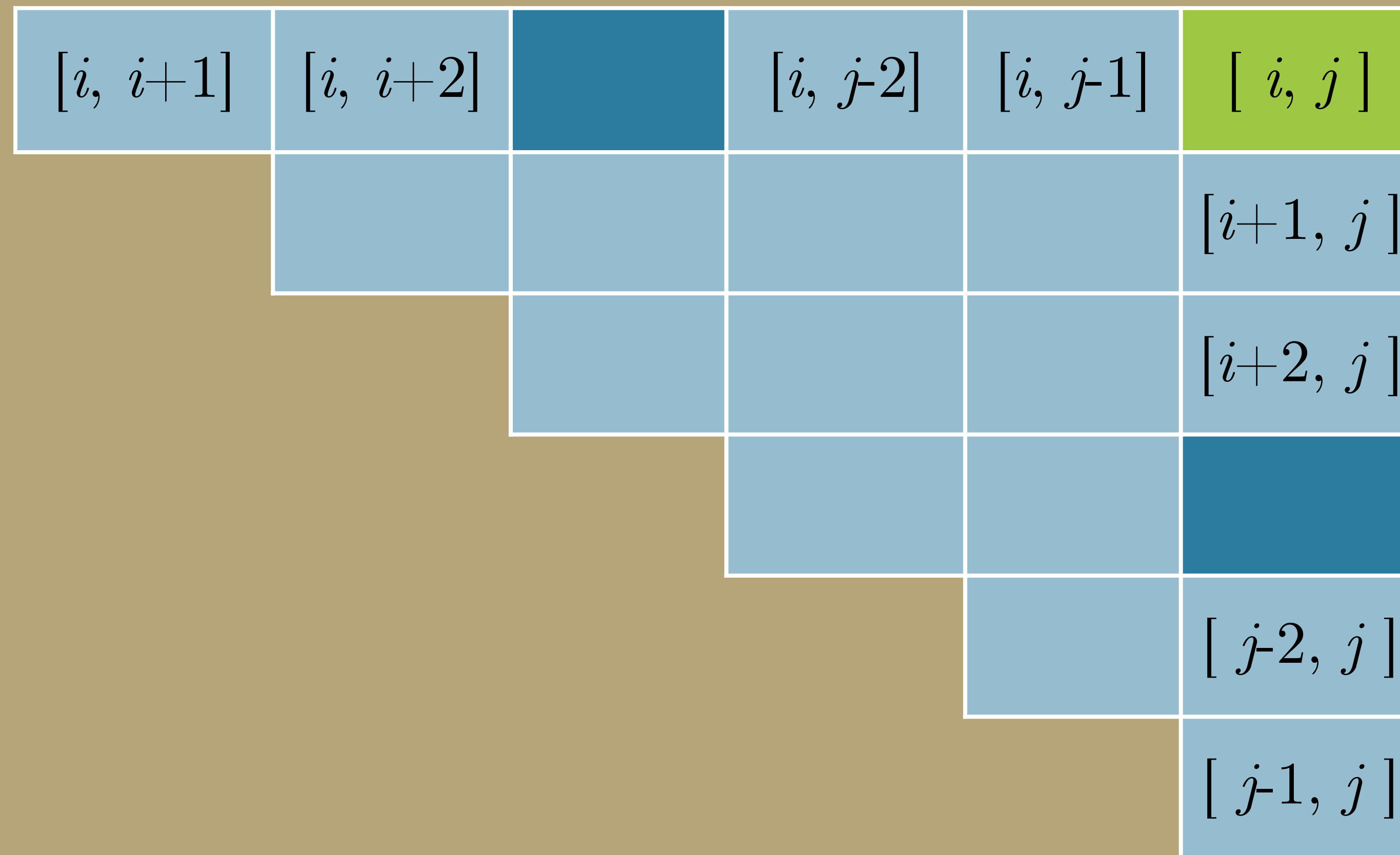
For cell $[i, j]$, find rule spanning cells $[i, k]$ and $[k, j]$ where $i < k < j$

Filling Cell $[i, j]$



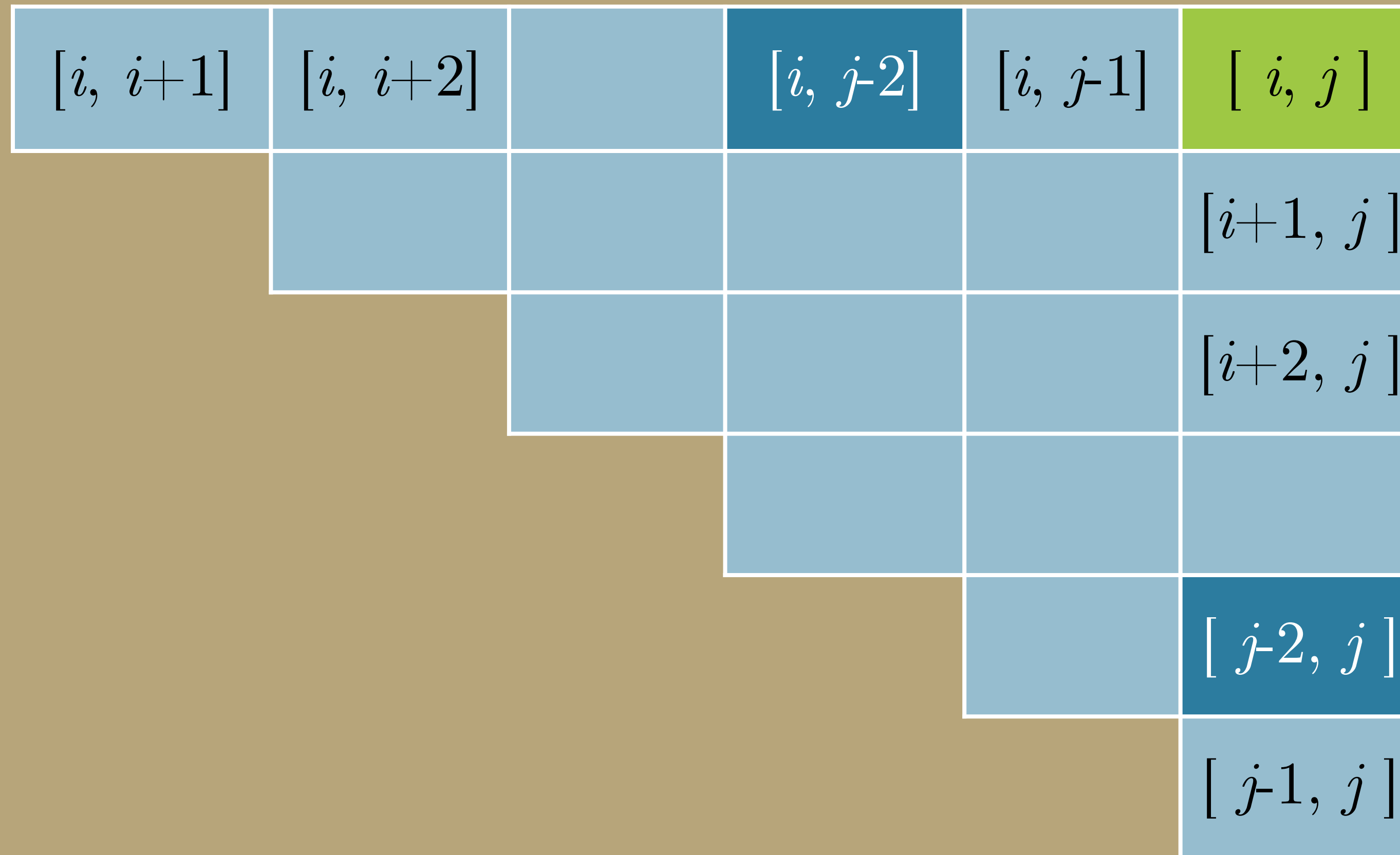
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Filling Cell [i, j]



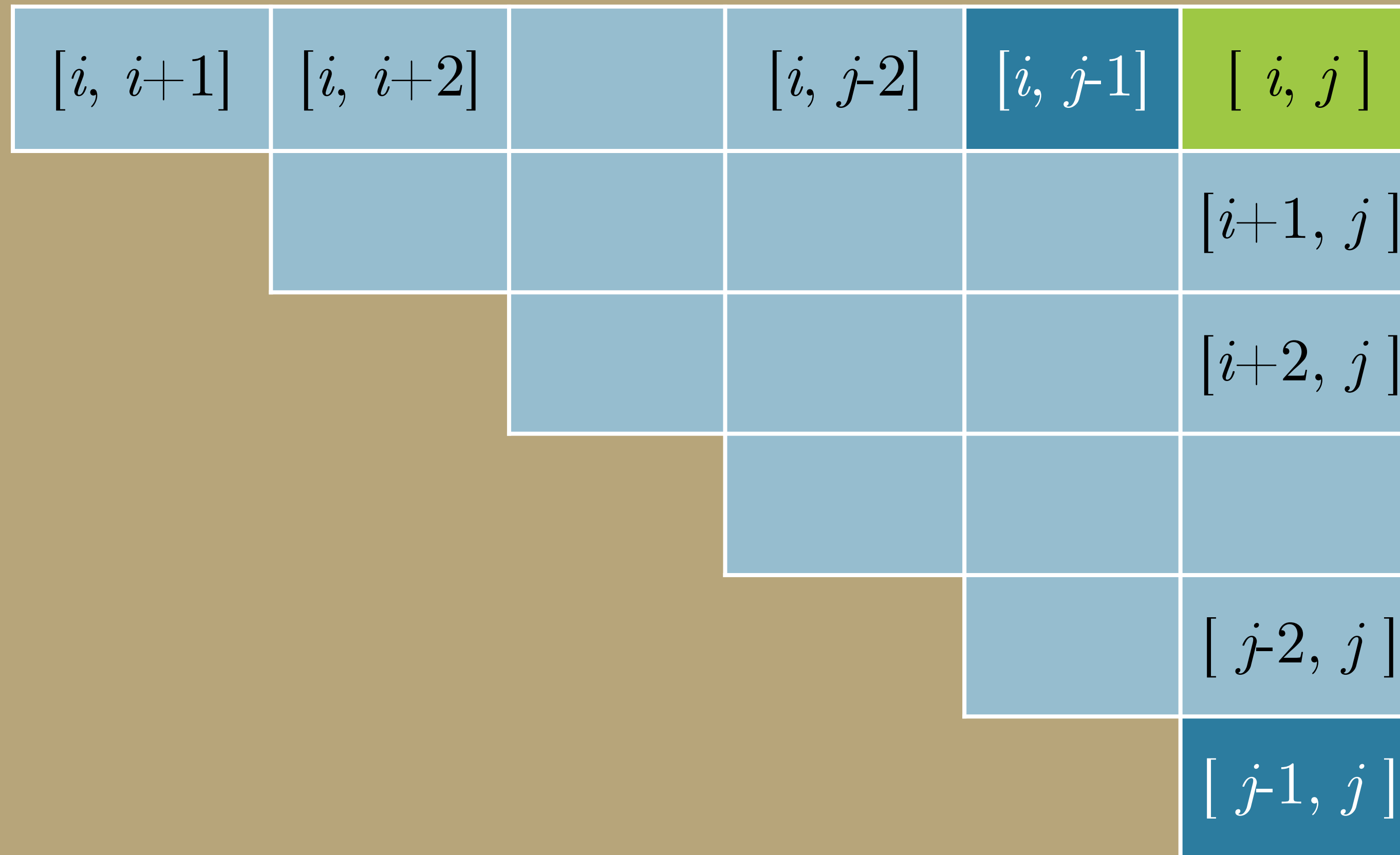
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Filling Cell $[i, j]$



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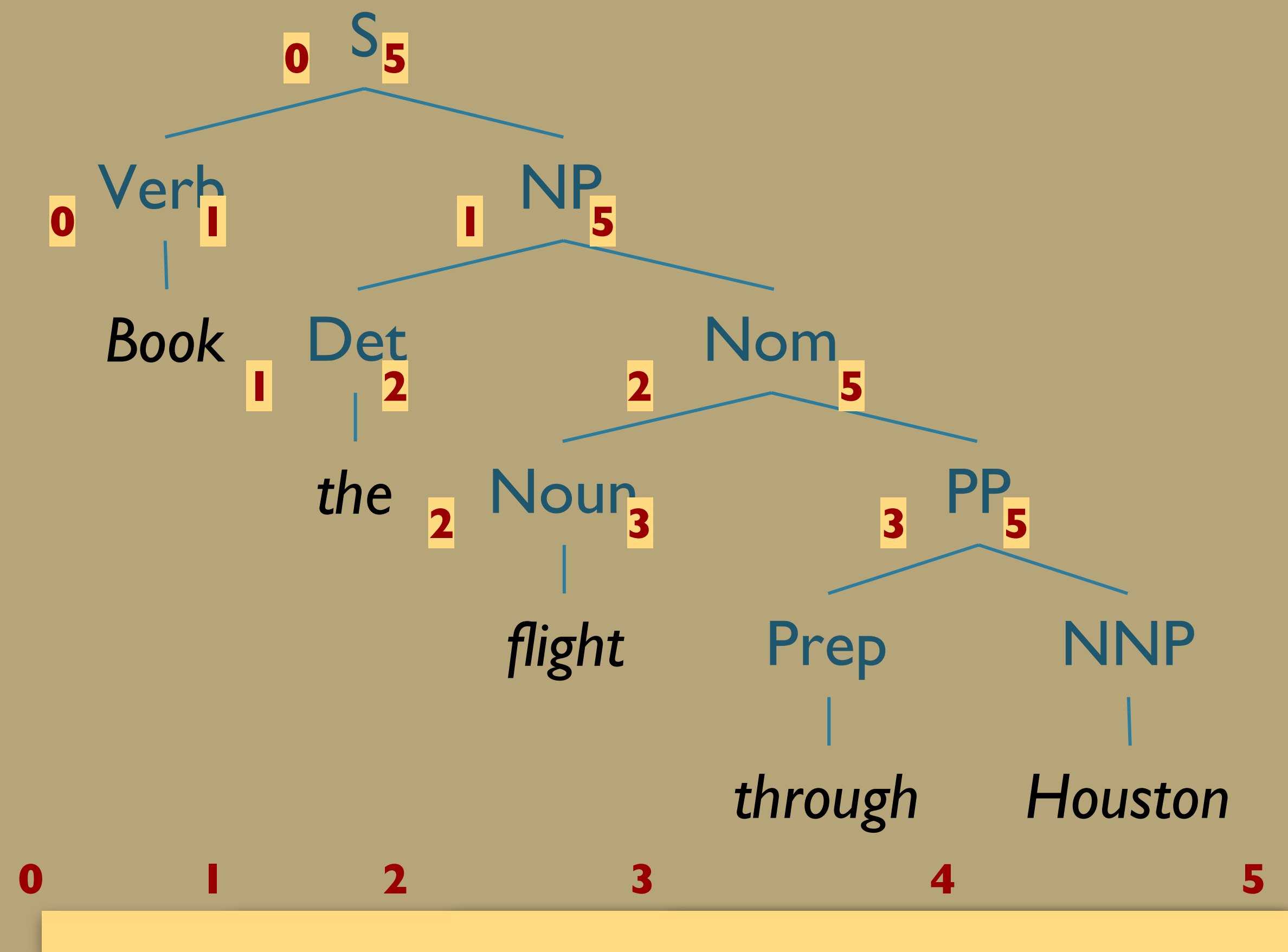
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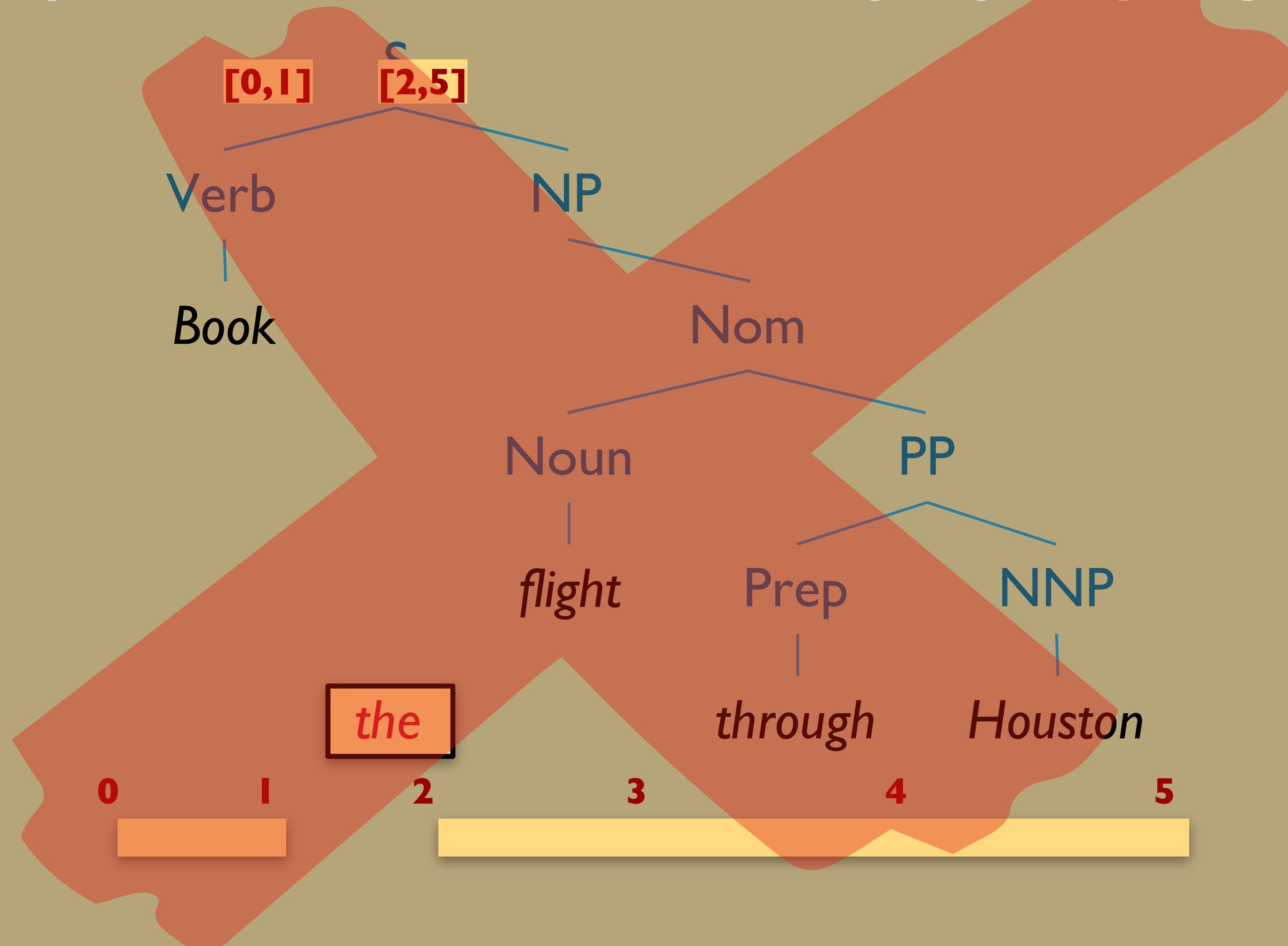
Why fill cells this way?

- For cell $[i, j]$, must be able to access all cells $[i, k]$ and $[k, j]$ where $i < k < j$



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- For cell $[i, j]$, must be able to access all cells $[i, k]$ and $[k, j]$ where $i < k < j$



CKY Parsing Example

\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- $Det \rightarrow that\ |\ this\ |\ a$
- $Noun \rightarrow book\ |\ flight\ |\ meal\ |\ money$
- $Pronoun \rightarrow I\ |\ she\ |\ me$
- $Proper-Noun \rightarrow Houston\ |\ TWA$
- $Aux \rightarrow does$
- $Preposition \rightarrow from\ |\ to\ |\ on\ |\ near\ |\ through$
- $Verb \rightarrow book\ |\ include\ |\ prefer$

0

1

2

3

4

5

6

I

prefer

a

flight

on

TWA

\mathcal{L}_1 Grammar

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 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
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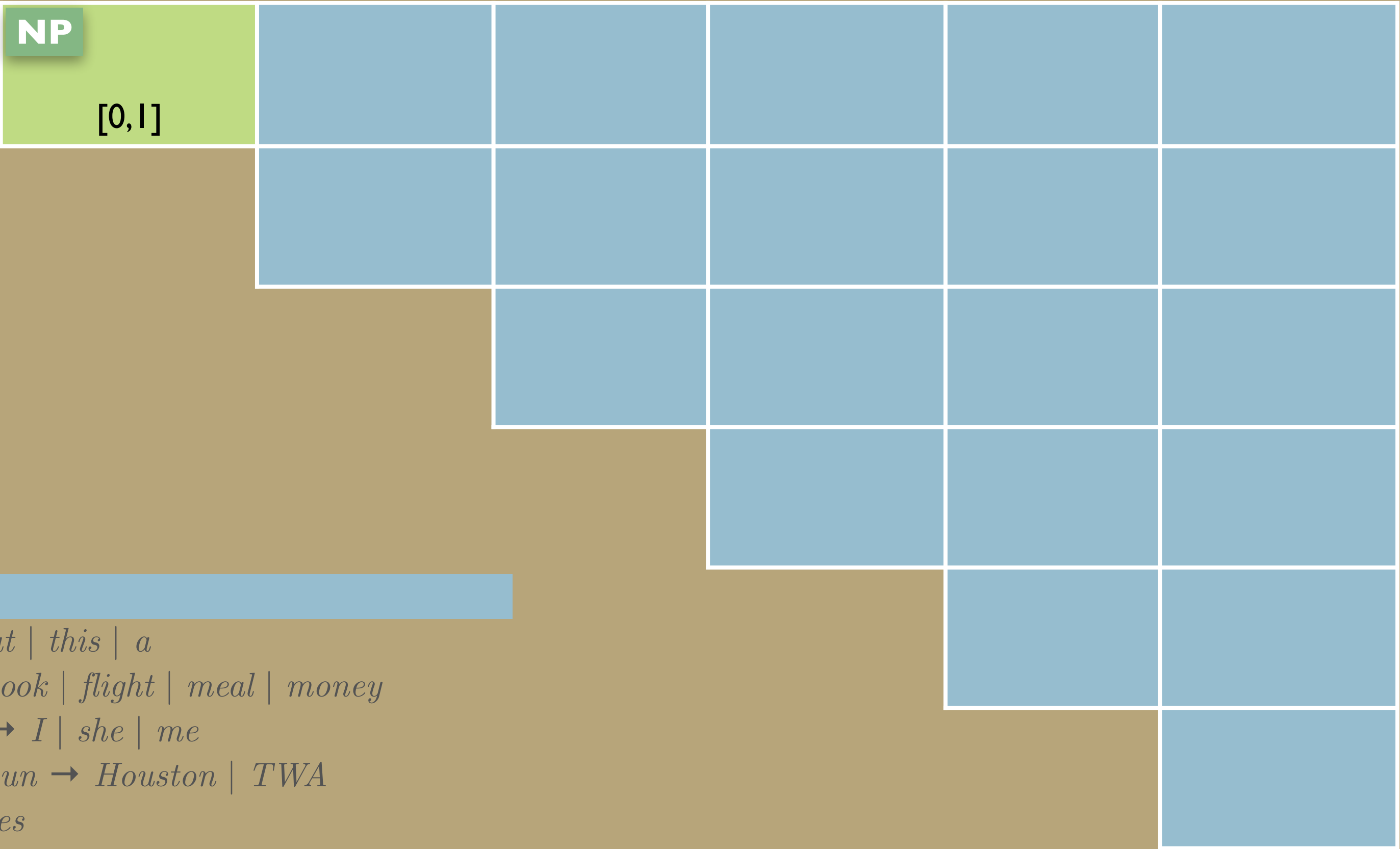
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Lexicon

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I

01

prefer

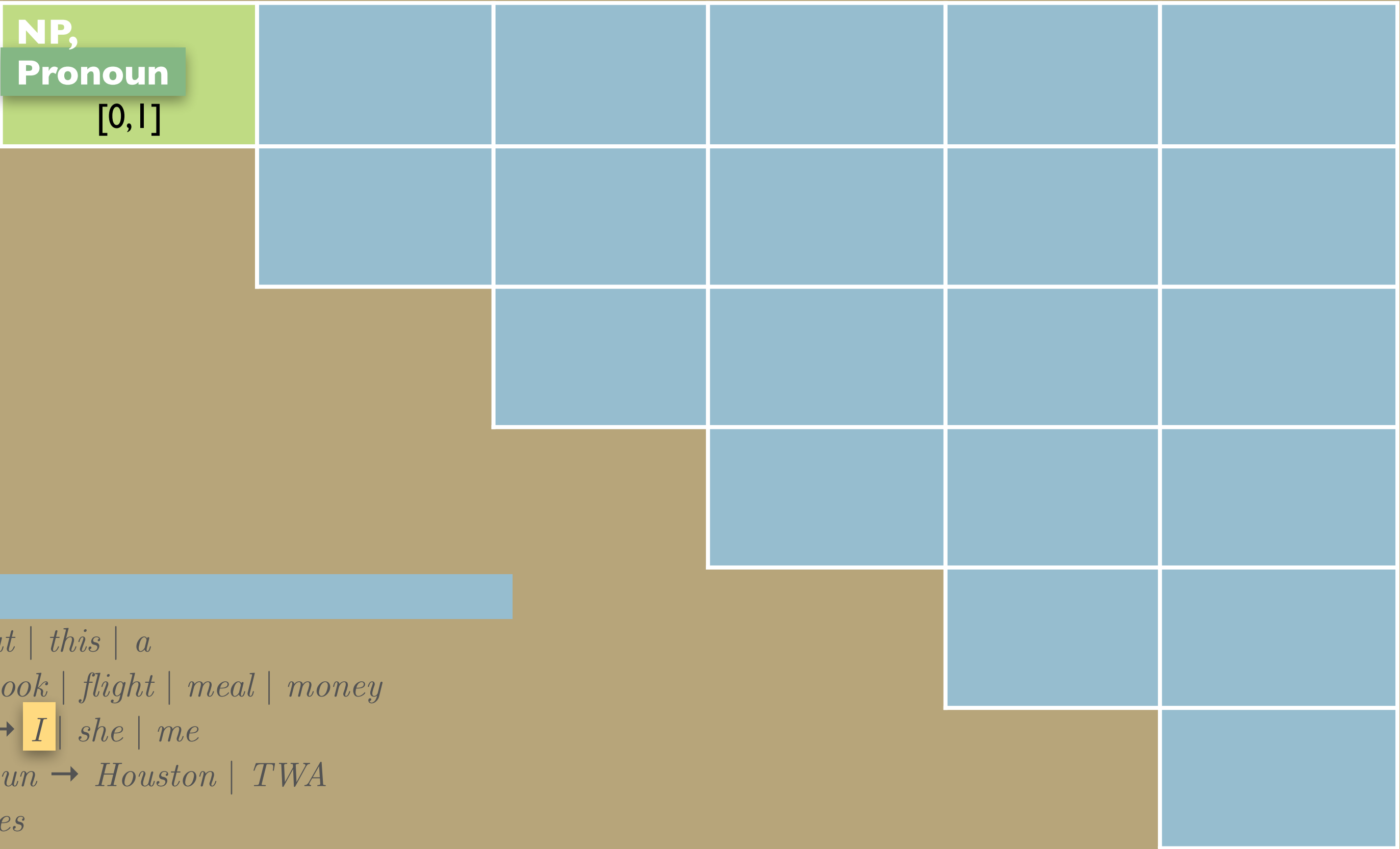
a

flight

on

TWA

23456



\mathcal{L}_1 Grammar

$S \rightarrow NP VP$
 $S \rightarrow X1 VP$
 $X1 \rightarrow Aux NP$
 $S \rightarrow book / include / prefer$
 $S \rightarrow Verb NP$
 $S \rightarrow X2 PP$
 $S \rightarrow Verb PP$
 $S \rightarrow VP PP$

$NP \rightarrow I / she / me$
 $NP \rightarrow TWA / Houston$
 $NP \rightarrow Det Nominal$

$Nominal \rightarrow book / flight / meal / money$
 $Nominal \rightarrow Nominal Noun$
 $Nominal \rightarrow Nominal PP$

$VP \rightarrow book / include / prefer$
 $VP \rightarrow Verb NP$
 $VP \rightarrow X2 PP$
 $X2 \rightarrow Verb NP$
 $VP \rightarrow Verb PP$
 $VP \rightarrow VP PP$

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Lexicon

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 $Noun \rightarrow book | flight | meal | money$
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 $Proper-Noun \rightarrow Houston | TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from | to | on | near | through$
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0

1

2

3

4

5

6

I

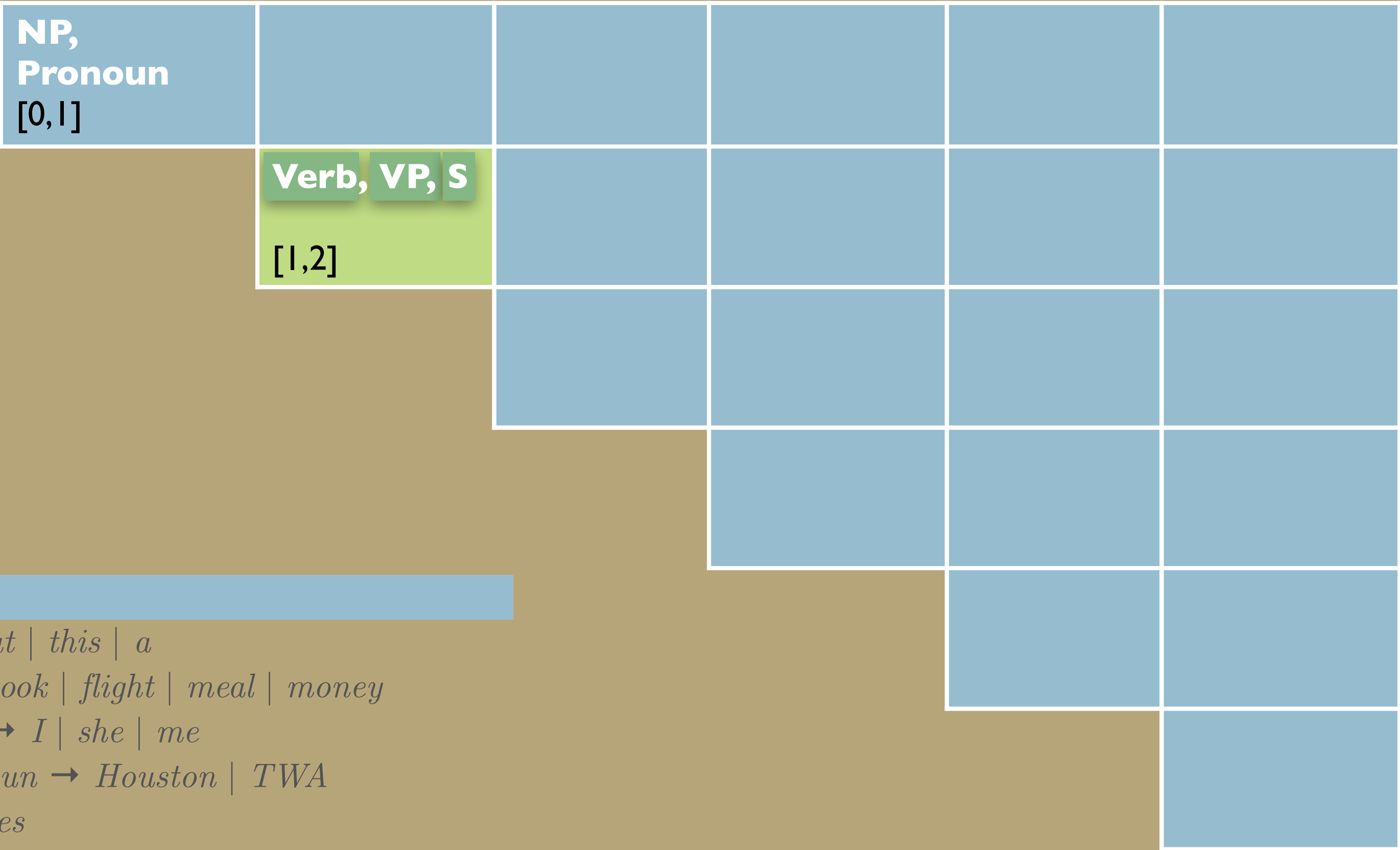
prefer

a

flight

on

TWA



\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \text{ VP}$$
$$X1 \rightarrow Aux NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X_2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP\ PP$$

$NP \rightarrow I / she / me$

NP → *TWA* / *Houston*

$NP \rightarrow Det \text{ Nominal}$

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Nominal \rightarrow *Nominal Noun*

Nominal \rightarrow *Nominal PP*

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$$VP \rightarrow Verb\ NP$$
$$VP \rightarrow X^2 PP$$
$$X2 \rightarrow Verb\ NP$$
$$VP \rightarrow Verb PP$$
$$VP \rightarrow VP PP$$

$PP \rightarrow$ *Preposition*

**NP,
Pronoun**
[0,1]

$[0,2]$

Verb, VP, S

[1,2]

Lexicon

Det → *that* | *this* | *a*

Noun → *book* | *flight* | *meal* | *money*

Pronoun → *I* | *she* | *me*

Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer

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Lexicon

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Noun → *book* | *flight* | *meal* | *money*

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Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer

NP, Pronoun [0,1]

$[0,2]$

Verb, VP, S

[1,2]

Diagram illustrating a 1D lattice with three sites labeled 0, 1, and 2. Site 0 is occupied by a red particle, site 1 by a green particle, and site 2 by a blue particle. A dashed line is drawn between sites 0 and 1, with the label I above it and *prefer* below it.

3

4

5

6

\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \text{ VP}$$
$$X1 \rightarrow Aux NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X_2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP \ PP$$

$NP \rightarrow I / she / me$

NP → *TWA* / *Houston*

$NP \rightarrow Det\ Nominal$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

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$PP \rightarrow$ *Preposition*

Lexicon

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Noun → *book* | *flight* | *meal* | *money*

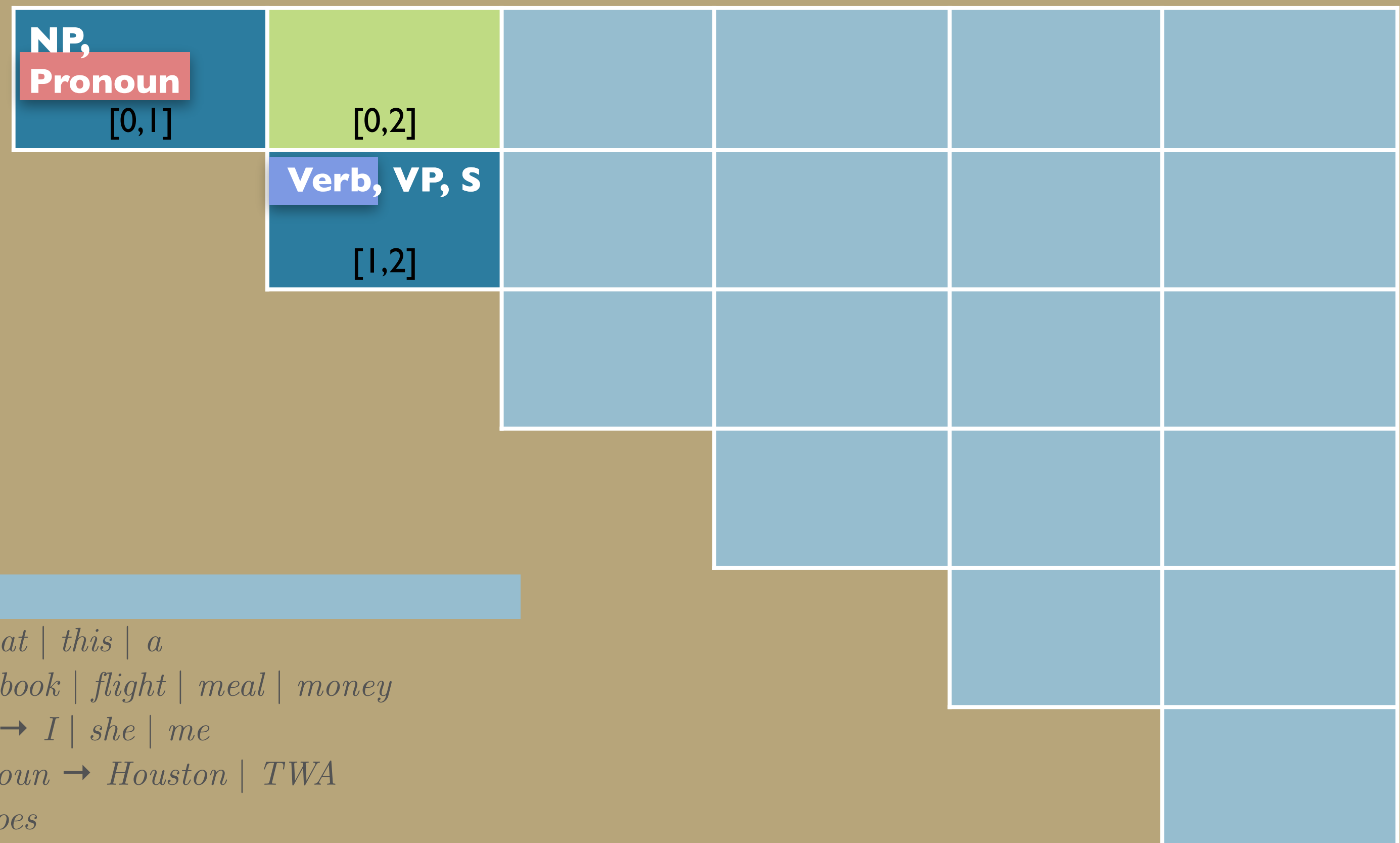
Pronoun → *I* | *she* | *me*

Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer



A diagram of a 1D lattice with three sites labeled 0, 1, and 2. Site 0 is represented by a red box, site 1 by a green box, and site 2 by a blue box. A dashed horizontal line runs above the boxes. The letter I is positioned above site 0, and the word *prefer* is positioned above site 1.

\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \text{ VP}$$
$$X1 \rightarrow Aux NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X_2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP \ PP$$

$NP \rightarrow I / she / me$

NP → *TWA* / *Houston*

$NP \rightarrow Det \text{ Nominal}$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

Nominal \rightarrow *Nominal Noun*

Nominal \rightarrow *Nominal PP*

$VP \rightarrow book / include / prefer$

$$VP \rightarrow Verb\ NP$$
$$VP \rightarrow X^2 PP$$
$$X2 \rightarrow Verb\ NP$$
$$VP \rightarrow Verb PP$$
$$VP \rightarrow VP PP$$

$PP \rightarrow$ *Preposition*

Lexicon

$Det \rightarrow that \mid this \mid a$

Noun → *book* | *flight* | *meal* | *money*

Pronoun → *I* | *she* | *me*

Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer

A diagram showing a horizontal line with a vertical line segment intersecting it at a point. The vertical segment is labeled with a single letter 'l'.

.....
prefer

a

flight

on

TWA

\mathcal{L}_1 Grammar

$S \rightarrow NP VP$

$$S \rightarrow X1 \quad VP$$
$$X1 \rightarrow Aux \ NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X^2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP \ PP$$

$NP \rightarrow I / she / me$

NP → *TWA* / *Houston*

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Nominal \rightarrow *book* / *flight* / *meal* / *money*

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$PP \rightarrow$ *Preposition*

1

prefer

a

flight

on

TWA

0

1

2

3

4

5

6

NP, Pronoun [0,1]

S

$[0,2]$

Verb, VP, S

[1,2]

Lexicon

Det → *that* | *this* | *a*

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Verb \rightarrow book | include | prefer

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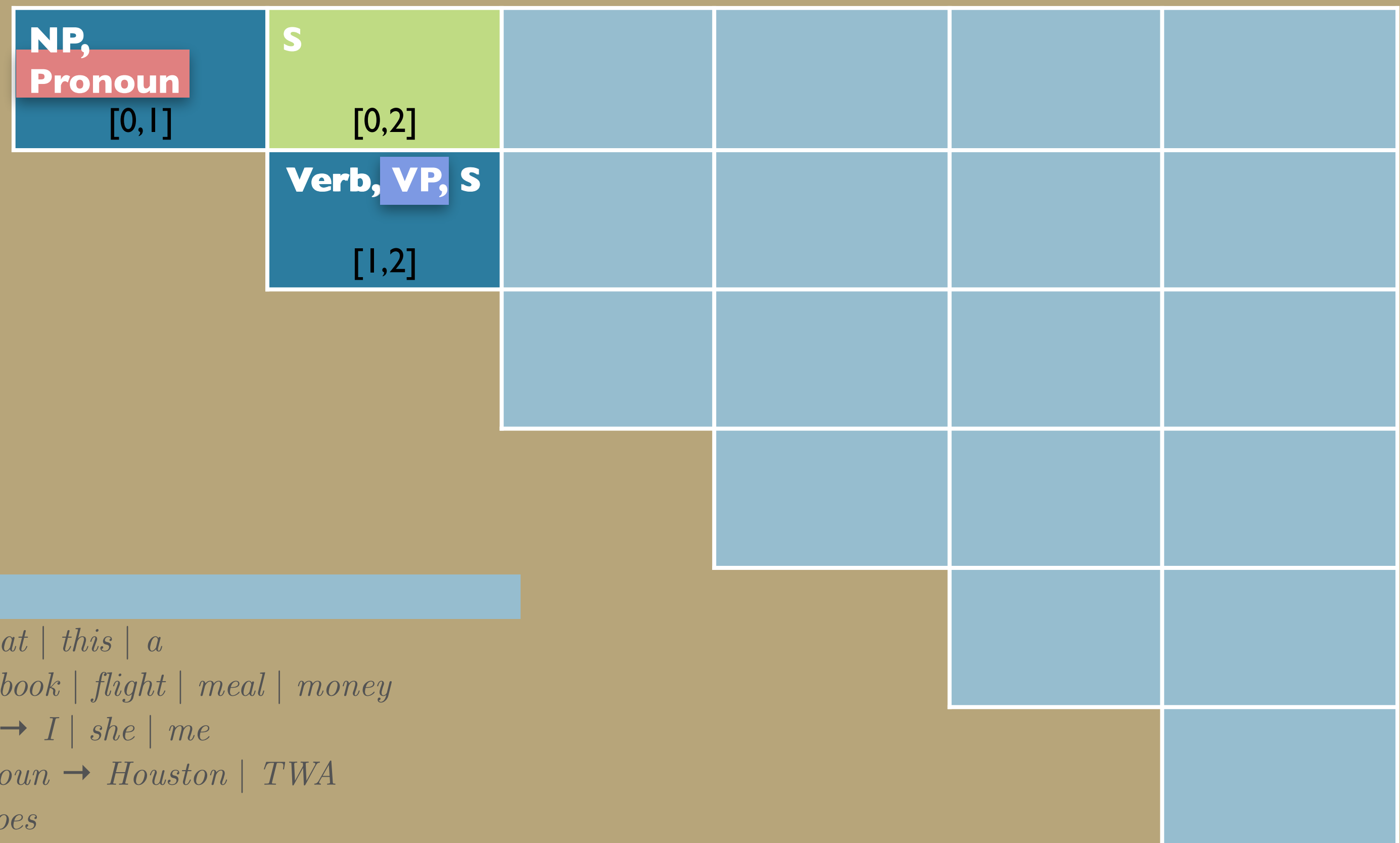
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Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer



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$$S \rightarrow X1 \quad VP$$
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$NP \rightarrow I \mid she \mid me$

NP → *TWA* / *Houston*

$$NP \rightarrow Det \textit{ Nominal}$$

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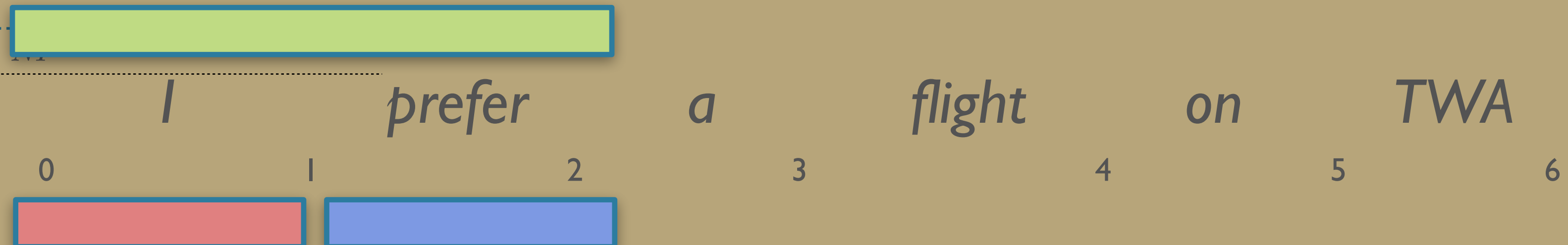
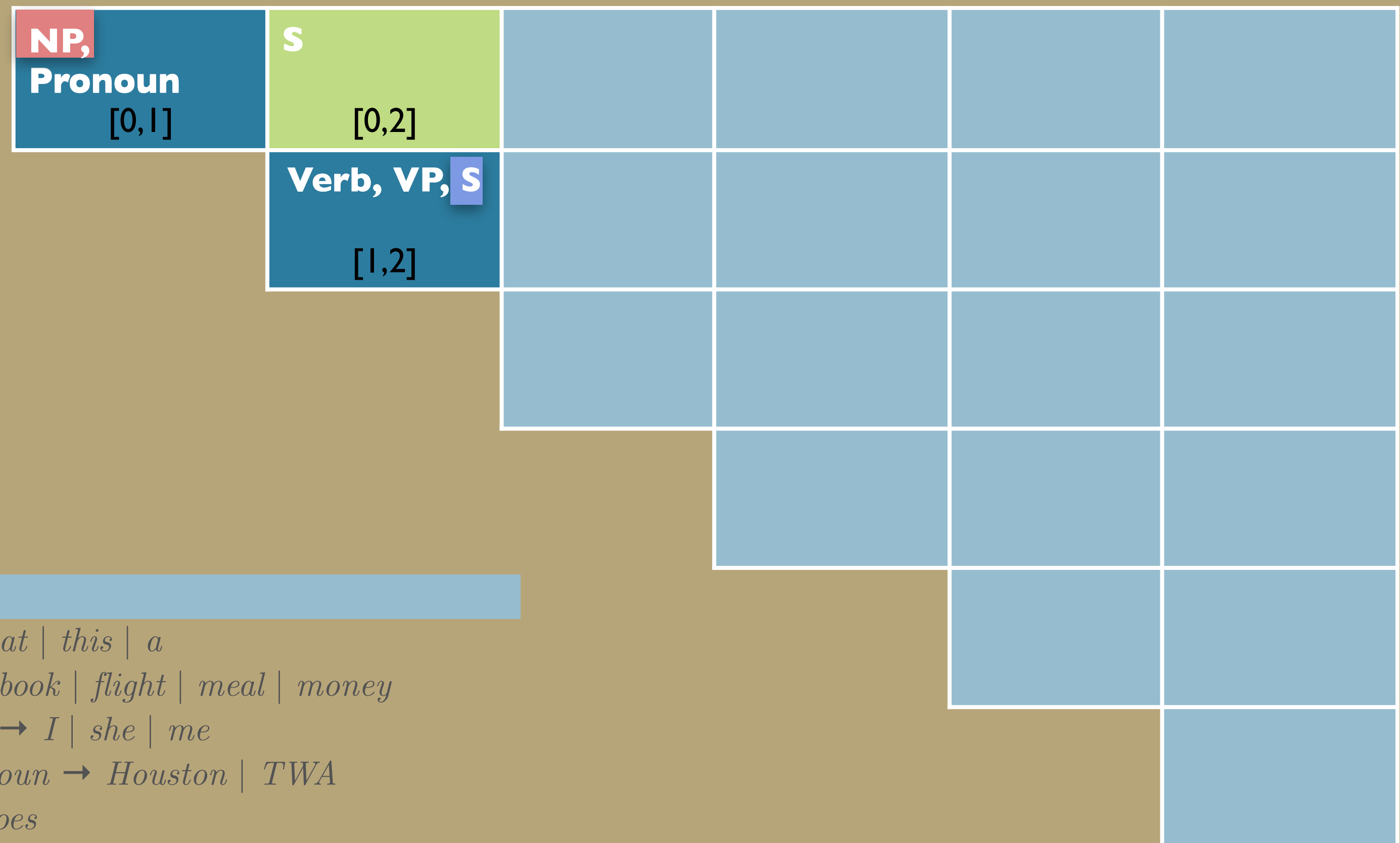
Pronoun → *I* | *she* | *me*

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Preposition → *from* | *to* | *on* | *near* | *through*

Verb → *book* | *include* | *prefer*



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$PP \rightarrow$ *Preposition*

NP,
Pronoun
[0,1]

[0,1]

S

$[0,2]$

[0,2]

Verb, VP, S

[1,2]

[1,2]

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Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer

A diagram illustrating the concept of a 'preference' in a game. It shows a horizontal axis with points 0, 1, and 2. A green bar is positioned above the axis, and a red bar is positioned below the axis. The word 'prefer' is written above the axis, indicating that the green bar represents a preferred outcome over the red bar.

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A diagram illustrating the concept of a 'preference' in a game. It shows a horizontal axis with points 0, 1, and 2. A green bar is positioned above the axis, and a red bar is positioned below the axis. The word 'prefer' is written above the axis, indicating a preference for a value greater than 1.

A diagram illustrating the concept of a 'preference' in a game. It shows a horizontal axis with points 0, 1, and 2. A green bar is positioned above the axis, and a red bar is positioned below the axis. The word 'prefer' is written above the axis, indicating that the green bar represents a preferred outcome over the red bar.

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\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* → that | this | **a**
- Noun* → book | flight | meal | money
- Pronoun* → I | she | me
- Proper-Noun* → Houston | TWA
- Aux* → does
- Preposition* → from | to | on | near | through
- Verb* → book | include | prefer

0 1 2 3 4 5 6

I prefer **a** flight on TWA

NP, Pronoun [0,1]	S [0,2]				
	Verb, VP, S [1,2]				
		Det [2,3]			

\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \text{ VP}$$
$$X1 \rightarrow Aux NP$$

$S \rightarrow \text{book} / \text{include} / \text{prefer}$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X_2^2 PP$$
$$S \rightarrow VP PP$$
$$NP \rightarrow I / she / me$$

$NP \rightarrow TWA / Houston$

$$NP \rightarrow Det\ Nominal$$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

Nominal → *Nominal Noun*

Nominal \rightarrow *Nominal PP*

$VP \rightarrow book / include / prefer$

$$VP \rightarrow Verb\ NP$$
$$VP \rightarrow X^2 PP$$
$$X_2 \rightarrow Verb\ NP$$
$$VP \rightarrow Verb PP$$

$VP \rightarrow VP PP$

$PP \rightarrow$ *Preposition NP*

Lexicon

Det \rightarrow *that* | *this* | *a*

Noun → *book* | *flight* | *meal* | *money*

Pronoun	\rightarrow	I		she		me
Person		1st		3rd		3rd
Number		Singular		Singular		Plural
Gender		Male		Female		Unknown

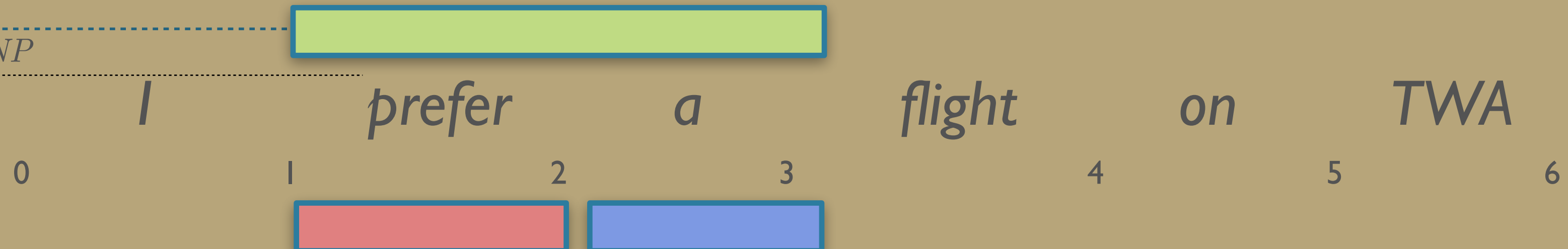
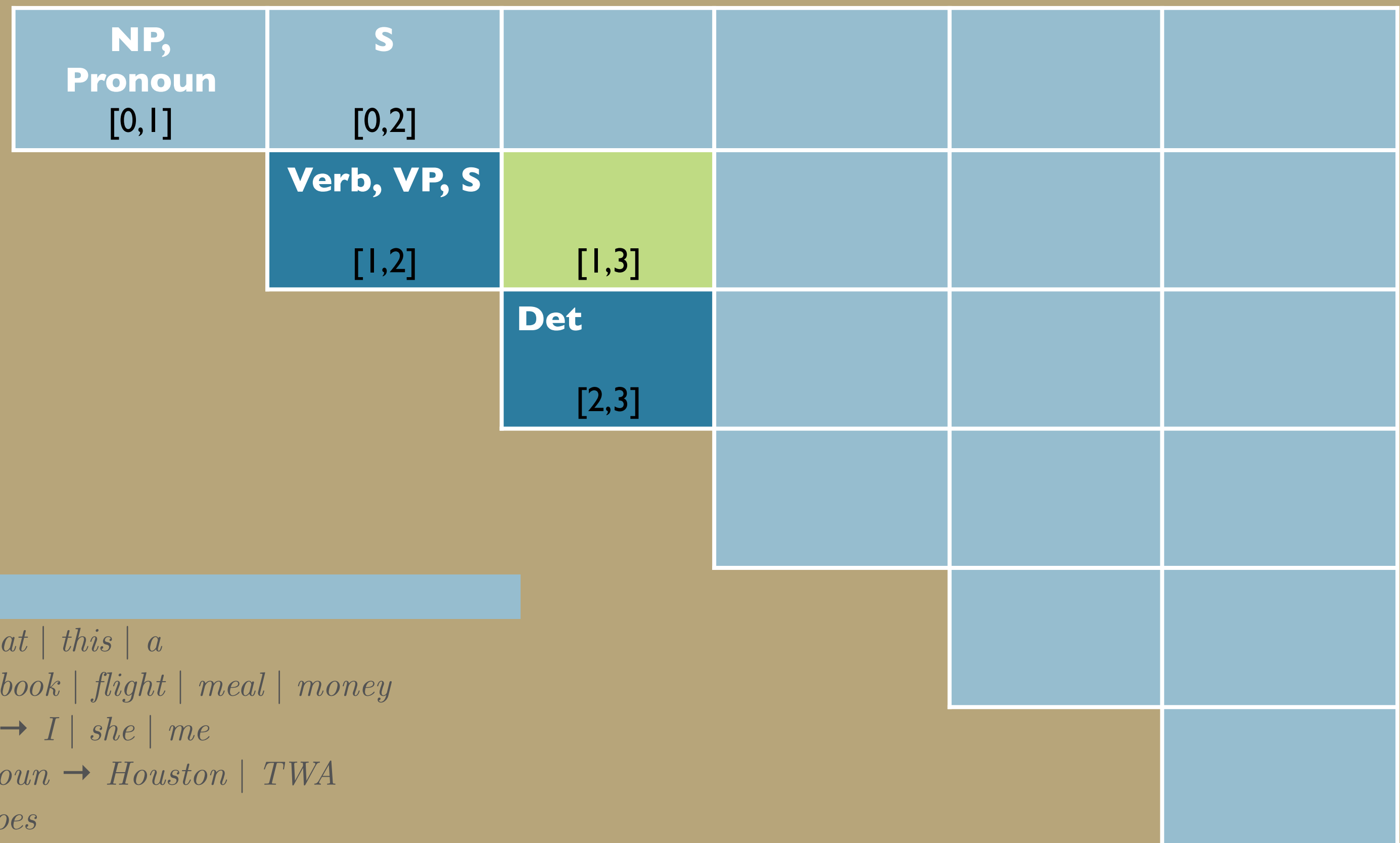
Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from | to | on | near | through*

Preposition → *from* | *to* | *on* | *near* | *through*
Verb → *back* | *include* | *prefer*

verb + book | include | prefer



\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \text{ VP}$$
$$X1 \rightarrow Aux\ NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X^2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP\ PP$$

$NP \rightarrow I / she / me$

NP → *TWA* / *Houston*

$NP \rightarrow Det \textit{ Nominal}$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

Nominal \rightarrow *Nominal Noun*

Nominal \rightarrow *Nominal PP*

$VP \rightarrow book / include / prefer$

$$VP \rightarrow Verb\ NP$$
$$VP \rightarrow X^2 PP$$
$$X2 \rightarrow Verb\ NP$$
$$VP \rightarrow Verb PP$$
$$VP \rightarrow VP PP$$

$PP \rightarrow \textit{Preposition NP}$

Lexicon

Det → *that* | *this* | *a*

Noun → *book* | *flight* | *meal* | *money*

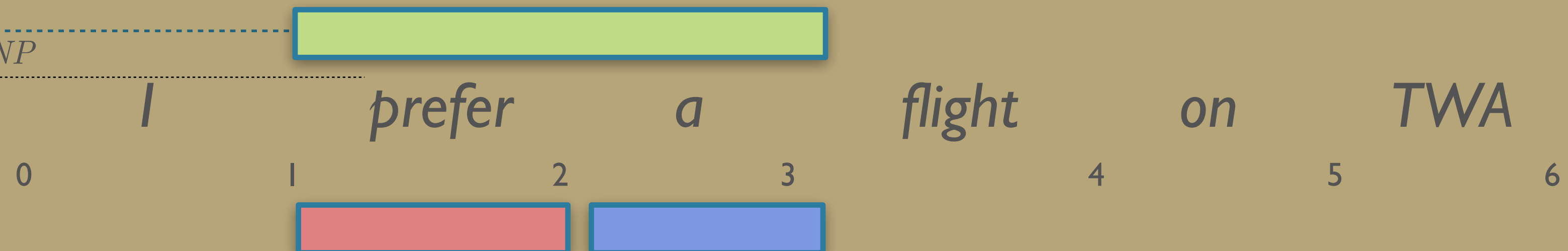
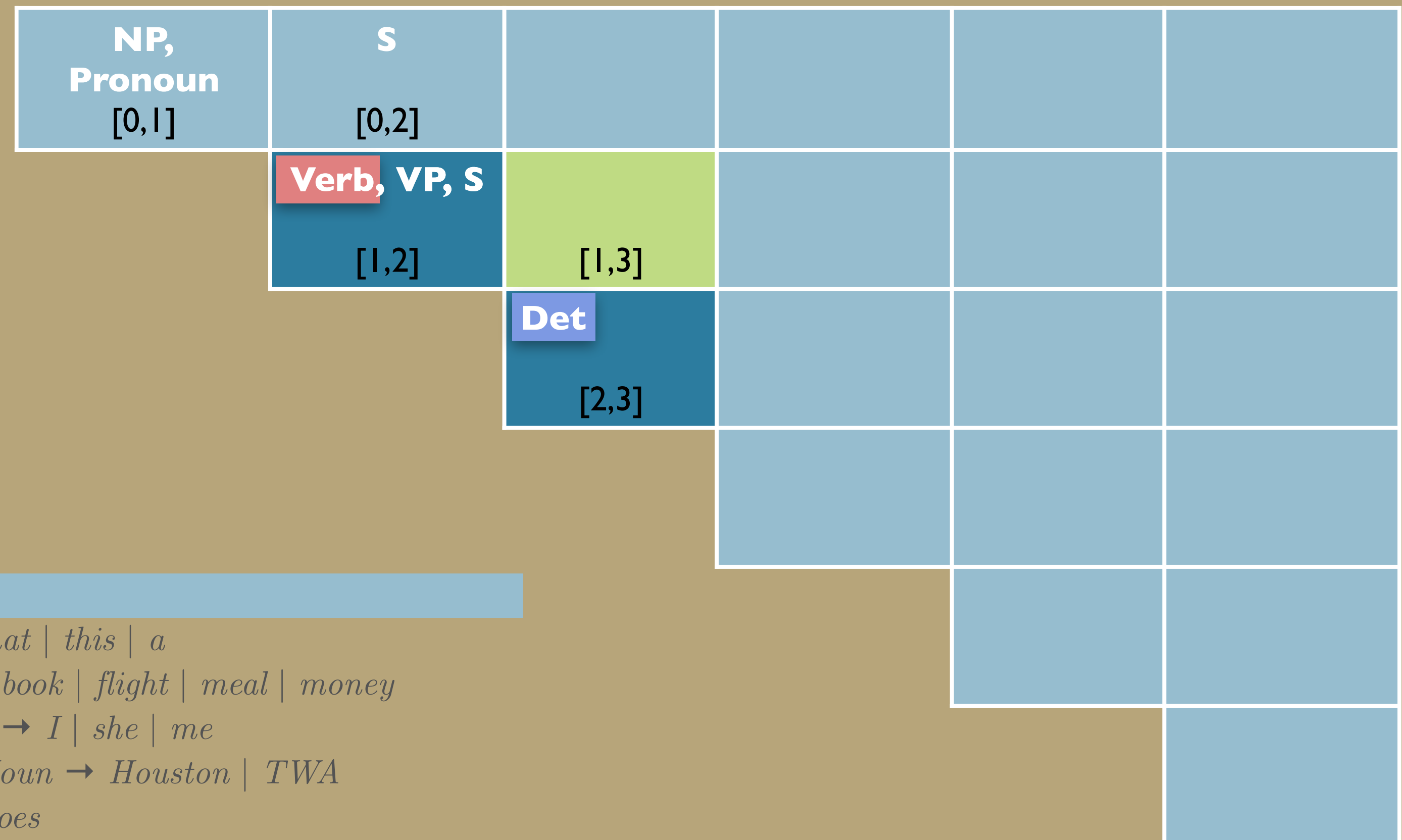
Pronoun → *I* | *she* | *me*

Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer



\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \text{ VP}$$
$$X1 \rightarrow Aux NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X^2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP\ PP$$

$NP \rightarrow I \mid she \mid me$

NP → *TWA* / *Houston*

$NP \rightarrow Det\ Nominal$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

Nominal \rightarrow *Nominal Noun*

Nominal \rightarrow *Nominal PP*

$VP \rightarrow book / include / prefer$

$$VP \rightarrow Verb\ NP$$
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$$X2 \rightarrow Verb\ NP$$
$$VP \rightarrow Verb PP$$
$$VP \rightarrow VP PP$$

$PP \rightarrow \textit{Preposition NP}$

Lexicon

$Det \rightarrow that \mid this \mid a$

Noun → *book* | *flight* | *meal* | *money*

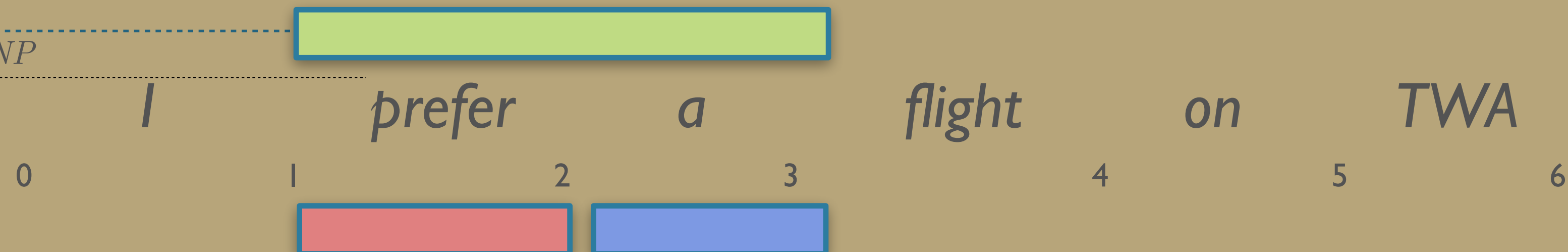
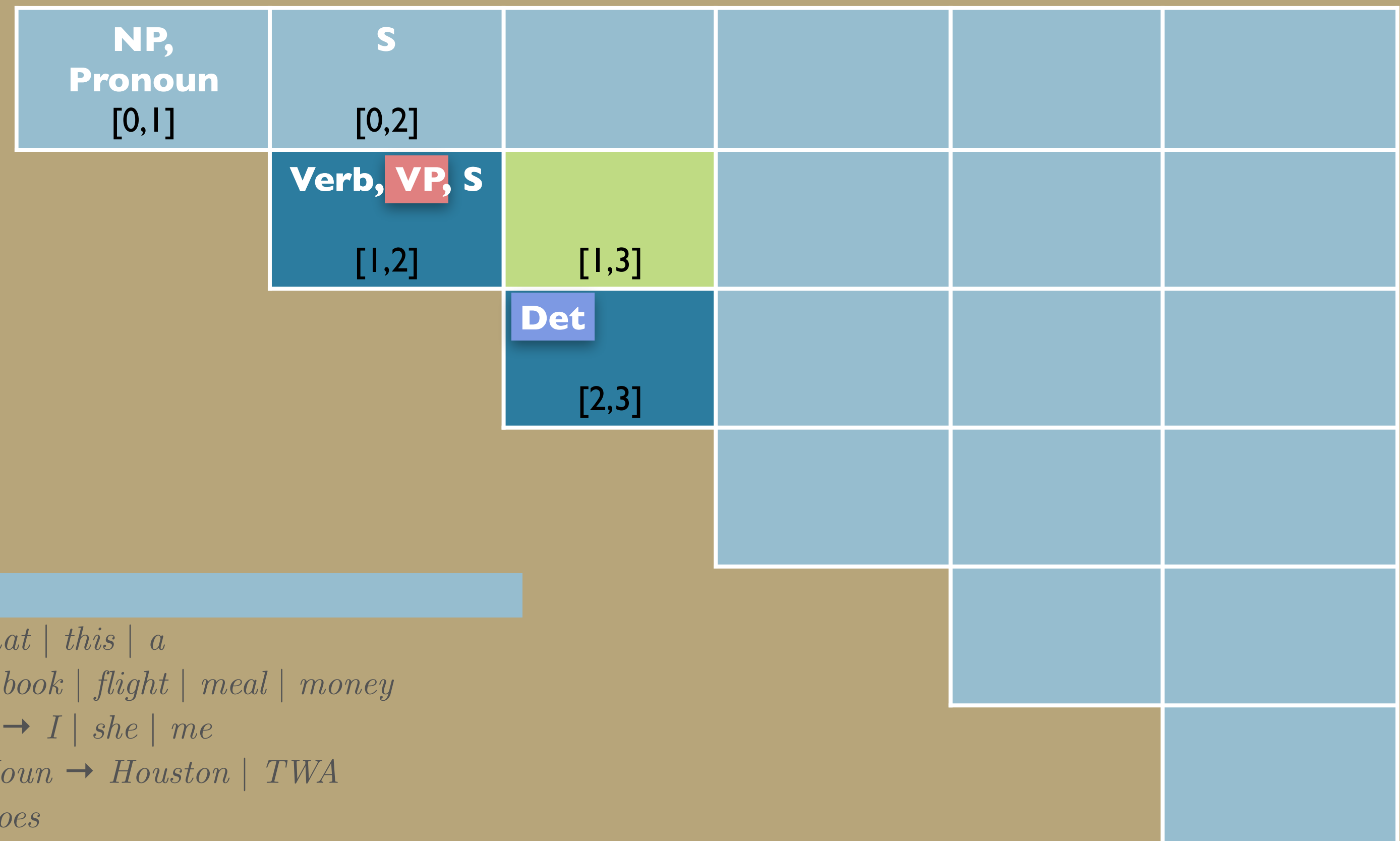
Pronoun \rightarrow I | she | me

Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer



\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \text{ } VP$$

$X1 \rightarrow Aux\ NP$

$S \rightarrow \text{book} / \text{include} / \text{prefer}$
 $G \rightarrow \text{H} / \text{L} / \text{ND}$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow \lambda^2 PP$$

$$S \rightarrow Verb PP$$
$$S \rightarrow VP PP$$

$$NP \rightarrow I \mid she \mid me$$

$NP \rightarrow TWA / Houston$

$$NP \rightarrow Det\ Nominal$$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

Nominal \rightarrow *Nominal Noun*

Nominal \rightarrow *Nominal PP*

$VP \rightarrow book / include / prefer$

$$VP \rightarrow Verb\ NP$$
$$VP \rightarrow X^2 PP$$
$$X2 \rightarrow Verb\ NP$$
$$VP \rightarrow Verb PP$$

$VP \rightarrow VP PP$

$PP \rightarrow$ *Preposition NP*

Lexicon

$Det \rightarrow that \mid this \mid a$

Noun \rightarrow *book* | *flight* | *meal* | *money*

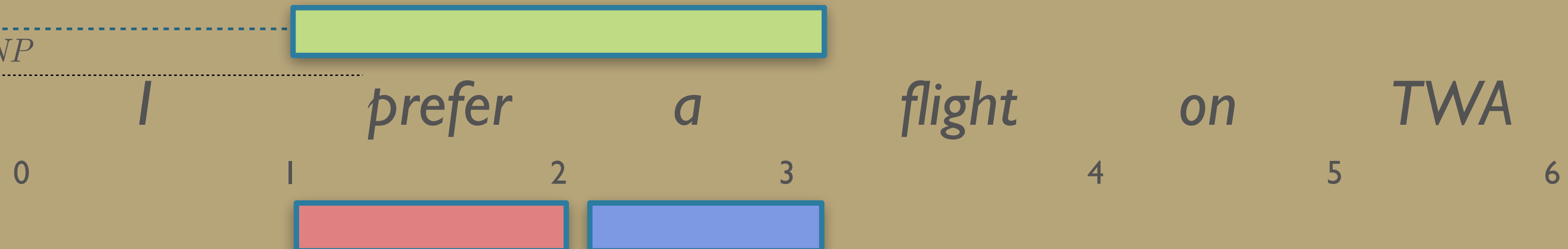
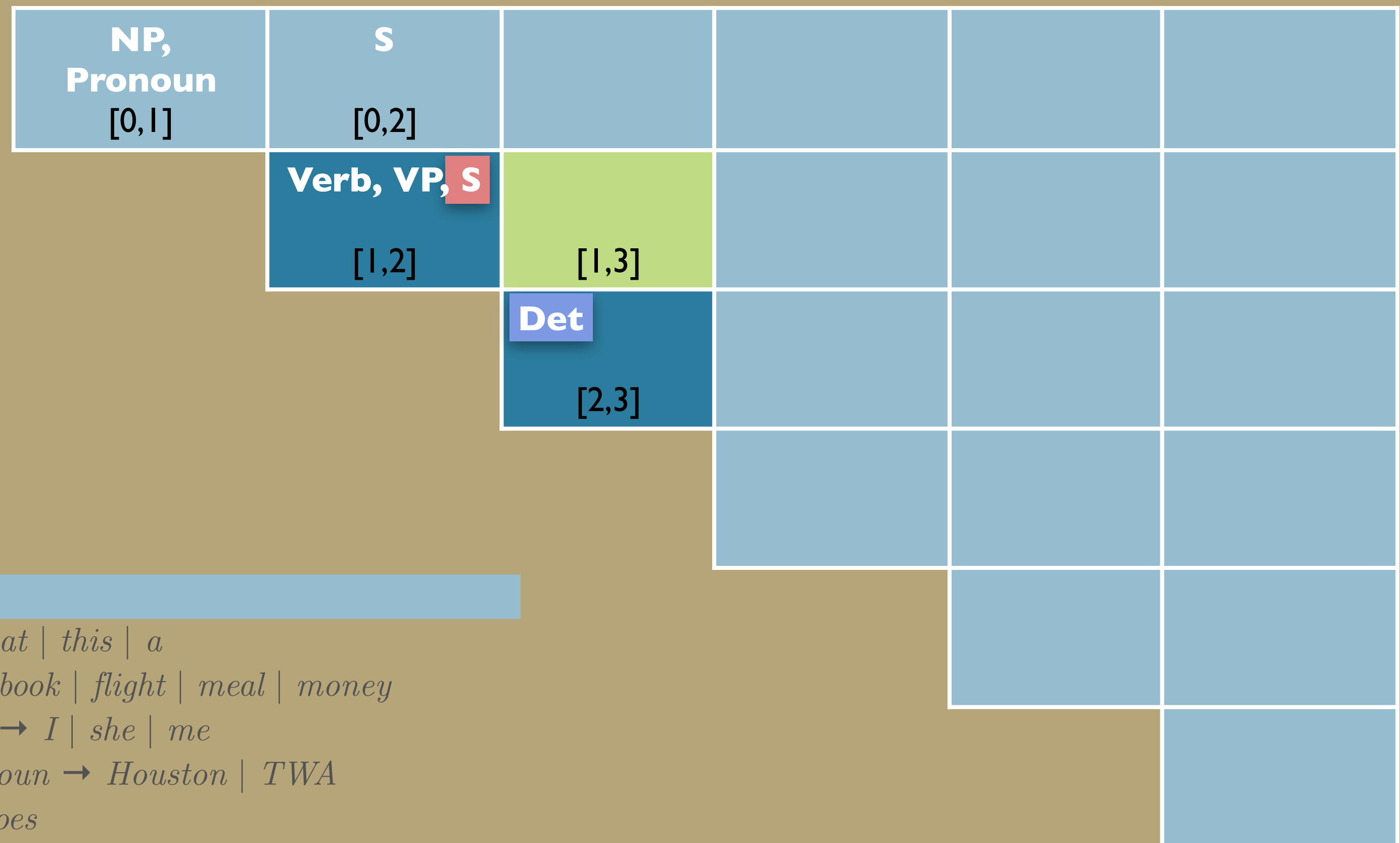
Pronoun → *I* | *she* | *me*

Proper-Noun → *Houston* | *TWA*

[illegible]

Preposition → *from* | *to* | *on* | *near* | *through*
Verb → *back* | *include* | *prefer*

verb → *book* | *include* | *prefer*



\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \text{ VP}$$
$$X1 \rightarrow Aux NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X_2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP\ PP$$

$NP \rightarrow I \mid she \mid me$

NP → *TWA* / *Houston*

$NP \rightarrow Det \text{ Nominal}$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

Nominal \rightarrow *Nominal Noun*

Nominal \rightarrow *Nominal PP*

$VP \rightarrow book / include / prefer$

$$VP \rightarrow Verb\ NP$$
$$VP \rightarrow X^2 PP$$
$$X2 \rightarrow Verb\ NP$$
$$VP \rightarrow Verb PP$$
$$VP \rightarrow VP PP$$

$PP \rightarrow \textit{Preposition NP}$

Lexicon

Det → *that* | *this* | *a*

Noun → *book* | *flight* | *meal* | *money*

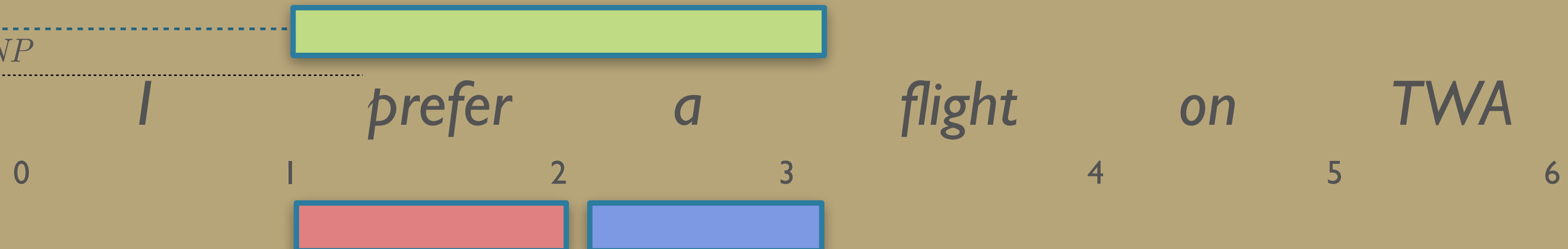
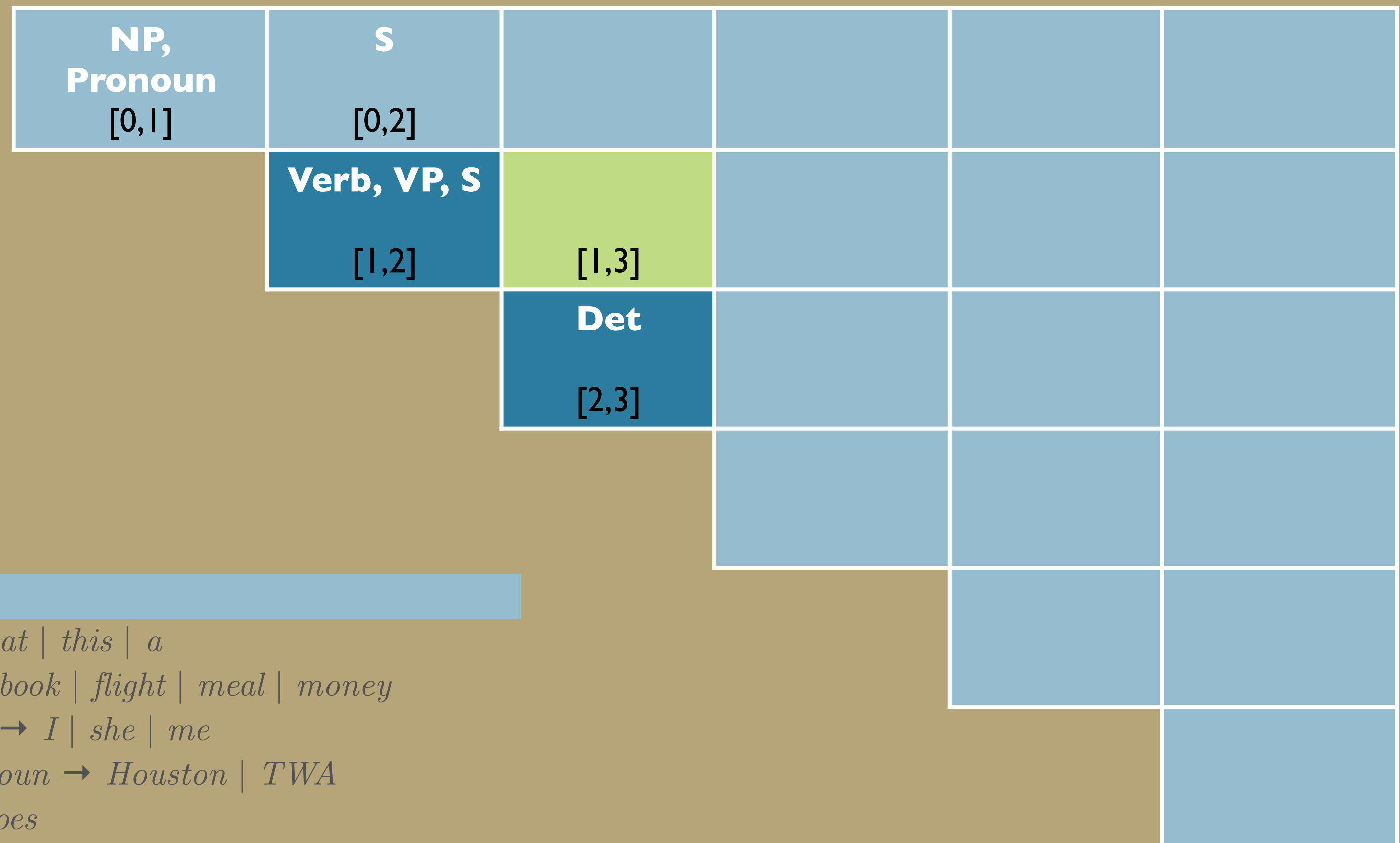
Pronoun → *I* | *she* | *me*

Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer



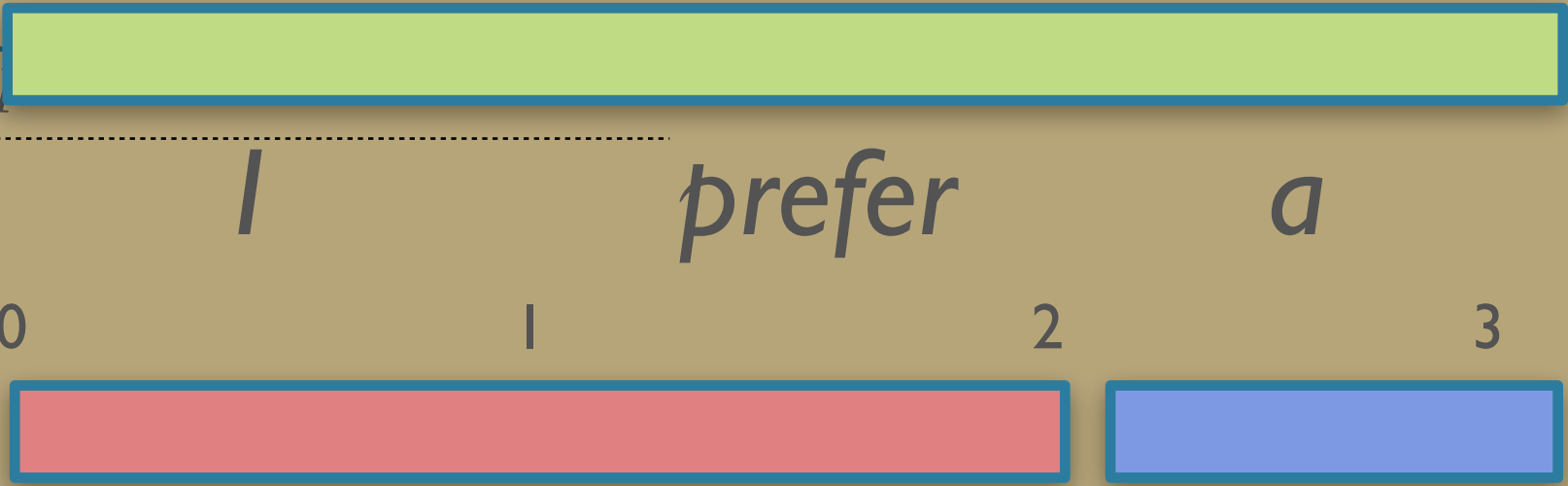
\mathcal{L}_1 Grammar

- $S \rightarrow NP\ VP$
- $S \rightarrow X1\ VP$
- $X1 \rightarrow Aux\ NP$
- $S \rightarrow book\ /\ include\ /\ prefer$
- $S \rightarrow Verb\ NP$
- $S \rightarrow X2\ PP$
- $S \rightarrow Verb\ PP$
- $S \rightarrow VP\ PP$
-
- $NP \rightarrow I\ /\ she\ /\ me$
- $NP \rightarrow TWA\ /\ Houston$
- $NP \rightarrow Det\ Nominal$
-
- $Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Nominal \rightarrow Nominal\ Noun$
- $Nominal \rightarrow Nominal\ PP$
-
- $VP \rightarrow book\ /\ include\ /\ prefer$
- $VP \rightarrow Verb\ NP$
- $VP \rightarrow X2\ PP$
- $X2 \rightarrow Verb\ NP$
- $VP \rightarrow Verb\ PP$
- $VP \rightarrow VP\ PP$
-
- $PP \rightarrow Preposition\ NP$

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]			
		Det [2,3]			

Lexicon

- $Det \rightarrow that\ /\ this\ /\ a$
- $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Pronoun \rightarrow I\ /\ she\ /\ me$
- $Proper-Noun \rightarrow Houston\ /\ TWA$
- $Aux \rightarrow does$
- $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
- $Verb \rightarrow book\ /\ include\ /\ prefer$



\mathcal{L}_1 Grammar

$$S \rightarrow NP VP$$
$$S \rightarrow X1 \quad VP$$
$$X1 \rightarrow Aux \ NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X^2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP \ PP$$

$NP \rightarrow I \mid she \mid me$

NP → *TWA* / *Houston*

$$NP \rightarrow Det \textit{ Nominal}$$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

Nominal \rightarrow *Nominal Noun*

Nominal \rightarrow *Nominal PP*

$VP \rightarrow book / include / prefer$

$$VP \rightarrow Verb\ NP$$
$$VP \rightarrow X^2 PP$$
$$X2 \rightarrow Verb NP$$
$$VP \rightarrow Verb PP$$
$$VP \rightarrow VP PP$$

$PP \rightarrow$ *Preposition Noun*

Lexicon

Det → *that* | *this* | *a*

Noun → *book* | *flight* | *meal* | *money*

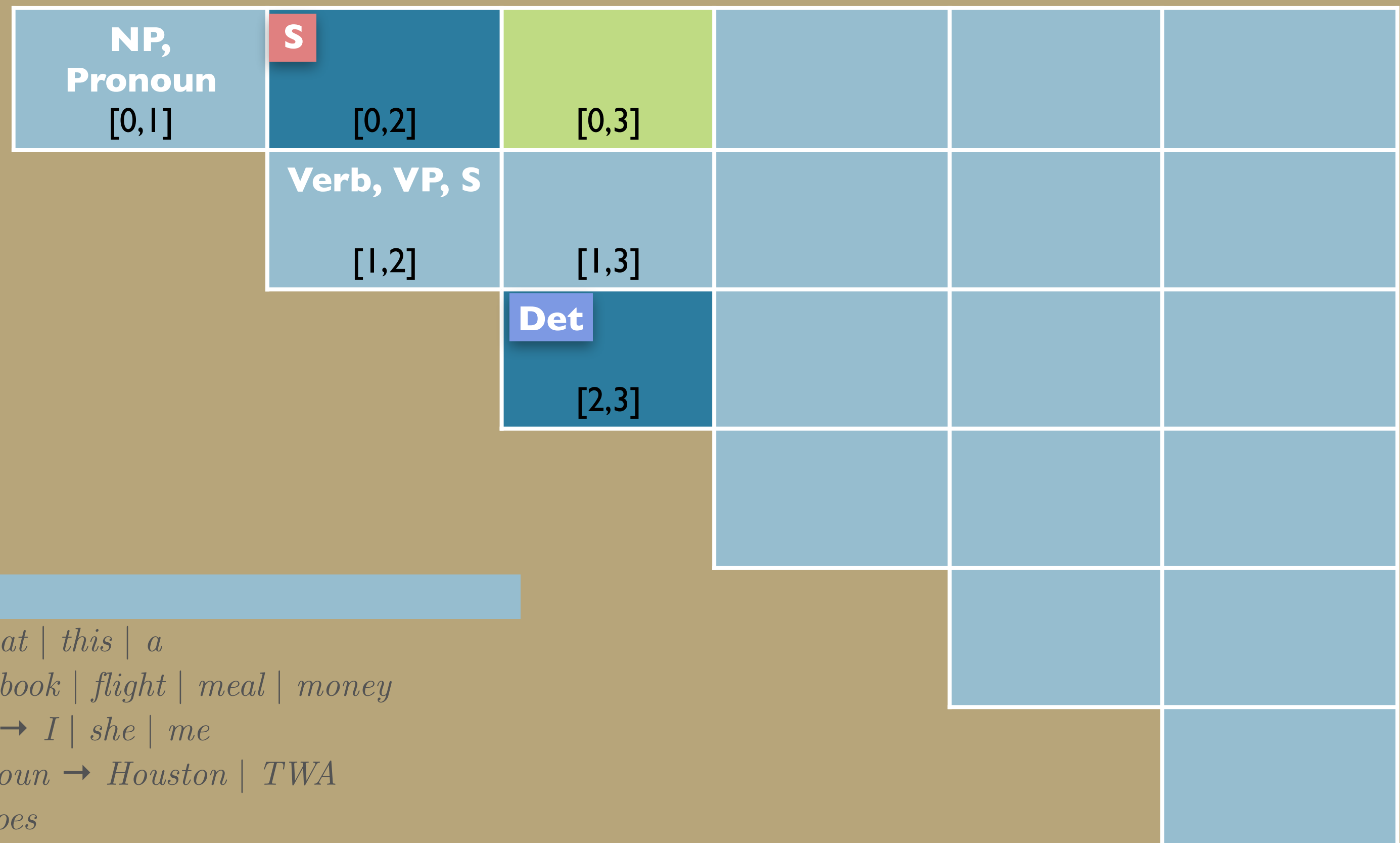
Pronoun \rightarrow *I* | *she* | *me*

Proper-Noun \rightarrow *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb → *book* | *include* | *prefer*



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* → that | this | a
Noun → book | flight | meal | money
Pronoun → I | she | me
Proper-Noun → Houston | TWA
Aux → does
Preposition → from | to | on | near | through
Verb → book | include | prefer

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]			
		Det [2,3]			
			Noun, Nom [3,4]		

IpreferaflightonTWA

0123456

\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

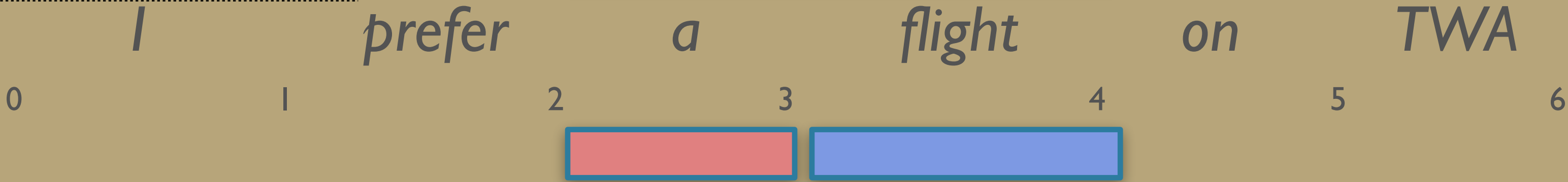
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* → *that* | *this* | *a*
Noun → *book* | *flight* | *meal* | *money*
Pronoun → *I* | *she* | *me*
Proper-Noun → *Houston* | *TWA*
Aux → *does*
Preposition → *from* | *to* | *on* | *near* | *through*
Verb → *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]			
		Det [2,3]	[2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

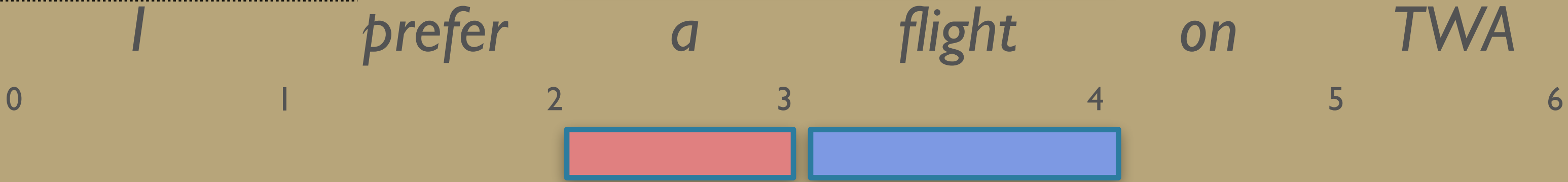
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

Det \rightarrow *that* | *this* | *a*
Noun \rightarrow *book* | *flight* | *meal* | *money*
Pronoun \rightarrow *I* | *she* | *me*
Proper-Noun \rightarrow *Houston* | *TWA*
Aux \rightarrow *does*
Preposition \rightarrow *from* | *to* | *on* | *near* | *through*
Verb \rightarrow *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]			
		Det [2,3]	[2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

 $NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

 $Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

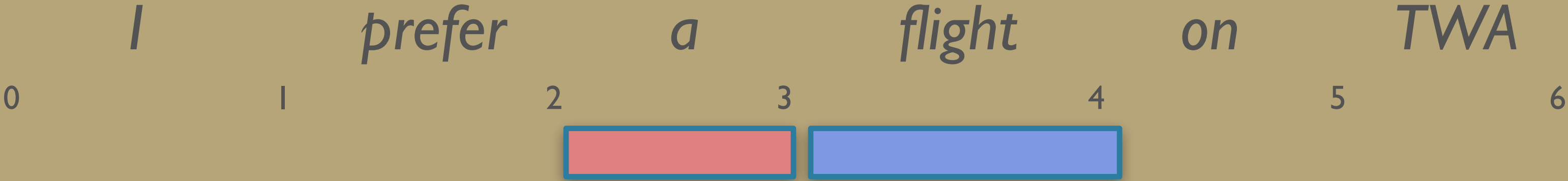
 $VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

 $PP \rightarrow Preposition\ NP$

Lexicon

- $Det \rightarrow that\ |\ this\ |\ a$
- $Noun \rightarrow book\ |\ flight\ |\ meal\ |\ money$
- $Pronoun \rightarrow I\ |\ she\ |\ me$
- $Proper-Noun \rightarrow Houston\ |\ TWA$
- $Aux \rightarrow does$
- $Preposition \rightarrow from\ |\ to\ |\ on\ |\ near\ |\ through$
- $Verb \rightarrow book\ |\ include\ |\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]			
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

- $S \rightarrow NP\ VP$
- $S \rightarrow X1\ VP$
- $X1 \rightarrow Aux\ NP$
- $S \rightarrow book\ /\ include\ /\ prefer$
- $S \rightarrow Verb\ NP$
- $S \rightarrow X2\ PP$
- $S \rightarrow Verb\ PP$
- $S \rightarrow VP\ PP$
-
- $NP \rightarrow I\ /\ she\ /\ me$
- $NP \rightarrow TWA\ /\ Houston$
- $NP \rightarrow Det\ Nominal$
-
- $Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Nominal \rightarrow Nominal\ Noun$
- $Nominal \rightarrow Nominal\ PP$
-
- $VP \rightarrow book\ /\ include\ /\ prefer$
- $VP \rightarrow Verb\ NP$
- $VP \rightarrow X2\ PP$
- $X2 \rightarrow Verb\ NP$
- $VP \rightarrow Verb\ PP$
- $VP \rightarrow VP\ PP$
-
- $PP \rightarrow Preposition\ NP$

NP, Pronoun [0,1]	S [0,2]				
	Verb, VP, S [1,2]				
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		

Lexicon

- $Det \rightarrow that\ /\ this\ /\ a$
- $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
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- $Aux \rightarrow does$
- $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
- $Verb \rightarrow book\ /\ include\ /\ prefer$



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
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 $S \rightarrow Verb\ NP$
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 $NP \rightarrow I\ /\ she\ /\ me$
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 $Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

 $VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

 $PP \rightarrow Preposition\ NP$

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Pronoun \rightarrow I\ /\ she\ /\ me$
 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]	VP [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
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 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* → that | this | a
Noun → book | flight | meal | money
Pronoun → I | she | me
Proper-Noun → Houston | TWA
Aux → does
Preposition → from | to | on | near | through
Verb → book | include | prefer

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]	VP, X2 [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

- $S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

 $NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

 $Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

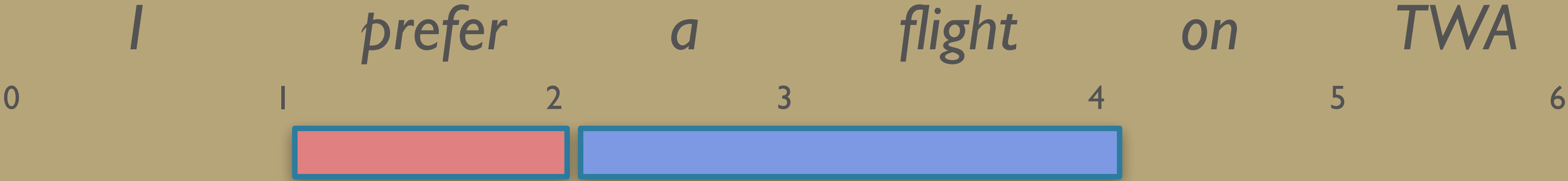
 $VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

 $PP \rightarrow Preposition\ NP$

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		

Lexicon

- $Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Pronoun \rightarrow I\ /\ she\ /\ me$
 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* \rightarrow *that* | *this* | *a*
Noun \rightarrow *book* | *flight* | *meal* | *money*
Pronoun \rightarrow *I* | *she* | *me*
Proper-Noun \rightarrow *Houston* | *TWA*
Aux \rightarrow *does*
Preposition \rightarrow *from* | *to* | *on* | *near* | *through*
Verb \rightarrow *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det → that | this | a
- Noun → book | flight | meal | money
- Pronoun → I | she | me
- Proper-Noun → Houston | TWA
- Aux → does
- Preposition → from | to | on | near | through
- Verb → book | include | prefer

NP, Pronoun [0,1]	S [0,2]				
	Verb, VP, S [1,2]		VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

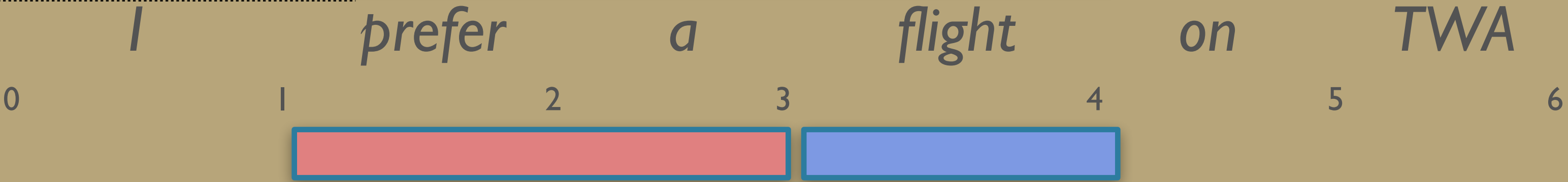
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* \rightarrow *that* | *this* | *a*
Noun \rightarrow *book* | *flight* | *meal* | *money*
Pronoun \rightarrow *I* | *she* | *me*
Proper-Noun \rightarrow *Houston* | *TWA*
Aux \rightarrow *does*
Preposition \rightarrow *from* | *to* | *on* | *near* | *through*
Verb \rightarrow *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

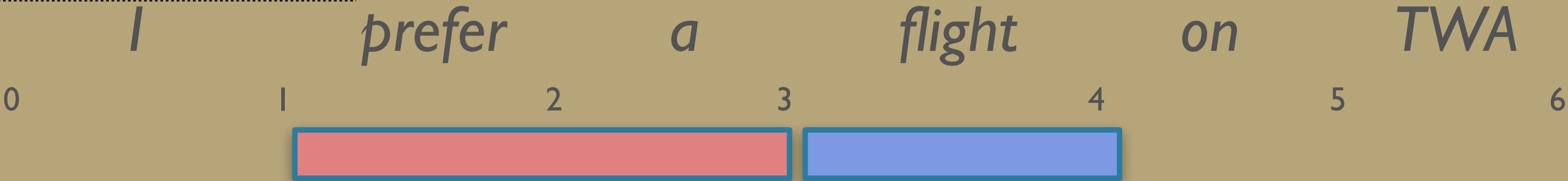
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Pronoun \rightarrow I\ /\ she\ /\ me$
 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]			
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



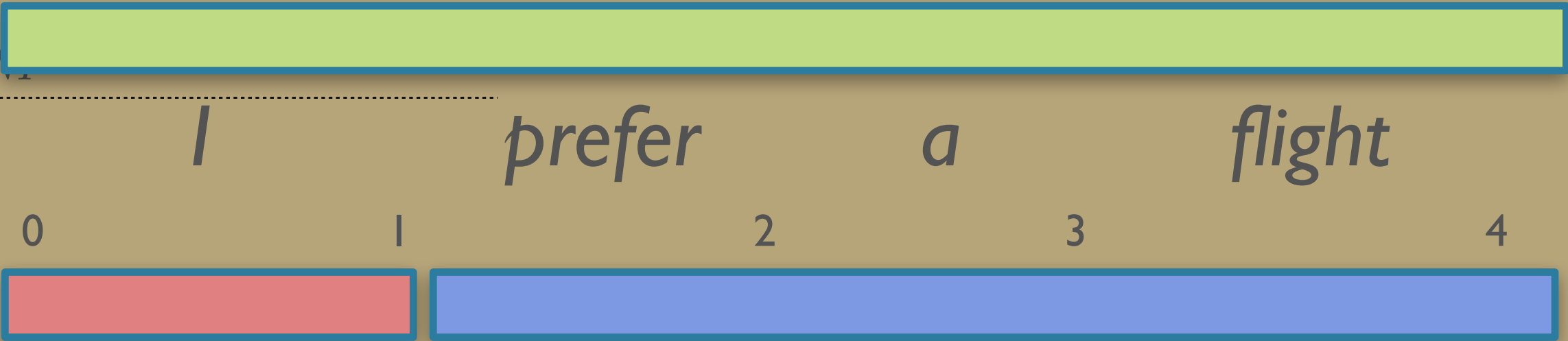
\mathcal{L}_1 Grammar

- $S \rightarrow NP\ VP$
- $S \rightarrow X1\ VP$
- $X1 \rightarrow Aux\ NP$
- $S \rightarrow book\ /\ include\ /\ prefer$
- $S \rightarrow Verb\ NP$
- $S \rightarrow X2\ PP$
- $S \rightarrow Verb\ PP$
- $S \rightarrow VP\ PP$
-
- $NP \rightarrow I\ /\ she\ /\ me$
- $NP \rightarrow TWA\ /\ Houston$
- $NP \rightarrow Det\ Nominal$
-
- $Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Nominal \rightarrow Nominal\ Noun$
- $Nominal \rightarrow Nominal\ PP$
-
- $VP \rightarrow book\ /\ include\ /\ prefer$
- $VP \rightarrow Verb\ NP$
- $VP \rightarrow X2\ PP$
- $X2 \rightarrow Verb\ NP$
- $VP \rightarrow Verb\ PP$
- $VP \rightarrow VP\ PP$
-
- $PP \rightarrow Preposition\ NP$

Lexicon

- $Det \rightarrow that\ /\ this\ /\ a$
- $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Pronoun \rightarrow I\ /\ she\ /\ me$
- $Proper-Noun \rightarrow Houston\ /\ TWA$
- $Aux \rightarrow does$
- $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
- $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	[0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP VP$

$$S \rightarrow X1 \quad VP$$
$$X1 \rightarrow Aux \ NP$$

$S \rightarrow book / include / prefer$

$$S \rightarrow Verb\ NP$$
$$S \rightarrow X^2 PP$$
$$S \rightarrow Verb PP$$
$$S \rightarrow VP \ PP$$

$NP \rightarrow I \mid she \mid me$

NP → *TWA* / *Houston*

$$NP \rightarrow Det \textit{ Nominal}$$

Nominal \rightarrow *book* / *flight* / *meal* / *money*

Nominal \rightarrow *Nominal Noun*

Nominal \rightarrow *Nominal PP*

$VP \rightarrow book / include / prefer$

$$VP \rightarrow Verb\ NP$$
$$VP \rightarrow X^2 PP$$
$$X2 \rightarrow Verb NP$$
$$VP \rightarrow Verb PP$$
$$VP \rightarrow VP \ PP$$

$PP \rightarrow$ *Preposition* N

NP, Pronoun [0,1]

S

[0,2]

Verb, VP, S

[1,2]

—

S

[0,4]

VP, X2, S

[1,4]

NP

[2.4]

Noun, Nom

Lexicon

Det → *that* | *this* | *a*

Noun → *book* | *flight* | *meal* | *money*

Pronoun \rightarrow *I* | *she* | *me*

Proper-Noun → *Houston* | *TWA*

Aux → *does*

Preposition → *from* | *to* | *on* | *near* | *through*

Verb \rightarrow book | include | prefer

Diagram illustrating a 1D lattice with 5 sites (0 to 4). The lattice is divided into two regions: a red region (sites 0 to 1) and a blue region (sites 2 to 4). The sites are labeled with their indices (0, 1, 2, 3, 4) and the corresponding fermion states (I , $prefer$, a , $flight$). A green bar is shown above the lattice, and a dashed line is drawn between the top and bottom bars.

I prefer a flight on TWA

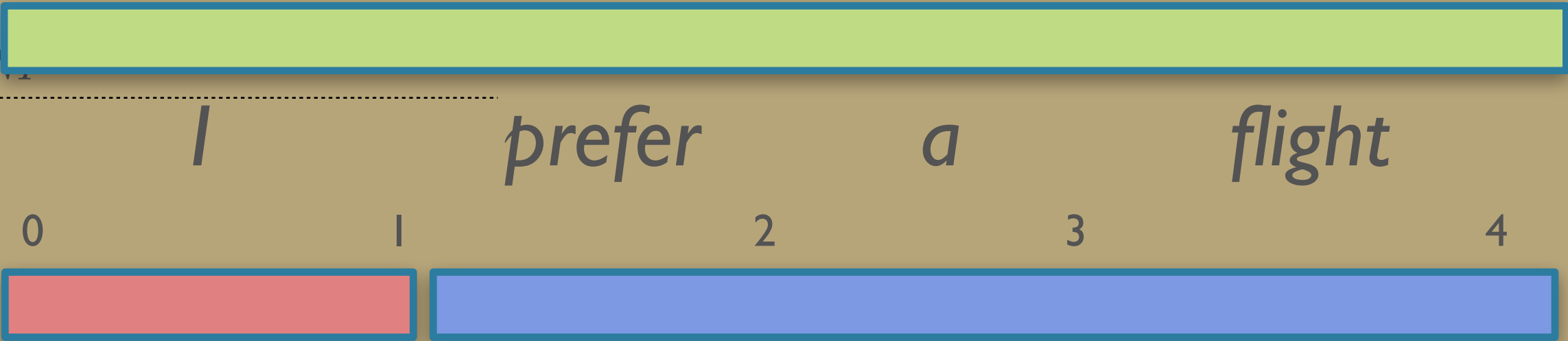
\mathcal{L}_1 Grammar

- $S \rightarrow NP\ VP$
- $S \rightarrow X1\ VP$
- $X1 \rightarrow Aux\ NP$
- $S \rightarrow book\ /\ include\ /\ prefer$
- $S \rightarrow Verb\ NP$
- $S \rightarrow X2\ PP$
- $S \rightarrow Verb\ PP$
- $S \rightarrow VP\ PP$
-
- $NP \rightarrow I\ /\ she\ /\ me$
- $NP \rightarrow TWA\ /\ Houston$
- $NP \rightarrow Det\ Nominal$
-
- $Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Nominal \rightarrow Nominal\ Noun$
- $Nominal \rightarrow Nominal\ PP$
-
- $VP \rightarrow book\ /\ include\ /\ prefer$
- $VP \rightarrow Verb\ NP$
- $VP \rightarrow X2\ PP$
- $X2 \rightarrow Verb\ NP$
- $VP \rightarrow Verb\ PP$
- $VP \rightarrow VP\ PP$
-
- $PP \rightarrow Preposition\ NP$

NP, Pronoun [0,1]	S [0,2]		S [0,4]		
	Verb, VP, S [1,2]		VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		

Lexicon

- $Det \rightarrow that\ /\ this\ /\ a$
- $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Pronoun \rightarrow I\ /\ she\ /\ me$
- $Proper-Noun \rightarrow Houston\ /\ TWA$
- $Aux \rightarrow does$
- $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
- $Verb \rightarrow book\ /\ include\ /\ prefer$



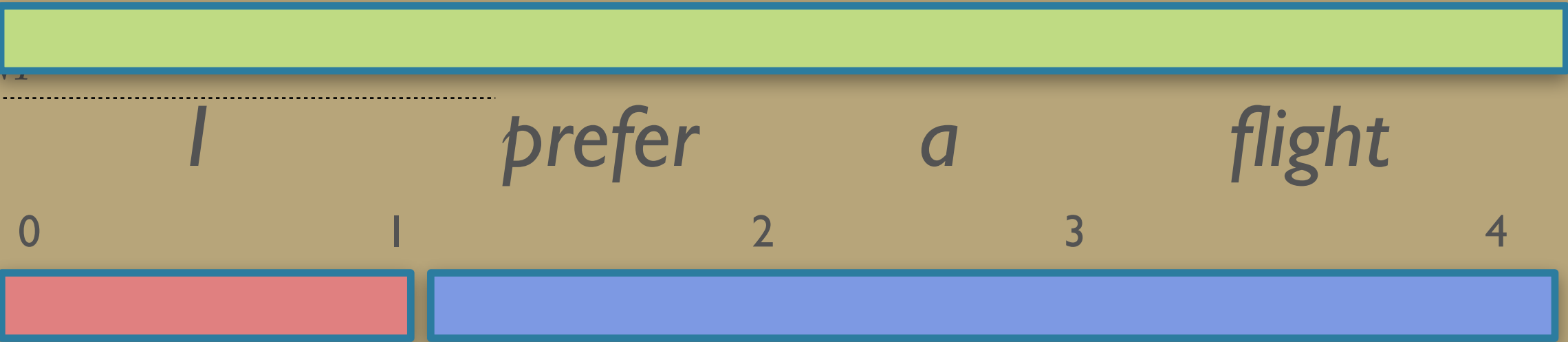
\mathcal{L}_1 Grammar

- $S \rightarrow NP\ VP$
- $S \rightarrow X1\ VP$
- $X1 \rightarrow Aux\ NP$
- $S \rightarrow book\ /\ include\ /\ prefer$
- $S \rightarrow Verb\ NP$
- $S \rightarrow X2\ PP$
- $S \rightarrow Verb\ PP$
- $S \rightarrow VP\ PP$
-
- $NP \rightarrow I\ /\ she\ /\ me$
- $NP \rightarrow TWA\ /\ Houston$
- $NP \rightarrow Det\ Nominal$
-
- $Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Nominal \rightarrow Nominal\ Noun$
- $Nominal \rightarrow Nominal\ PP$
-
- $VP \rightarrow book\ /\ include\ /\ prefer$
- $VP \rightarrow Verb\ NP$
- $VP \rightarrow X2\ PP$
- $X2 \rightarrow Verb\ NP$
- $VP \rightarrow Verb\ PP$
- $VP \rightarrow VP\ PP$
-
- $PP \rightarrow Preposition\ NP$

Lexicon

- $Det \rightarrow that\ /\ this\ /\ a$
- $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
- $Pronoun \rightarrow I\ /\ she\ /\ me$
- $Proper-Noun \rightarrow Houston\ /\ TWA$
- $Aux \rightarrow does$
- $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
- $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]		S [0,4]		
	Verb, VP, S [1,2]		VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

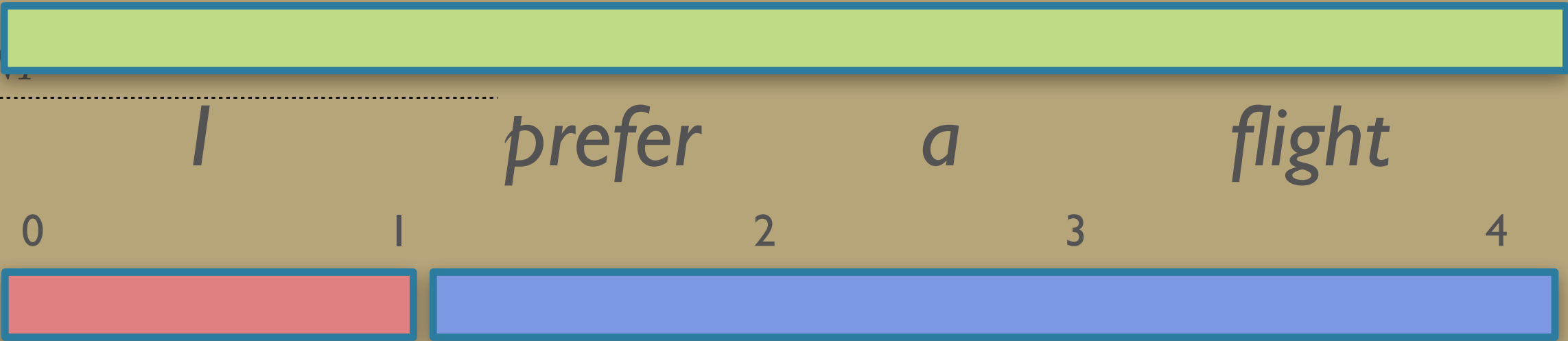
$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Pronoun \rightarrow I\ /\ she\ /\ me$
 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$



NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		

\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
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 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

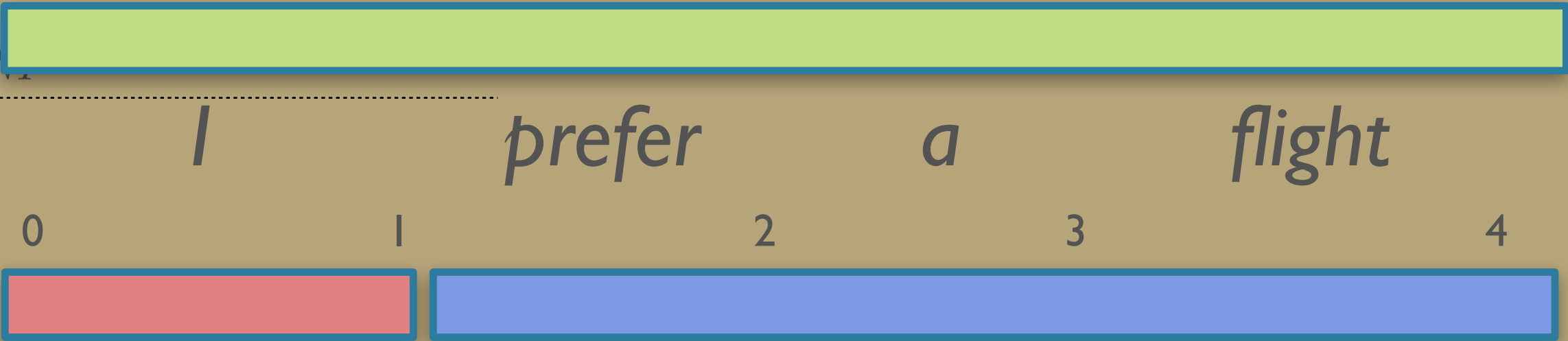
$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Pronoun \rightarrow I\ /\ she\ /\ me$
 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$



NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		

\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
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 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
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 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
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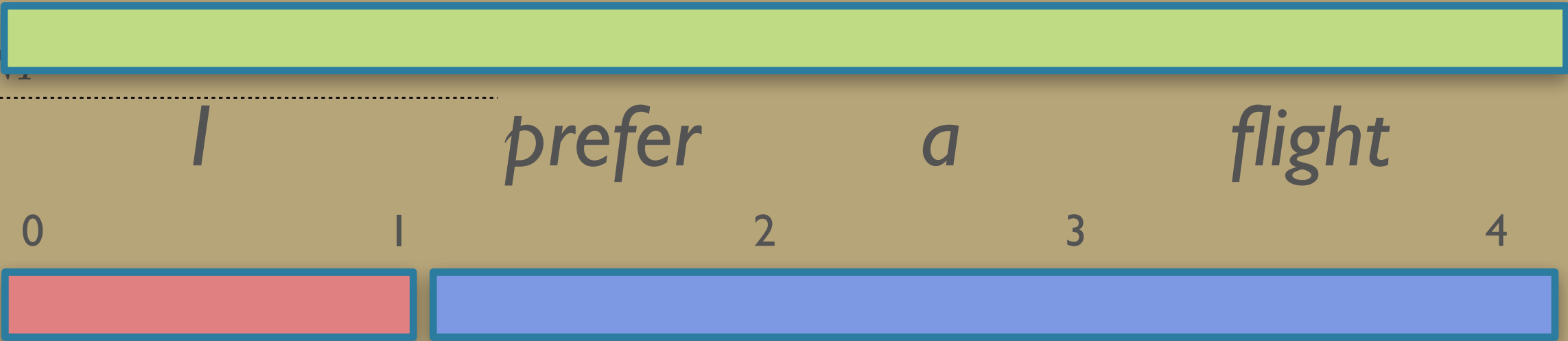
$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
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 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

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 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
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 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$



NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		

\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
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 $NP \rightarrow Det\ Nominal$

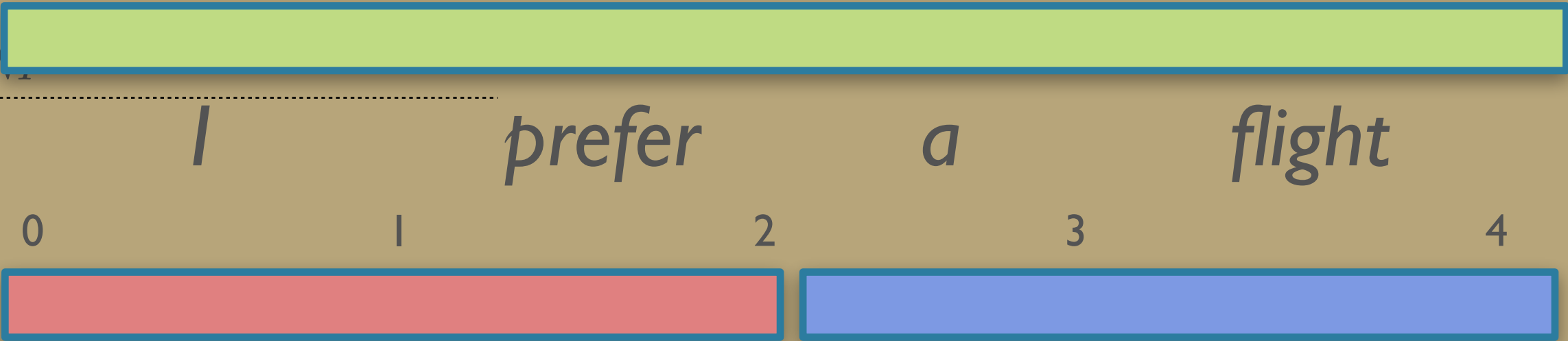
$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
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 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$



NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		

\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
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$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

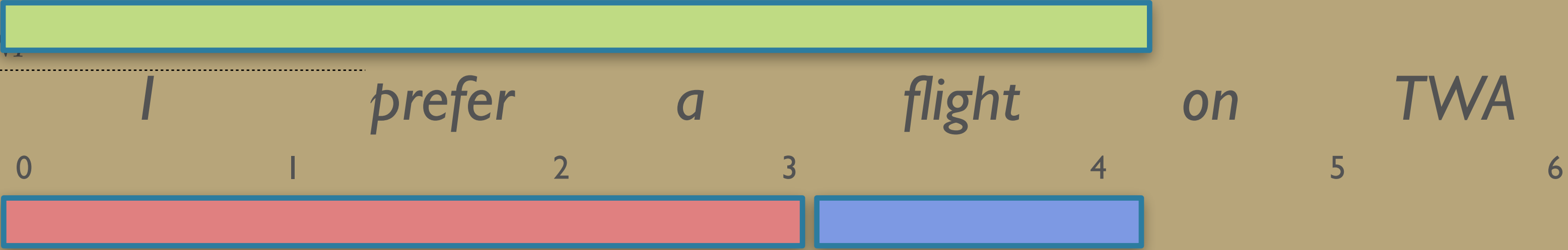
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Pronoun \rightarrow I\ /\ she\ /\ me$
 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

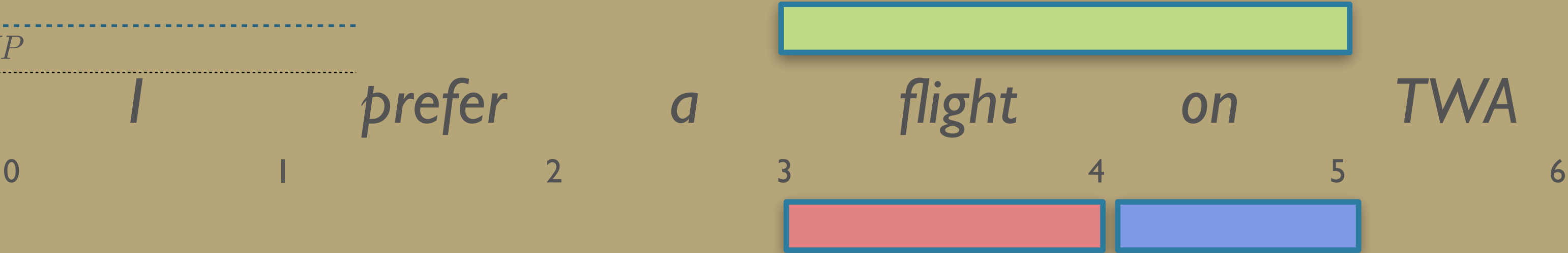
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Pronoun \rightarrow I\ /\ she\ /\ me$
 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

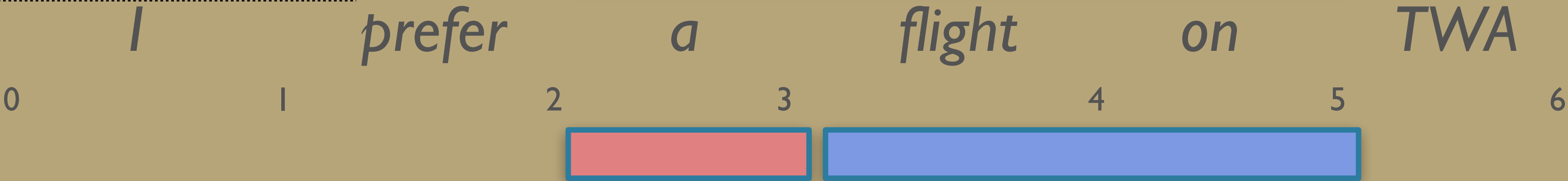
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* → *that* | *this* | *a*
Noun → *book* | *flight* | *meal* | *money*
Pronoun → *I* | *she* | *me*
Proper-Noun → *Houston* | *TWA*
Aux → *does*
Preposition → *from* | *to* | *on* | *near* | *through*
Verb → *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]	[2,5]	
			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

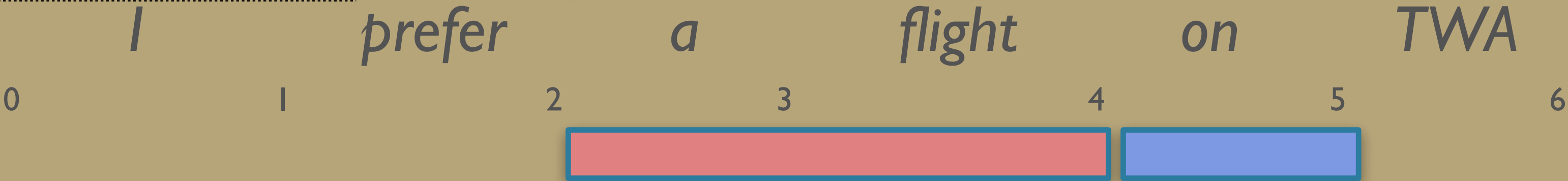
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* \rightarrow *that* | *this* | *a*
Noun \rightarrow *book* | *flight* | *meal* | *money*
Pronoun \rightarrow *I* | *she* | *me*
Proper-Noun \rightarrow *Houston* | *TWA*
Aux \rightarrow *does*
Preposition \rightarrow *from* | *to* | *on* | *near* | *through*
Verb \rightarrow *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		
				Prep [4,5]	



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$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
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$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* → *that* | *this* | *a*
Noun → *book* | *flight* | *meal* | *money*
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Proper-Noun → *Houston* | *TWA*
Aux → *does*
Preposition → *from* | *to* | *on* | *near* | *through*
Verb → *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		
				Prep [4,5]	



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$S \rightarrow NP\ VP$
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$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
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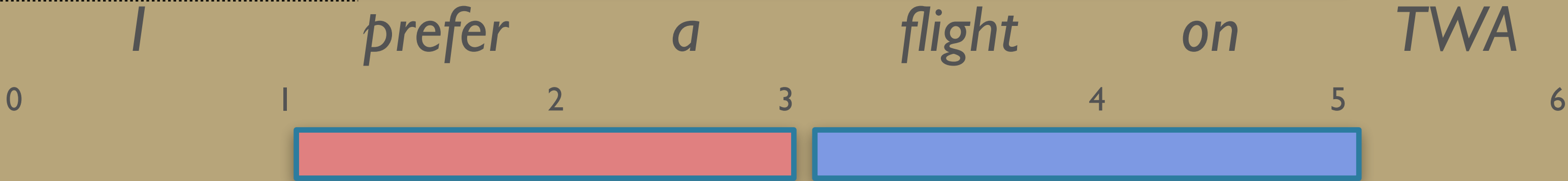
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det \rightarrow that | this | a
- Noun \rightarrow book | flight | meal | money
- Pronoun \rightarrow I | she | me
- Proper-Noun \rightarrow Houston | TWA
- Aux \rightarrow does
- Preposition \rightarrow from | to | on | near | through
- Verb \rightarrow book | include | prefer

NP, Pronoun [0,1]	S [0,2]		S [0,4]		
	Verb, VP, S [1,2]		VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		
				Prep [4,5]	



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
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$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

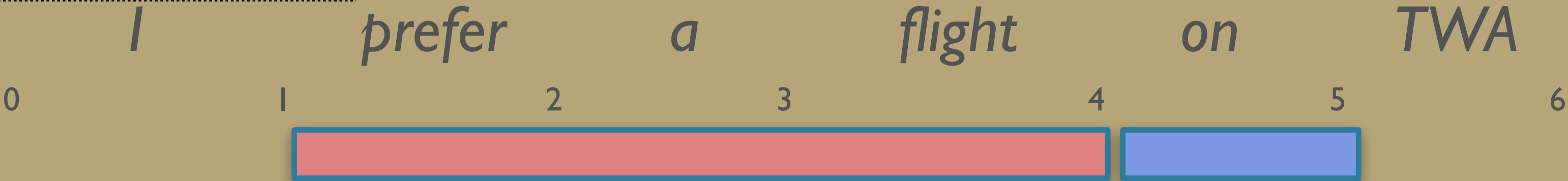
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* $\rightarrow that\ |\ this\ |\ a$
Noun $\rightarrow book\ |\ flight\ |\ meal\ |\ money$
Pronoun $\rightarrow I\ |\ she\ |\ me$
Proper-Noun $\rightarrow Houston\ |\ TWA$
Aux $\rightarrow does$
Preposition $\rightarrow from\ |\ to\ |\ on\ |\ near\ |\ through$
Verb $\rightarrow book\ |\ include\ |\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]		
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		
				Prep [4,5]	



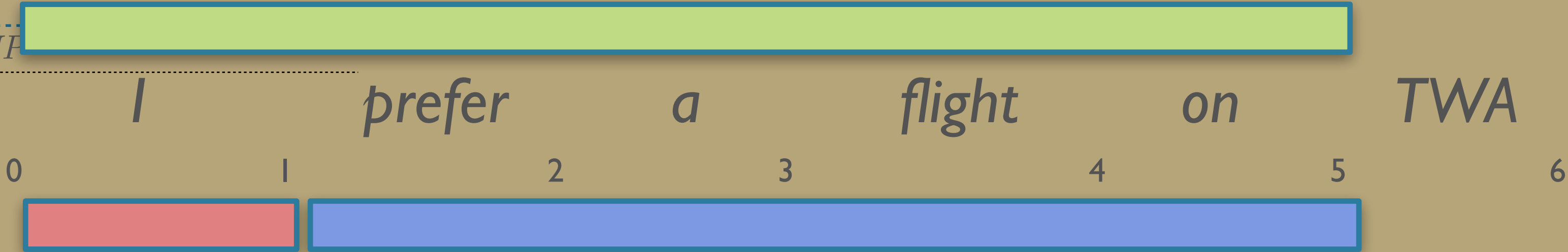
\mathcal{L}_1 Grammar

- $S \rightarrow NP VP$
- $S \rightarrow X1 VP$
- $X1 \rightarrow Aux NP$
- $S \rightarrow book \mid include \mid prefer$
- $S \rightarrow Verb NP$
- $S \rightarrow X2 PP$
- $S \rightarrow Verb PP$
- $S \rightarrow VP PP$
-
- $NP \rightarrow I \mid she \mid me$
- $NP \rightarrow TWA \mid Houston$
- $NP \rightarrow Det Nominal$
-
- $Nominal \rightarrow book \mid flight \mid meal \mid money$
- $Nominal \rightarrow Nominal Noun$
- $Nominal \rightarrow Nominal PP$
-
- $VP \rightarrow book \mid include \mid prefer$
- $VP \rightarrow Verb NP$
- $VP \rightarrow X2 PP$
- $X2 \rightarrow Verb NP$
- $VP \rightarrow Verb PP$
- $VP \rightarrow VP PP$
-
- $PP \rightarrow Preposition NP$

Lexicon

- $Det \rightarrow that \mid this \mid a$
- $Noun \rightarrow book \mid flight \mid meal \mid money$
- $Pronoun \rightarrow I \mid she \mid me$
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- $Aux \rightarrow does$
- $Preposition \rightarrow from \mid to \mid on \mid near \mid through$
- $Verb \rightarrow book \mid include \mid prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	



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 $X1 \rightarrow Aux\ NP$
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 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

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 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

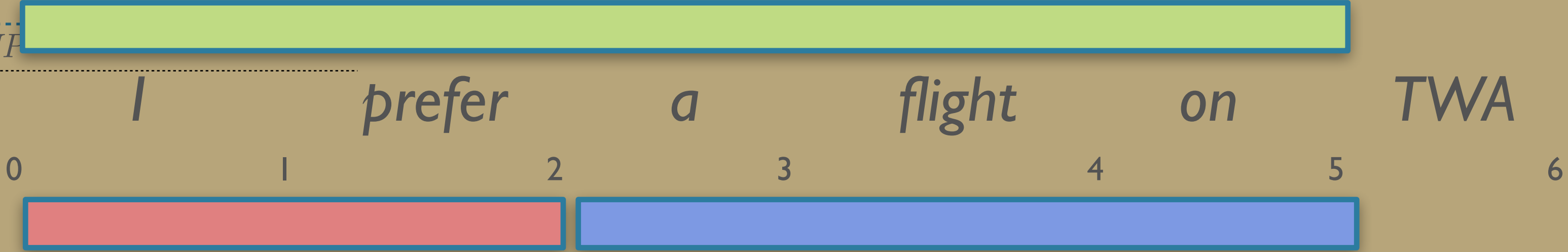
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

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 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	



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$S \rightarrow NP\ VP$
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 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

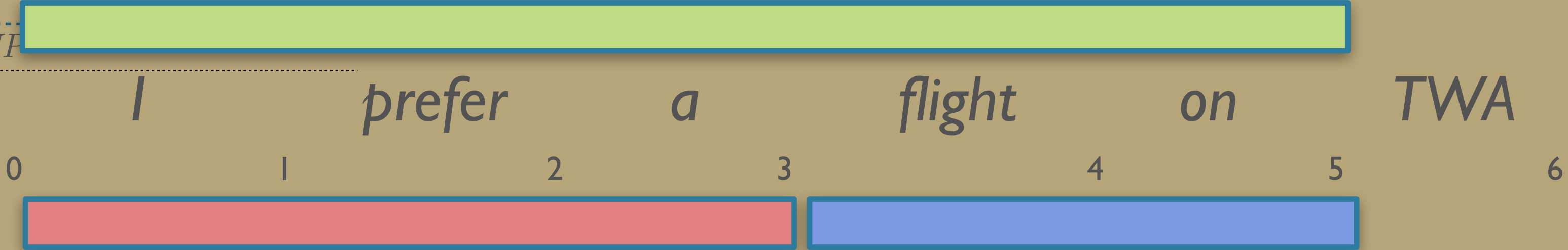
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Pronoun \rightarrow I\ /\ she\ /\ me$
 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
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 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

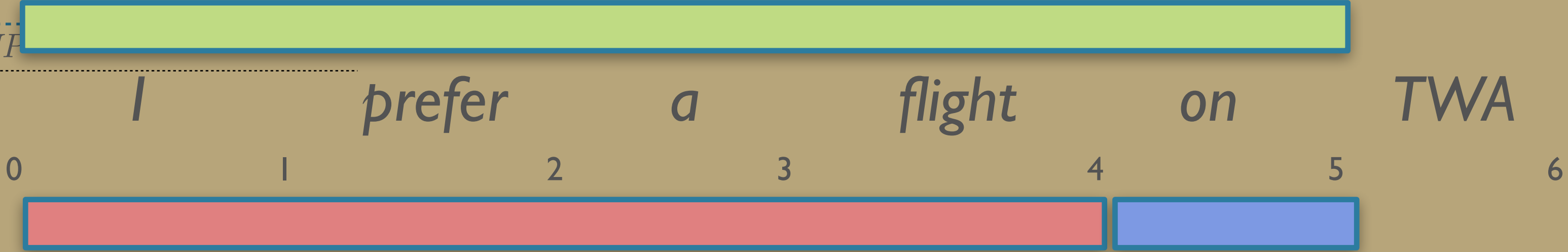
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

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 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
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 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
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$NP \rightarrow I\ /\ she\ /\ me$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

Det \rightarrow *that* | *this* | *a*
Noun \rightarrow *book* | *flight* | *meal* | *money*
Pronoun \rightarrow *I* | *she* | *me*
Proper-Noun \rightarrow *Houston* | *TWA*
Aux \rightarrow *does*
Preposition \rightarrow *from* | *to* | *on* | *near* | *through*
Verb \rightarrow *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			Noun, Nom [3,4]	[3,5]	
				Prep [4,5]	
					NNP, NP [5,6]

IpreferaflightonTWA

0123456

\mathcal{L}_1 Grammar

$S \rightarrow NP VP$
 $S \rightarrow X1 VP$
 $X1 \rightarrow Aux NP$
 $S \rightarrow book \mid include \mid prefer$
 $S \rightarrow Verb NP$
 $S \rightarrow X2 PP$
 $S \rightarrow Verb PP$
 $S \rightarrow VP PP$

 $NP \rightarrow I \mid she \mid me$
 $NP \rightarrow TWA \mid Houston$
 $NP \rightarrow Det Nominal$

 $Nominal \rightarrow book \mid flight \mid meal \mid money$
 $Nominal \rightarrow Nominal Noun$
 $Nominal \rightarrow Nominal PP$

 $VP \rightarrow book \mid include \mid prefer$
 $VP \rightarrow Verb NP$
 $VP \rightarrow X2 PP$
 $X2 \rightarrow Verb NP$
 $VP \rightarrow Verb PP$
 $VP \rightarrow VP PP$

 $PP \rightarrow$

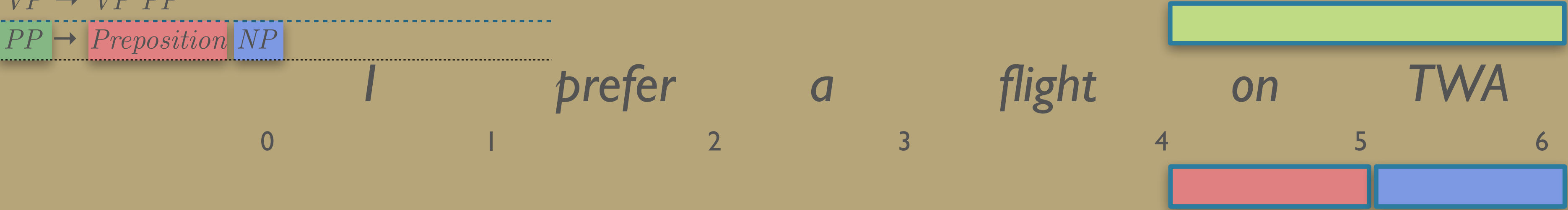
Preposition

NP

NP, Pronoun [0,1]	S [0,2]		S [0,4]		
	Verb, VP, S [1,2]		VP, X2, S [1,4]		
		Det [2,3]	NP [2,4]		
			Noun, Nom [3,4]		
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

Lexicon

$Det \rightarrow that \mid this \mid a$
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 $Nominal \rightarrow Nominal\ Noun$
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$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

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Lexicon

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Aux \rightarrow does
Preposition \rightarrow from | to | on | near | through
Verb \rightarrow book | include | prefer

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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 $VP \rightarrow Verb\ NP$
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 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

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Noun \rightarrow book | flight | meal | money
Pronoun \rightarrow I | she | me
Proper-Noun \rightarrow Houston | TWA
Aux \rightarrow does
Preposition \rightarrow from | to | on | near | through
Verb \rightarrow book | include | prefer

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
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 $S \rightarrow book\ /\ include\ /\ prefer$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
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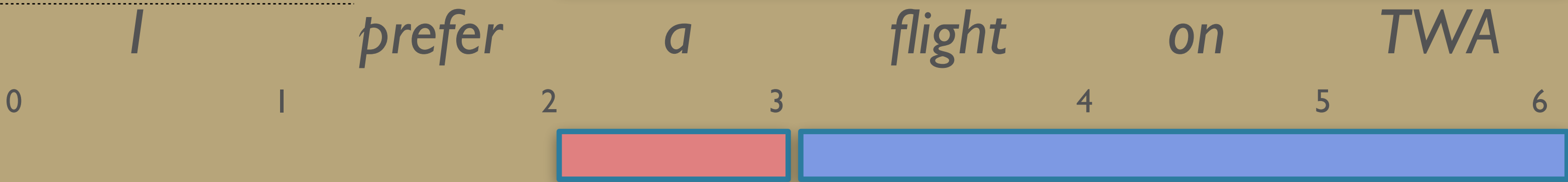
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

Det → *that* | *this* | *a*
Noun → *book* | *flight* | *meal* | *money*
Pronoun → *I* | *she* | *me*
Proper-Noun → *Houston* | *TWA*
Aux → *does*
Preposition → *from* | *to* | *on* | *near* | *through*
Verb → *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

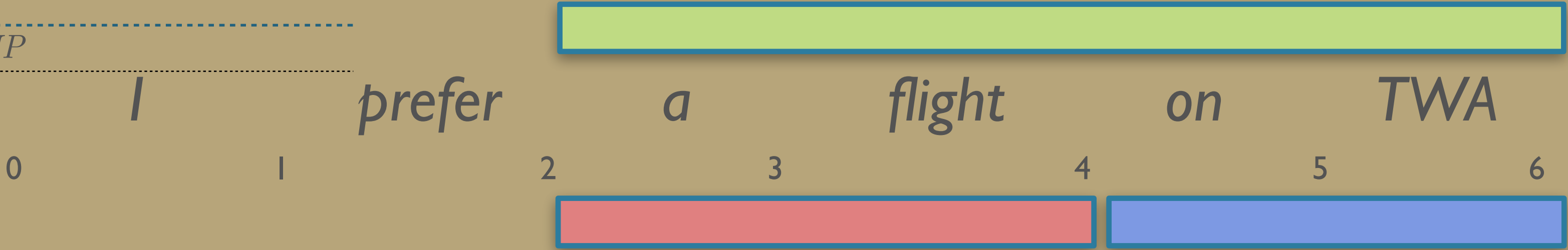
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* → *that* | *this* | *a*
Noun → *book* | *flight* | *meal* | *money*
Pronoun → *I* | *she* | *me*
Proper-Noun → *Houston* | *TWA*
Aux → *does*
Preposition → *from* | *to* | *on* | *near* | *through*
Verb → *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

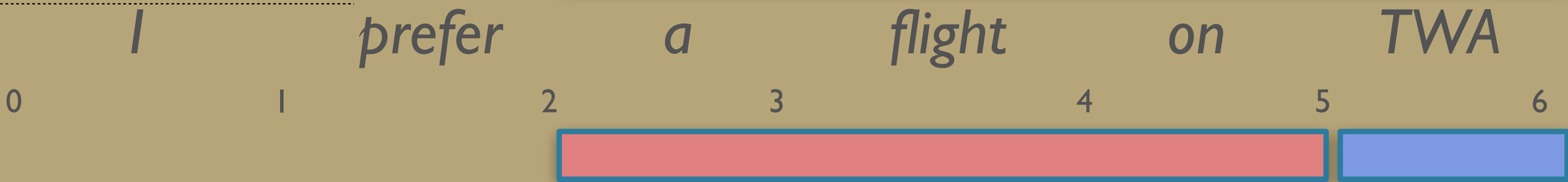
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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 $S \rightarrow book\ /\ include\ /\ prefer$
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$NP \rightarrow I\ /\ she\ /\ me$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

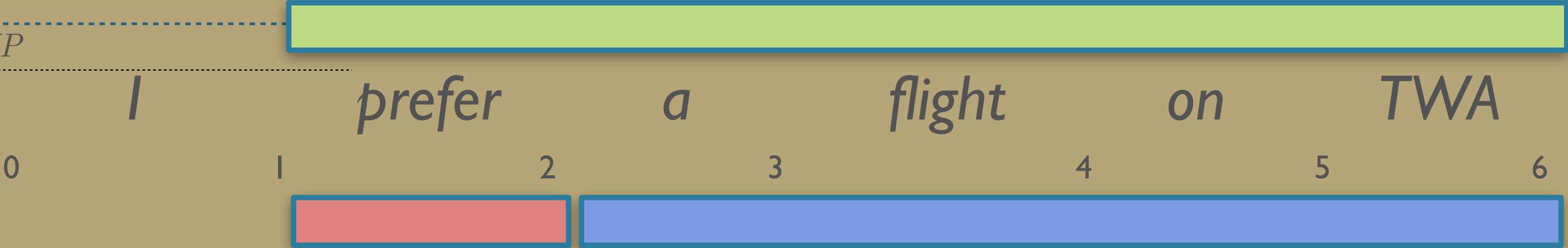
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	[1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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$S \rightarrow NP\ VP$
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 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
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 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
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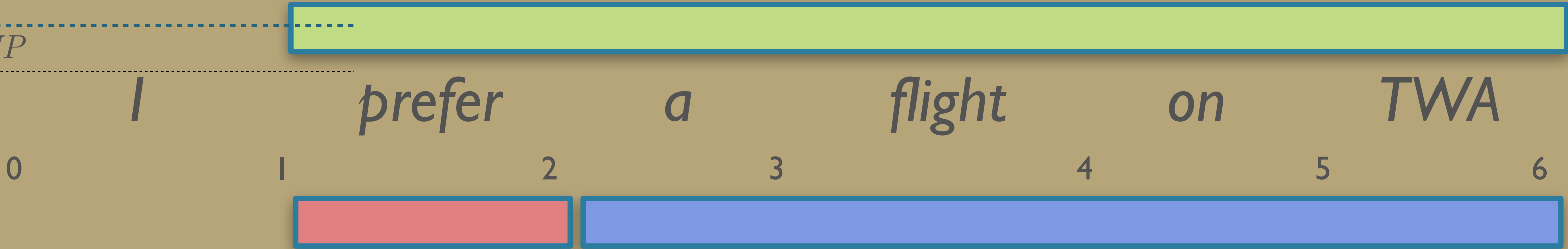
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
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 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2 [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
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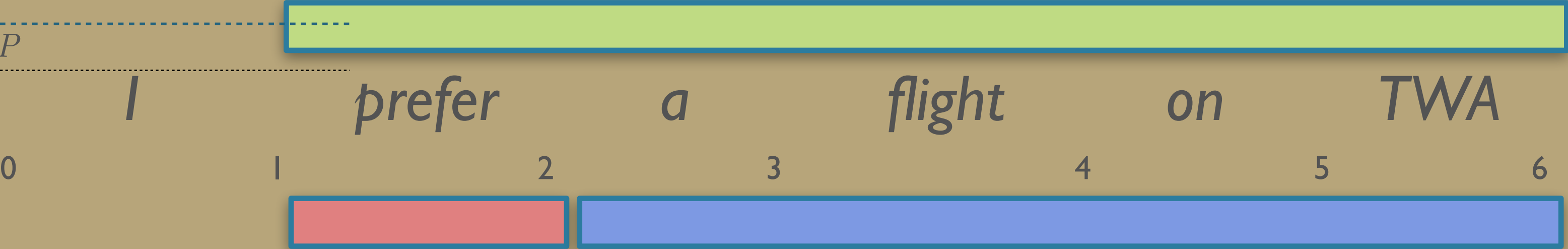
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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$S \rightarrow NP\ VP$
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$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
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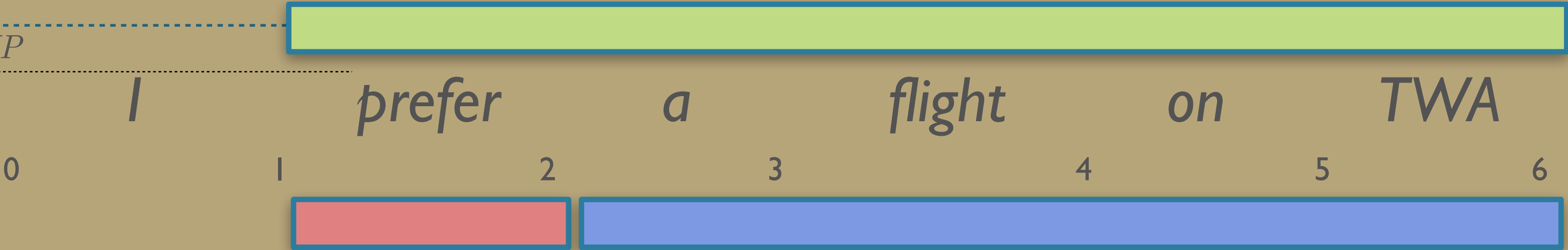
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

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 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP , S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
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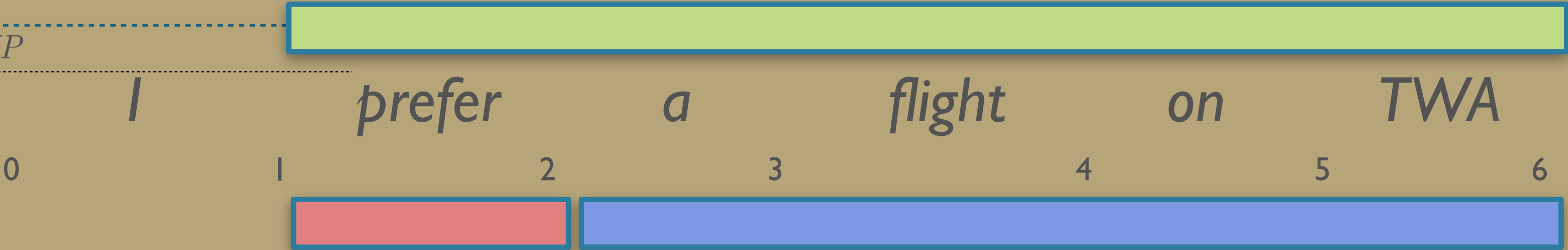
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

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 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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 $S \rightarrow VP\ PP$

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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

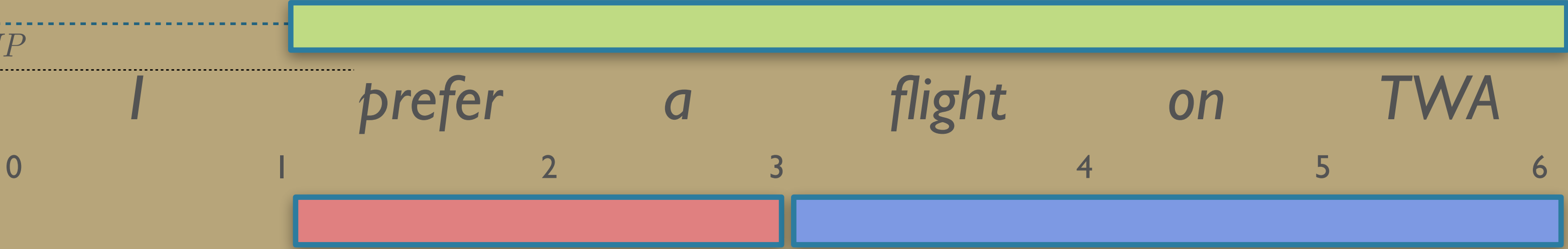
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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 $S \rightarrow book\ /\ include\ /\ prefer$
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 $S \rightarrow X2\ PP$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
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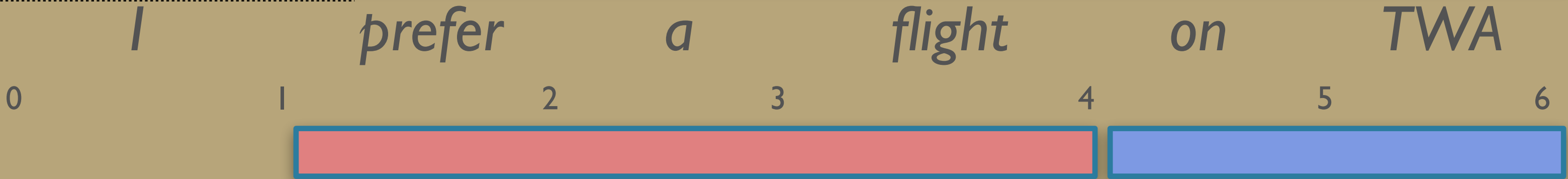
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
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 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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 $S \rightarrow Verb\ PP$

$S \rightarrow VP\ PP$

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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
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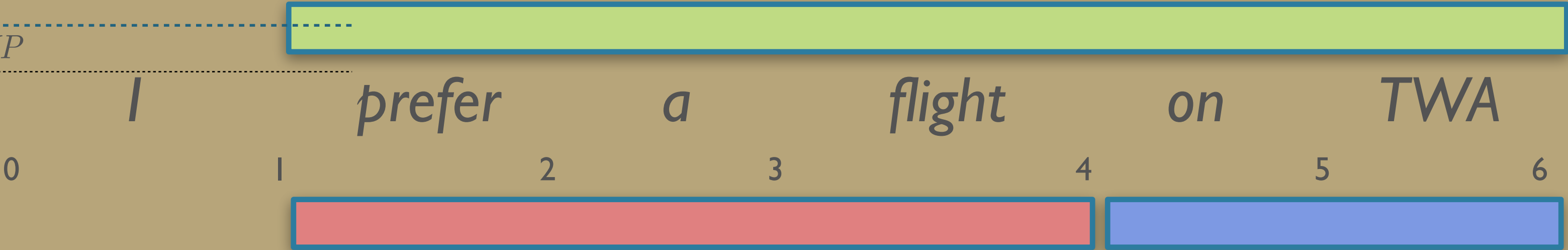
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
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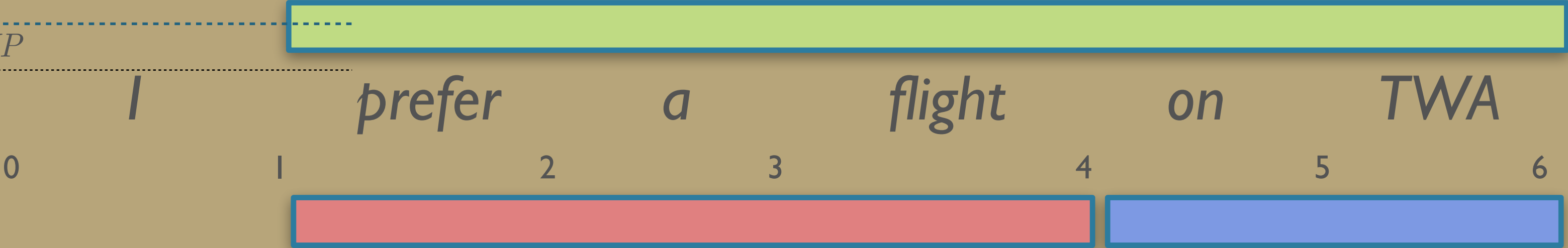
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

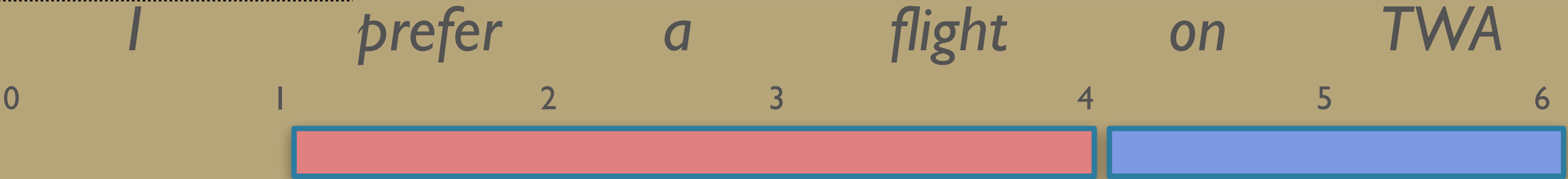
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

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 $Proper-Noun \rightarrow Houston\ |\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ |\ to\ |\ on\ |\ near\ |\ through$
 $Verb \rightarrow book\ |\ include\ |\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



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 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

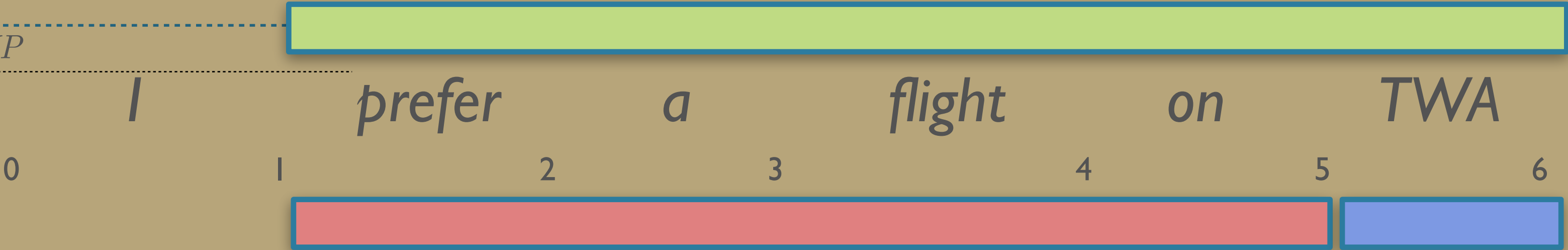
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

- Det* → *that* | *this* | *a*
Noun → *book* | *flight* | *meal* | *money*
Pronoun → *I* | *she* | *me*
Proper-Noun → *Houston* | *TWA*
Aux → *does*
Preposition → *from* | *to* | *on* | *near* | *through*
Verb → *book* | *include* | *prefer*

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
 $S \rightarrow Verb\ NP$
 $S \rightarrow X2\ PP$
 $S \rightarrow Verb\ PP$
 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

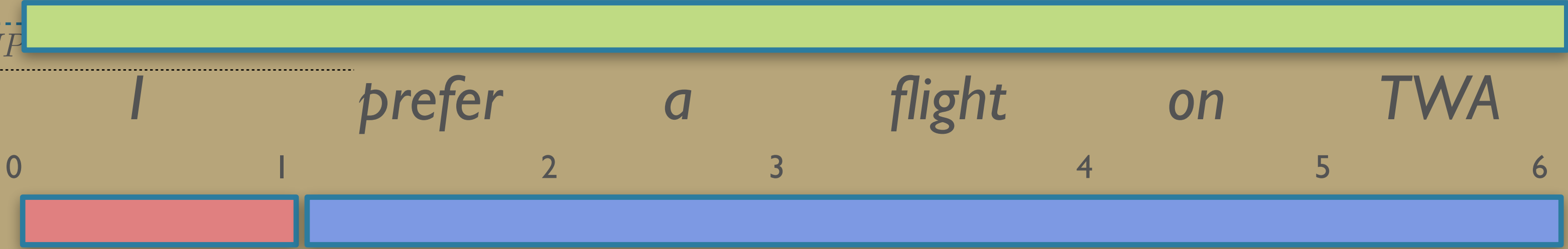
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 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	[0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



\mathcal{L}_1 Grammar

$S \rightarrow NP VP$

$S \rightarrow X1 VP$

$X1 \rightarrow Aux NP$

$S \rightarrow book \mid include \mid prefer$

$S \rightarrow Verb NP$

$S \rightarrow X2 PP$

$S \rightarrow Verb PP$

$S \rightarrow VP PP$

$NP \rightarrow I \mid she \mid me$

$NP \rightarrow TWA \mid Houston$

$NP \rightarrow Det Nominal$

$Nominal \rightarrow book \mid flight \mid meal \mid money$

$Nominal \rightarrow Nominal Noun$

$Nominal \rightarrow Nominal PP$

$VP \rightarrow book \mid include \mid prefer$

$VP \rightarrow Verb NP$

$VP \rightarrow X2 PP$

$X2 \rightarrow Verb NP$

$VP \rightarrow Verb PP$

$VP \rightarrow VP PP$

$PP \rightarrow Preposition NP$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

Lexicon

$Det \rightarrow that \mid this \mid a$

$Noun \rightarrow book \mid flight \mid meal \mid money$

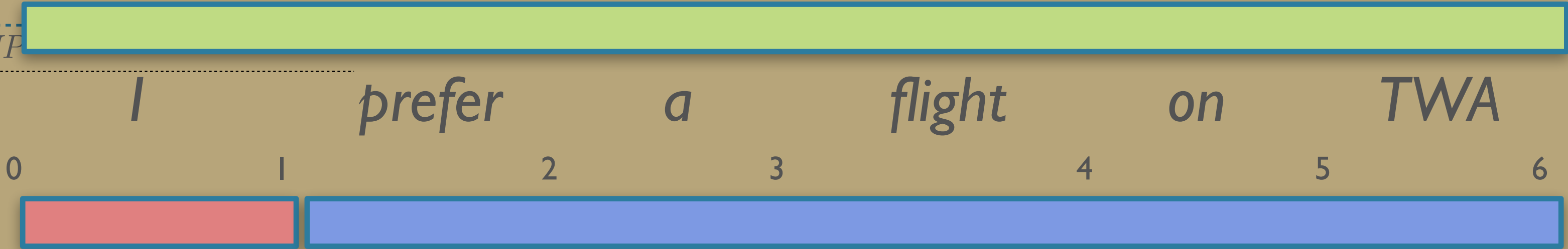
$Pronoun \rightarrow I \mid she \mid me$

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$Aux \rightarrow does$

$Preposition \rightarrow from \mid to \mid on \mid near \mid through$

$Verb \rightarrow book \mid include \mid prefer$



\mathcal{L}_1 Grammar

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 $S \rightarrow X1\ VP$
 $X1 \rightarrow Aux\ NP$
 $S \rightarrow book\ /\ include\ /\ prefer$
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 $NP \rightarrow Det\ Nominal$

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 $Nominal \rightarrow Nominal\ Noun$
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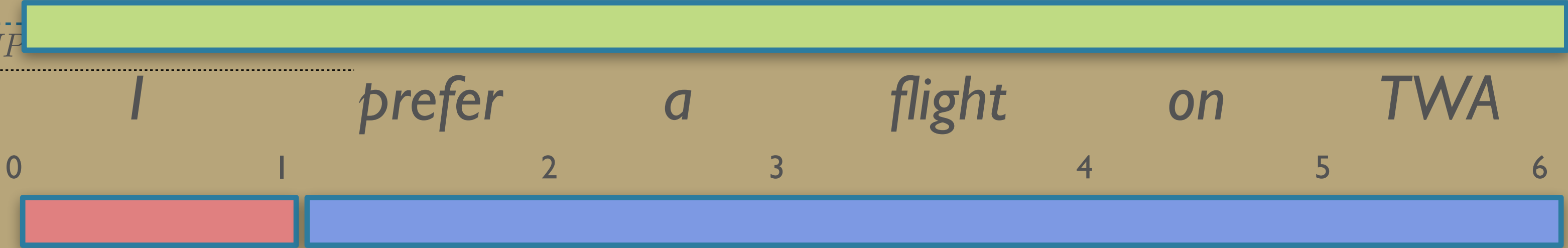
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 $X2 \rightarrow Verb\ NP$
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 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
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 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
 $NP \rightarrow TWA\ /\ Houston$
 $NP \rightarrow Det\ Nominal$

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 $Nominal \rightarrow Nominal\ Noun$
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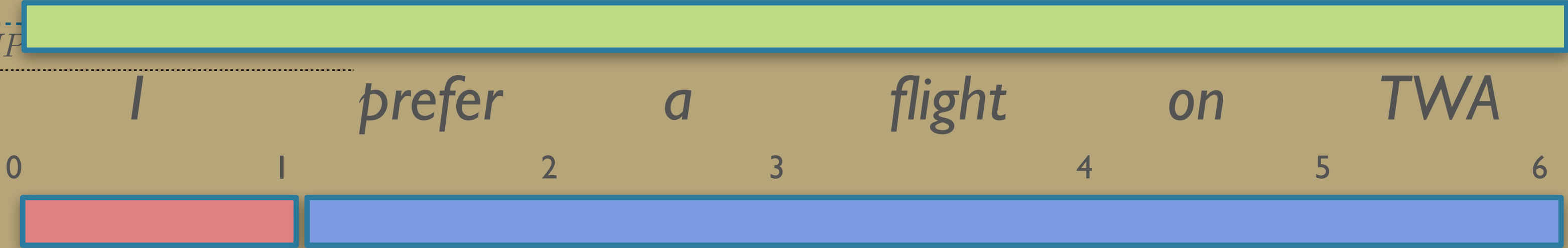
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 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

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Lexicon

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 $Proper-Noun \rightarrow Houston\ /\ TWA$
 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]		S [0,4]		S [0,6]
	Verb, VP, S [1,2]		VP, X2, S [1,4]		VP, X2, S [1,6]
		Det [2,3]	NP [2,4]		NP [2,6]
			Noun, Nom [3,4]		Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



\mathcal{L}_1 Grammar

$S \rightarrow NP\ VP$
 $S \rightarrow X1\ VP$
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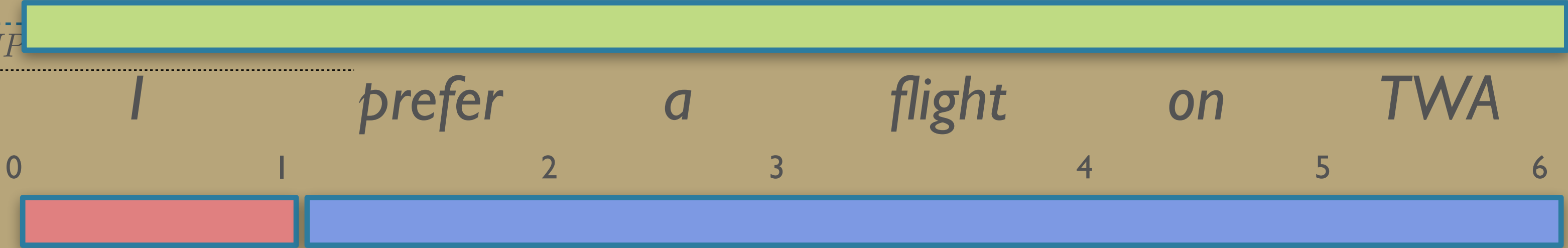
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 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

$PP \rightarrow Preposition\ NP$

Lexicon

$Det \rightarrow that\ /\ this\ /\ a$
 $Noun \rightarrow book\ /\ flight\ /\ meal\ /\ money$
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 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
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NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
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$S \rightarrow NP\ VP$
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 $S \rightarrow VP\ PP$

$NP \rightarrow I\ /\ she\ /\ me$
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 $NP \rightarrow Det\ Nominal$

$Nominal \rightarrow book\ /\ flight\ /\ meal\ /\ money$
 $Nominal \rightarrow Nominal\ Noun$
 $Nominal \rightarrow Nominal\ PP$

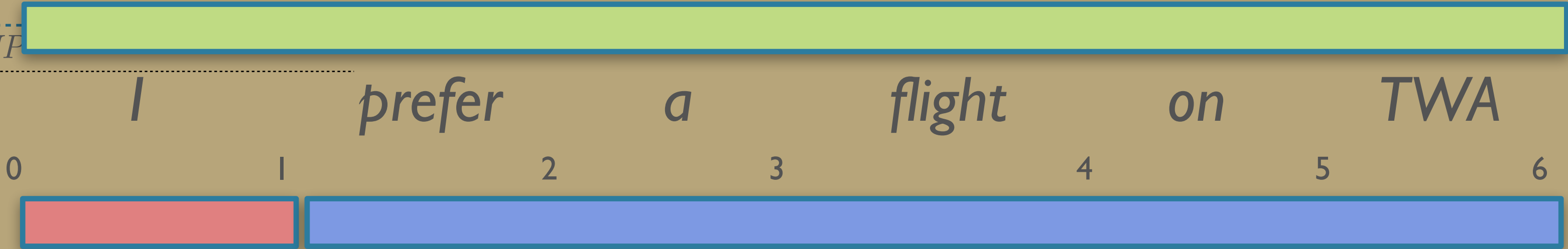
$VP \rightarrow book\ /\ include\ /\ prefer$
 $VP \rightarrow Verb\ NP$
 $VP \rightarrow X2\ PP$
 $X2 \rightarrow Verb\ NP$
 $VP \rightarrow Verb\ PP$
 $VP \rightarrow VP\ PP$

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Lexicon

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 $Aux \rightarrow does$
 $Preposition \rightarrow from\ /\ to\ /\ on\ /\ near\ /\ through$
 $Verb \rightarrow book\ /\ include\ /\ prefer$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	S [0,6]
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP, X2, S [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]



Current CKY Algorithm

- Limitations:
- Only stores non-terminals in cell
 - Not rules or cells corresponding to RHS
- Stores SETS of non-terminals
 - Multiple rules with same LHS collide
- Currently only *acceptance/recognition*

CKY *Parser*

- Pair each nonterminal with back-pointer to cells from which it was derived
- Last step:
 - construct trees from back-pointers in $[0, n]$

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

VP → *VP* *PP*

```
bp_1 = BackPointer()  
bp_1.l_child = [VP, (1,4)]  
bp_1.r_child = [PP, (4,6)]
```

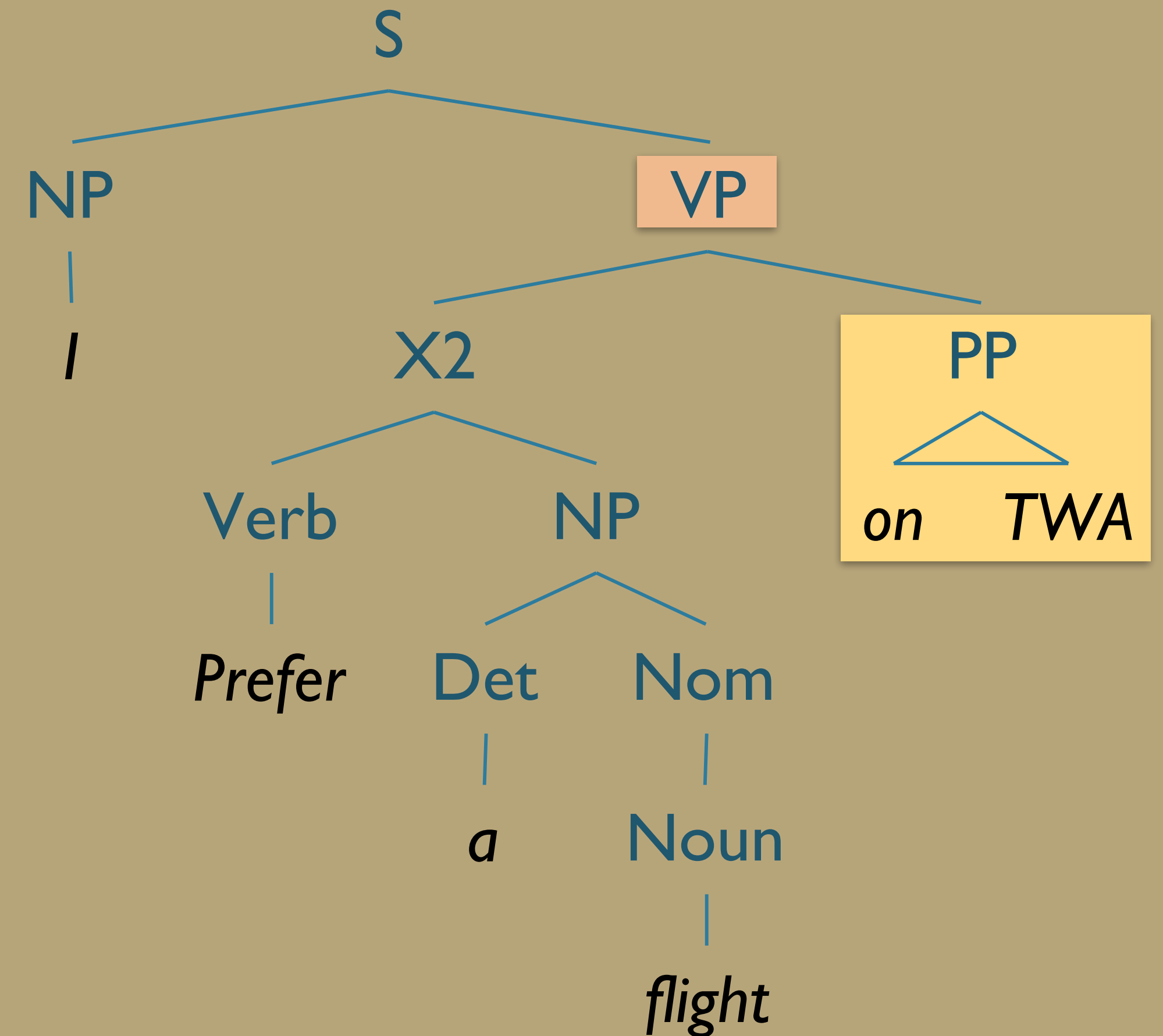
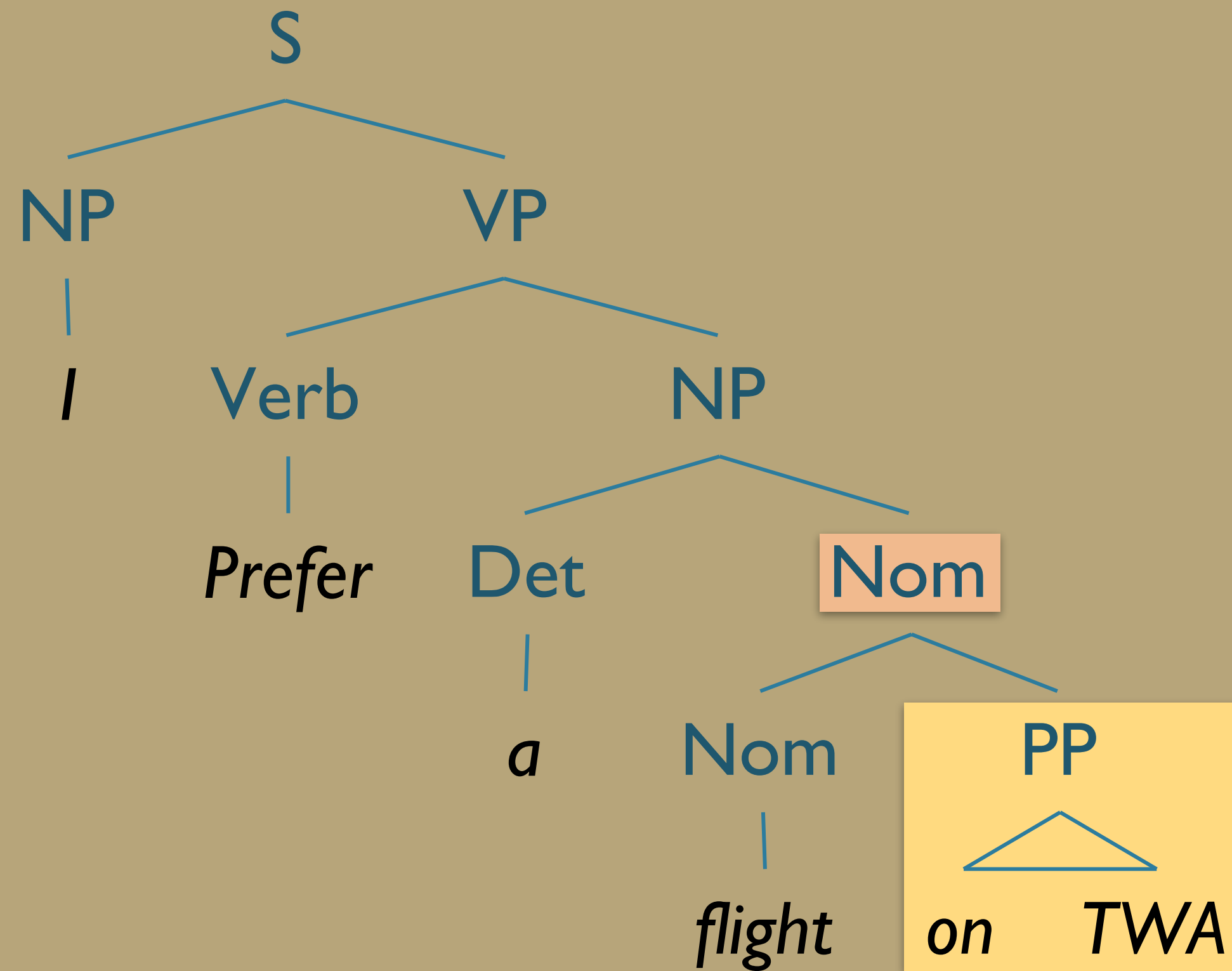
NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2, S [1,4]	[1,5]	VP [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

VP → *X2* *PP*

```
bp_2 = BackPointer()
bp_2.l_child = [X2, (1,4)]
bp_2.r_child = [PP, (4,6)]
```

NP, Pronoun [0,1]	S [0,2]	[0,3]	S [0,4]	[0,5]	
	Verb, VP, S [1,2]	[1,3]	VP, X2 , S [1,4]	[1,5]	VP [1,6]
		Det [2,3]	NP [2,4]	[2,5]	NP [2,6]
			Noun, Nom [3,4]	[3,5]	Nom [3,6]
				Prep [4,5]	PP [4,6]
					NNP, NP [5,6]

Resulting Parses



CKY Discussion

- Running time:
 - $O(n^3)$ where n is the length of the input string
 - Inner loop grows as square of # of non-terminals
- Expressiveness:
 - As implemented, requires CNF
 - Weak equivalence to original grammar
 - Doesn't capture full original structure
 - Back-conversion?
 - Can do binarization, terminal conversion
 - Unit productions requires change in CKY

Parsing: PCFGs

Ling 571

Deep Processing Techniques for NLP

October 7, 2019

Probabilistic Context-free Grammars: Roadmap

- Motivation: Ambiguity
- Approach:
 - Definition
 - Disambiguation
 - Parsing
 - Evaluation
 - Enhancements

Motivation

- What about ambiguity?
- Current algorithm can *represent* it
- ...can't resolve it.

Probabilistic Parsing

- Provides strategy for solving disambiguation problem
 - Compute the probability of all analyses
 - Select the most probable
- Employed in language modeling for speech recognition
 - N-gram grammars predict words, constrain search
 - Also, constrain generation, translation

PCFGs: Formal Definition

N	a set of non-terminal symbols (or variables)
Σ	a set of terminal symbols (disjoint from N)
R	a set of rules of productions, each of the form $A \rightarrow \beta[p]$, where A is a non-terminal where A is a non-terminal, β is a string of symbols from the infinite set of strings $(\Sigma \cup N)^*$ and p is a number between 0 and 1 expressing $P(\beta A)$
S	a designated start symbol

PCFGs

- Augment each production with probability that LHS will be expanded as RHS
 - $P(A \rightarrow \beta)$
 - $P(A \rightarrow \beta | A)$
 - $P(\beta | A)$
 - $P(RHS \mid LHS)$
- NB: the first is often used; but the latter are what's really meant.

PCFGs

- Sum over all possible expansions is 1

$$\sum_{\beta} P(A \rightarrow \beta) = 1$$

- A PCFG is ***consistent*** if sum of probabilities of all sentences in language is 1
- Recursive rules often yield inconsistent grammars ([*Booth & Thompson, 1973*](#))

Example PCFG: Augmented \mathcal{L}_1

Grammar		Lexicon
$S \rightarrow NP VP$	[.80]	$Det \rightarrow that [.10] \mid a [.30] \mid the [.60]$
$S \rightarrow Aux NP VP$	[.15]	$Noun \rightarrow book [.10] \mid flight [.30] \mid meal [.15] \mid money [0.5]$
$S \rightarrow VP$	[.05]	$\mid flights [0.40] \mid dinner [.10]$
$NP \rightarrow Pronoun$	[.35]	$Verb \rightarrow book [.30] \mid include [.30] \mid prefer [.40]$
$NP \rightarrow Proper-Noun$	[.30]	$Pronoun \rightarrow I [.40] \mid she [.05] \mid me [.15] \mid you [.40]$
$NP \rightarrow Det Nominal$	[.20]	$Proper-Noun \rightarrow Houston [.60] \mid NWA [.40]$
$NP \rightarrow Nominal$	[.15]	$Aux \rightarrow does [.60] \mid can [.40]$
$Nominal \rightarrow Noun$	[.75]	$Preposition \rightarrow from [.30] \mid to [.30] \mid on [.20] \mid near [.15]$
$Nominal \rightarrow Nominal Noun$	[.20]	$\mid through [.05]$
$Nominal \rightarrow Nominal PP$	[.05]	
$VP \rightarrow Verb$	[.35]	
$VP \rightarrow Verb NP$	[.20]	
$VP \rightarrow Verb NP PP$	[.10]	
$VP \rightarrow Verb PP$	[.15]	
$VP \rightarrow Verb NP NP$	[.05]	
$VP \rightarrow VP PP$	[.15]	
$PP \rightarrow Preposition NP$	[1.0]	

Example PCFG: Augmented \mathcal{L}_1

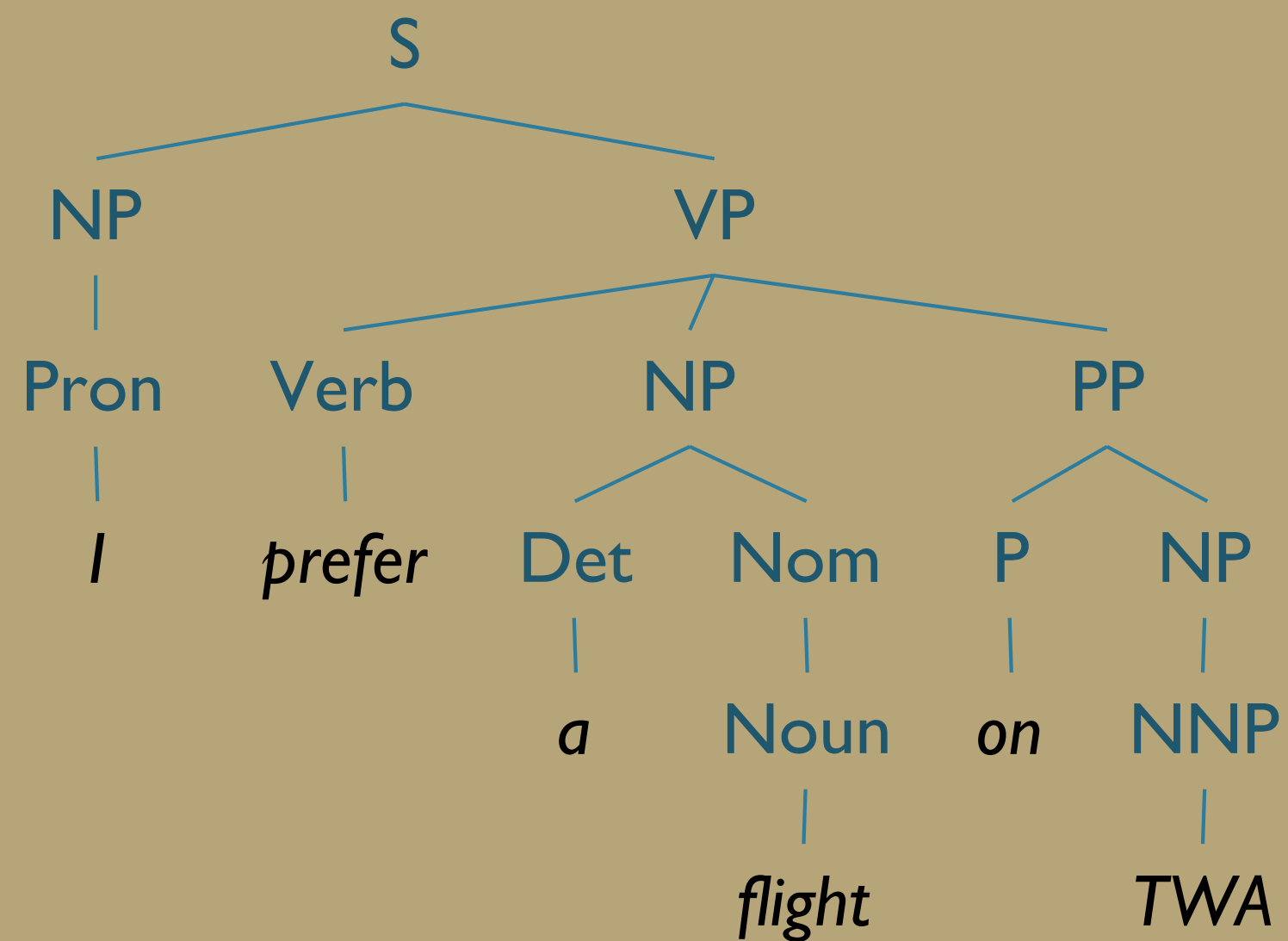
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$S \rightarrow VP$	[.05]	$\mid flights [0.40] \mid dinner [.10]$
$NP \rightarrow Pronoun$	[.35]	$Verb \rightarrow book [.30] \mid include [.30] \mid prefer [.40]$
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$Nominal \rightarrow Nominal PP$	[.05]	
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$VP \rightarrow Verb NP$	[.20]	
$VP \rightarrow Verb NP PP$	[.10]	
$VP \rightarrow Verb PP$	[.15]	
$VP \rightarrow Verb NP NP$	[.05]	
$VP \rightarrow VP PP$	[.15]	
$PP \rightarrow Preposition NP$	[1.0]	

Disambiguation

- A PCFG assigns probability to each parse tree T for input S
- Probability of T : product of all rules used to derive T

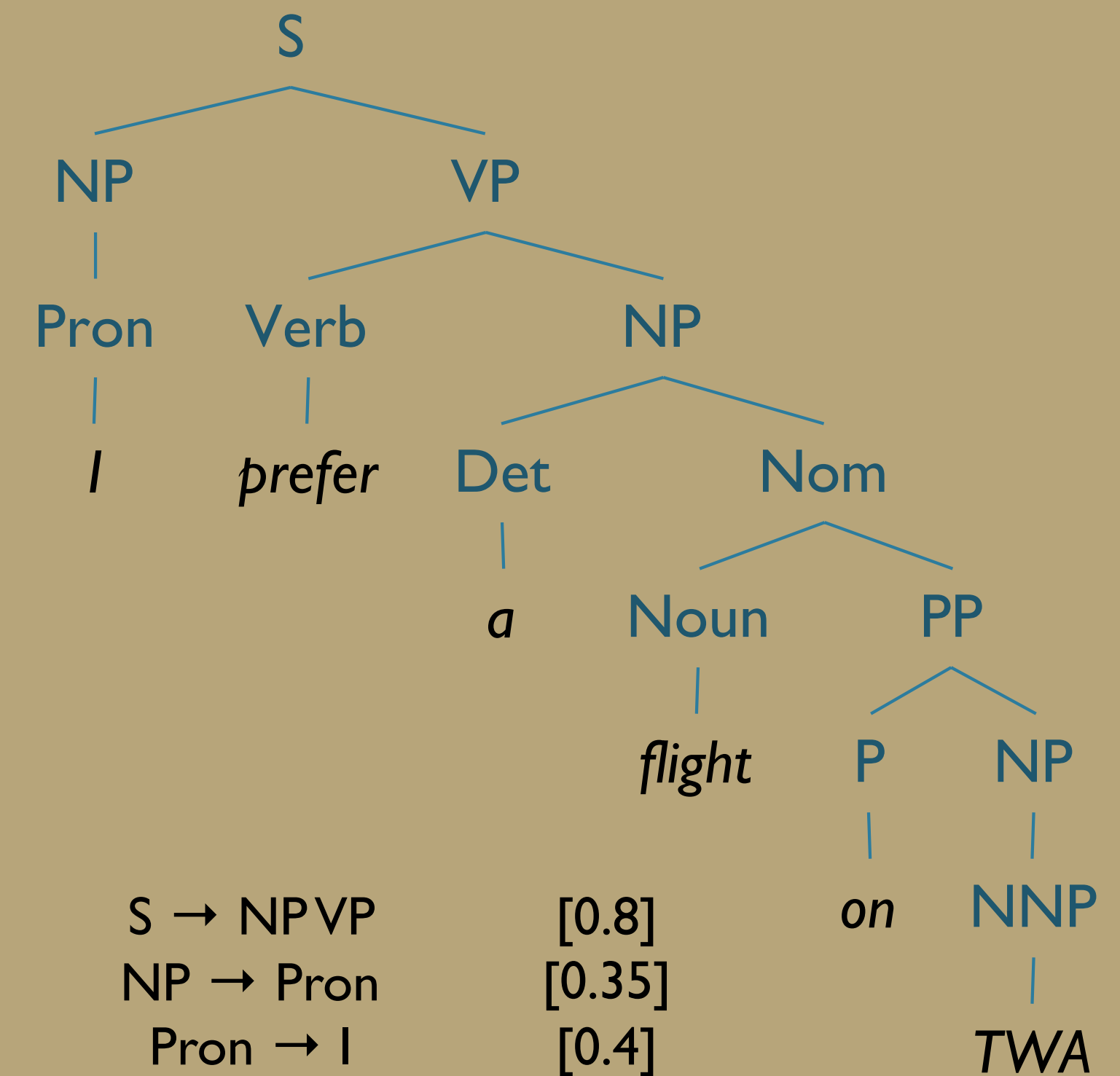
$$P(T, S) = \prod_{i=1}^n P(RHS_i | LHS_i)$$

$$P(T, S) = P(T)P(S | T) = P(T)$$



$S \rightarrow NP VP$	[0.8]
$NP \rightarrow Pron$	[0.35]
$Pron \rightarrow I$	[0.4]
$VP \rightarrow V NP PP$	[0.1]
$V \rightarrow prefer$	[0.4]
$NP \rightarrow Det Nom$	[0.2]
$Det \rightarrow a$	[0.3]
$Nom \rightarrow N$	[0.75]
$N \rightarrow flight$	[0.3]
$PP \rightarrow P NP$	[1.0]
$P \rightarrow on$	[0.2]
$NP \rightarrow NNP$	[0.3]
$NNP \rightarrow NWA$	[0.4]

$\sim 1.452 \times 10^{-6}$



$S \rightarrow NP VP$	[0.8]
$NP \rightarrow Pron$	[0.35]
$Pron \rightarrow I$	[0.4]
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$V \rightarrow prefer$	[0.4]
$NP \rightarrow Det Nom$	[0.2]
$Det \rightarrow a$	[0.3]
$Nom \rightarrow Nom PP$	[0.05]
$Nom \rightarrow N$	[0.75]
$N \rightarrow flight$	[0.3]
$PP \rightarrow P NP$	[1.0]
$P \rightarrow on$	[0.2]
$NP \rightarrow NNP$	[0.3]
$NNP \rightarrow NWA$	[0.4]

$\sim 1.452 \times 10^{-7}$

Parsing Problem for PCFGs

- Select T such that (*s.t.*)

$$\hat{T}(S) = \operatorname{argmax}_{T \text{ s.t. } S=\text{yield}(T)} P(T)$$

- String of words S is *yield* of parse tree
- Select the tree \hat{T} that maximizes the probability of the parse

Unit Testing

Unit Testing

- Strategy of testing individual pieces of code in isolation
- Helps ensure:
 - Basic functionality in isolation
 - Complex functionality when individual components are combined
- In many industry jobs, you can't commit code without unit tests!

Unit Testing in Python

- Many good tutorials on the web [[here's one](#)]
- In a nutshell:

```
from unittest import TestCase
```

```
class longTests(TestCase):
```

```
    def test_three(self):
```

```
        length_3_rule = parse_productions('A -> B C D')
```

```
        target_rules = parse_productions(''''A -> B _X0_  
                                          _X0_ -> C D''')
```

```
        self.assertEqual(set(target_rules),  
                          set(fix_long_rules(length_3_rule)))
```

Unit Testing in Python

- “nose” is a test running framework that will detect all `test_*` method functions

```
python3 -m nose hw2.py
```

```
AssertionError: Items in the first set but not the second:
```

```
    A -> B _X0_  
    _X0_ -> C D
```

```
Items in the second set but not the first:
```

```
    A -> B C
```

```
-----  
Ran 12 tests in 0.040s
```

```
FAILED (failures=1)
```

Unit Testing

- Good practice:
 - Save input that crashes your program for a unit test