Deflationism, Gaps, and Expressive Power

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1 The Incompatibilist Argument

1.1 Deflationism

(E1) S is equivalent to 'S' *is true*

(E2) *not*-S is equivalent to 'S' *is false*

A *core deflationist* accepts the following Intersubstitutivity Principle:

(IP) If B' is a sentence resulting from B by substituting any number of occurrences of sub-sentences S for 'S' *is true* (in non-opaque contexts), then B' and B are either both acceptable, rejectable, or ill-formed.

1.2 Gappiness

A proponent of *gappiness*:

(G) There are meaningful, declarative sentences of English that are neither true nor false.

Reasons for thinking that (G) obtains:

- Frege: singular terms which fail to refer
- Ayer: moral/aesthetic emotivism
- Soames' smidget

1.3 The Argument

Compatibilists: deflationism and gappiness are consistent. Incompatibilists: no they aren't. The incompatibilist's shockingly simple argument:

(1) $\neg T(`S') \wedge \neg F(`S')$ [instance of (G)]

(2) $\neg T(`S') \wedge \neg T(`\neg S')$ [falsity as truth of negation]

(3) $\neg S \wedge \neg\neg S$ [two applications of (IP)]

2 Lack of Expressive Power

Most common response: somehow reject the inference from (2) to (3). My response: do not accept (1), while still accepting (G).

I will use the so-called strong-Kleene logic K_3, which has two properties:

Fact 1. *Let φ be a formula built using K_3 conjunction, disjunction, negation, and universal quantification. If all of the atomic subformulas of φ get value N, then so too does φ.*

Corollary 1. *In strong Kleene three-valued logic (both propositional and first-order), there are no theorems, i.e. no valid formulas.*

My argument contains two steps:

(a) A deflationist should primarily be committed to the bi-inferential version of the T-scheme instead of *any* biconditional (material or not) version.

(b) Kripke's construction provides a model acceptable to a deflationist which license (a) and blocks the incompatibilist argument at (1).

For (a):

- The truth predicate is a logical constant.
- The meanings of logical constants are explicated independently of one another.
- In this case, for instance, by:

(T+) $S \vdash T(\ulcorner S \urcorner)$
(T-) $T(\ulcorner S \urcorner) \vdash S$

Truth so explicated can still play the expressive role deflationists claim it does: consider Literalese. Objection: you need a conditional to express generalizations about truth. Two replies:

- A Literalese sentence may have such a logical form even if the spoken language has no conditional.
- The generalizations are *restricted* quantification, not unrestricted quantification over a conditional.

For (b): consider Kripke's definition of *groundedness*: a sentence is grounded iff it receives a truth value (i.e. its code ends up in the extension or antiextension of the truth predicate) at the minimal fixed point. Such sentences have a 'path of dependence' down to T-free sentences. This cashes out the deflationist idea that the truth predicate is just a device for semantic ascent/descent: it cannot be used to say anything about the world which could not be said without it.

A logic that (i) validates the T-inferences, (ii) interprets truth partially, and (iii) interprets the logical connectives (at least negation and conjunction) according to the strong-Kleene rules can be used to block the incompatibilist argument at (1). I have argued that a core deflationist can avail herself of the logic of a Kripkean fixed point to do exactly this.

2.1 Gaps or Gluts?

Field 2008: we can't identify 'is a gap' with 'has semantic value N' on the proposed interpretation because

$$\neg(S \vee \neg S) \vDash\Dashv S \wedge \neg S$$

where the left-hand side is a statement of gappiness and the right-hand side is a statement of gluttiness. Two responses:

- Orthogonal to the present issue
- Gluttiness can come apart from the expression above

2.2 The Battle Over Strong Negation

The most pertinent objection is that our interpretation does not allow one to truly assert any instances of (G). Just use strong negation to do so:

p	$\neg_s p$
T	F
N	T
F	T

Table 1: Strong Negation

Once equipped with strong negation, we can reformulate the incompatibilist argument as:

(1) $\neg_s T(\ulcorner S \urcorner) \wedge \neg_s F(\ulcorner S \urcorner)$

(2) $\neg_s T(\ulcorner S \urcorner) \wedge \neg_s T(\ulcorner \neg S \urcorner)$

(3) $\neg_s S \wedge \neg_s \neg S$

But this conclusion is not inconsistent.

Problem 1: *ungroundedness* is now definable, inviting revenge.

$$U(p) =_{df} \neg_s p \wedge \neg_s \neg p$$

It's easy to show that $U(p)$ has the following truth table: We can then define a

p	$U(p)$
T	F
N	T
F	F

Table 2: Truth Table for $U(p)$

determinate truth predicate as

$$DT(p) =_{df} T(\ulcorner p \urcorner) \wedge \neg U(p)$$

This, however, allows for the introduction of a strengthened Liar:

(λ^*) $\neg DT(\ulcorner \lambda^* \urcorner)$

But: *must one* reject strong negation? Maudlin thinks so: the fact and corollary above are constraints on any logical system. His arguments essentially generalize what the deflationist says about the truth predicate to all the other logical constants.

A1: Strong-Kleene Connectives

$\neg$	
T	F
N	N
F	T

$\wedge$	T	N	F
T	T	N	F
N	N	N	F
F	F	F	F

$\wedge$	T	N	F
T	T	T	T
N	T	N	N
F	T	N	F

A2: Kripke's Construction

Kripke's construction runs basically as follows: we start with a basic first-order language and extend it with a truth predicate T. This predicate is interpreted *partially*: it has a positive extension, T_1 and an antiextension, T_2 which are mutually exclusive but not necessarily jointly exhaustive. Kripke initially assigns $T_1 = T_2 = \varnothing$. At the first stage, all true sentences of the base language are added to T_1 and all false sentences of the base language to T_2, using Kleene's strong rules for the logical connectives.[1] Writing $\mathcal{L}(T_1, T_2)$ to denote the extended language with the predicate T interpreted as (T_1, T_2), Kripke generalizes the idea of the first stage to define truth predicates at all transfinite stages by:

$$\begin{aligned} T_0 &= (\varnothing, \varnothing) \\ T_{\alpha+1} &= \left(T'_{\alpha,1}, T'_{\alpha,2}\right) \\ T_\lambda &= \left(\bigcup_{\alpha<\lambda} T_{\alpha,1}, \bigcup_{\alpha<\lambda} T_{\alpha,2}\right) \end{aligned}$$

where $T'_{\alpha,1}$ consists of all true sentences of $\mathcal{L}(T_{\alpha,1}, T_{\alpha,2})$ and $T'_{\alpha,2}$ consists of all false sentences of $\mathcal{L}(T_{\alpha,1}, T_{\alpha,2})$.

Because the operator extending T this way is monotone, Kripke is able to show using standard cardinality arguments that a fixed point must be reached. In other words, there is a (countable, in fact) ordinal α such that $T_\alpha = T_{\alpha+1}$. The fixed point reached with this interpretation of T_0 is the unique *minimal fixed point*. The truth predicate at this fixed point has two notable properties for present purposes: (i) it validates (T+) and (T-) unrestrictedly, and (ii) it defines a *partial* truth predicate, in the sense that $T_1 \cup T_2$ does not contain all the sentences of the language.

[1] I leave implicit qualifiers to codes of sentences here and in what follows.